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# THE MOVING-COIL VOLTAGE REGULATOR

By E. T. NORRIS, Member.

[*Paper first received* 31*st May and in final form* 20*th October*, 1937; *read before* THE INSTITUTION 3*rd February*, *before the* SCOTTISH CENTRE 8*th February*, *before the* NORTH MIDLAND CENTRE, 1*st March*, *before the* TEES-SIDE SUB-CENTRE 2*nd March*, *and before the* NORTH IRELAND SUB-CENTRE 15*th March*, 1938.]

## SUMMARY

The general problem of voltage regulation under load is reviewed and the principal technical and operating characteristics of existing voltage-regulators are summarized. The problem has not been found easy of solution, as the more simple devices are restricted in scope and have other technical limitations, whilst apparatus of more general application is relatively expensive and complicated.

A recent development is described which in principle involves an isolated and short-circuited coil moving up and down the leg of a laminated iron core over fixed coils. Smooth voltage variation over any desired range is obtained without switches, contacts, flexible leads, or windings in slots. After a discussion of the principal technical and constructional characteristics of the new regulator, the special features and applications of regulators are considered under the separate headings of voltage control (*a*) in transmission and distribution circuits, and (*b*) for industrial and laboratory purposes.

## LIST OF SYMBOLS

$v_1$ = primary or input voltage of regulator.
$I_1$ = primary or input current of regulator.
$v_2$ = secondary or output voltage of regulator.
$I_2$ = secondary or output current of regulator.
$v_a$, $v_b$, etc. = voltage of coils *a*, *b*, etc.
$I_a$, $I_b$, etc. = current in coils *a*, *b*, etc.

In the foregoing symbols small letters represent no-load values and capital letters represent values with the regulator on load.

$N$ = ratio of turns in coils *n* and *a*.
$K$ = ratio of turns in coils *k* and *b*.
$N_1$ = ratio of turns in coils *b* and *a*.
$a$ = equivalent individual reactance of coil *a*.
$b$ = equivalent individual reactance of coil *b*.
*ab*, *ac*, etc. = leakage reactance between coils *a–b*, *a–c*, etc.
$Z$ = internal load impedance of regulator.
$R$ = internal load resistance of regulator.
$X$ = internal load reactance of regulator.

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## (1) INTRODUCTION

Controlled variation and adjustment of the voltage in an electrical circuit is required for a large number of purposes, and during the last 40 years considerable attention has been given to the invention and development of suitable voltage-control apparatus. The general problem is to devise means which will give a variation (preferably smooth and uniform) of voltage without interrupting or disturbing the load current. This problem is not easy of solution; it will be found that simple devices are limited in scope and have other technical disadvantages, whilst apparatus of a more general application is relatively expensive and complicated.

Voltage variation may be required directly as a means of obtaining a particular value of voltage (e.g. for insulation-testing purposes, electrostatic field control, or valve grid- and anode-potential control). Voltage variation is required indirectly for a large number of industrial and laboratory purposes in the control of power consumption, temperature values, and many electrochemical and electrolytic processes. Voltage control is also required in transmission and distribution systems to assist in the maintenance of constant voltage under all load conditions at consumers' terminals. This last application of voltage regulators covers so wide a field that it is convenient to divide the consideration of voltage-regulator applications and the corresponding regulator characteristics into two branches, namely (*a*) voltage control in transmission and distribution circuits; (*b*) industrial and laboratory applications.

This division will be made later in the paper, after consideration has been given to general principles and technical characteristics.

## (2) TYPES OF VOLTAGE REGULATOR

The term "voltage regulator" is commonly applied to two quite distinct classes of apparatus, namely (*a*) Apparatus for controlling and regulating the generator voltage in constant-potential systems. (Such apparatus usually functions by operating on the field excitation of the alternators.) (*b*) Apparatus for varying at will the voltage of a particular circuit or load, or for automatically maintaining it at or near a prescribed value, in either case without interrupting the flow of load current.

The present paper deals entirely with the latter class of apparatus.

The principal methods of achieving voltage variation in a circuit may be grouped under two headings: (i) Variation of impedance voltage-drop. (ii) Variation of induced voltage.

The simplest example of the first group is, at first sight, the variable resistor or reactor connected in series with the load. The voltage variation obtained is determined not only by the particular value of resistance or reactance but equally by the load-current value. Such devices cannot be considered true voltage-regulators, as their operation is not independently determinable but involves an external characteristic (the load current or impedance). Other regulators in this group include shunt and series capacitors, and circuits involving inductances with saturated iron cores. Most of these arrangements are subject to the same restrictions, in that their operation is mainly dependent upon certain external circuit conditions. The principal exceptions are potentiometer-type resistors or reactors, and these may be considered true voltage regulators, although their output voltage is partly determined by the load current. However, although simple in construction, this type of regulator is limited in size and rating virtually to small laboratory applications.

The second group is based on the variation of an electromagnetically induced voltage in a circuit, and includes all kinds of variable-ratio transformers. Since any induced voltage is proportional to the product of interlinking flux and interlinked turns, the variation may be obtained by changing either or both of these components. The common example of variable-turn regulation is on-load tap-changing gear in conjunction with tapped transformer or auto-transformer windings. This application has developed rapidly during the last 10 years in two directions: First, voltage regulation in transmission and distribution circuits of all ratings where variations up to 20 % or so are involved (e.g. most of the transformers on the C.E.B. grid system are provided with on-load tap-changing gear for 20 % range of voltage control; similar gear is becoming common on smaller units, even down to distribution transformers). Secondly, voltage control up to 100 % or more in relatively low-power light-current circuits, permitting direct connection to the bare turns of the regulating winding.

The second alternative of flux variation may be subdivided into variation of flux magnitude and variation of flux phase-displacement. The single-phase and the 3-phase induction regulators respectively are the best-known examples of these methods.

**Table 1**

PRINCIPAL TYPES OF VOLTAGE REGULATOR

| Principle of operation | Typical examples |
|---|---|
| 1. *Variable impedance* | |
| (*a*) Variable resistance | Potentiometer-type rheostats |
| (*b*) Variable reactance | Magneto voltage regulators<br>Potentiometer-type reactors |
| 2. *Variable induced voltage* | |
| (*a*) Variable turns .. | On-load tap-changing gear<br>Switch-type voltage regulators |
| (*b*) Variable flux magnitude | Single-phase induction voltage-regulators |
| (*c*) Variable flux phase-displacement | Three-phase induction voltage-regulators |

Table 1 summarizes the voltage-regulator classification just described, and gives examples typical of each class.

The best choice of voltage regulator from the existing types shown in Table 1 will, of course, depend upon operating conditions. The desirable characteristics are summarized in Table 2, with an indication of the most suitable type of regulator in each case. This Table also shows the characteristics and performance of the ideal voltage-regulator. It is incomplete in that price is not considered. Even a broad comparison of cost is difficult, as so much is dependent on local commercial and manufacturing conditions and upon standardization and production possibilities. Individual judgment of relative costs may be made by comparison of constructional features and of the basic material requirements of each type of regulator.

Whilst all of the characteristics given in the Table would be possessed by the ideal regulator, only a few may be necessary for any given application or purpose. For example, losses are relatively unimportant in small low-power regulators for laboratory work, and in many cases phase displacement is immaterial. The absence of moving parts is given as a desirable characteristic, but this is in the nature of an ideal, as some motion seems to be practically necessary for the control of any independent voltage-regulator as defined earlier in this Section. Instantaneous operation is also an ideal. High operating speed is not, necessarily, the same kind of thing even in degree. The matter will be discussed in more detail in Section (6)(*e*). The object of Table 2 is to facilitate assessment of the merits and demerits of any given voltage-regulator, including that described later in the paper. A critical comparison on this basis is given in Appendix 4.

## (3) THE MOVING-COIL VOLTAGE REGULATOR

### (a) Principle of Operation

Although most of the desirable characteristics mentioned in Table 2 are fulfilled by one or other existing type of regulator, any single type falls far short of compliance with the whole Table. There is therefore scope for the development of improved and more comprehensive apparatus. Moreover, its commercial justification will be ensured by the general demand for voltage regulation and control. This is especially the case for regulators controlling transmission and distribution circuits. The rapid growth during recent years of electricity consumption, and particularly the domestic load, has accentuated the difficulty of maintaining good voltage regulation. The demand for automatic regulators has increased greatly, not only as an easy means of curing bad cases of voltage-drop but on purely economic grounds to reduce capital expenditure on cables and substations in suburban and rural networks.

**Table 2**

Characteristics of Voltage Regulators

| Characteristic | Best type of regulator, from Table 1 |
|---|---|
| *Simplicity and reliability* | |
| (i) No moving parts . . . . | — |
| (ii) Isolated moving parts . . . . | 1(*b*) |
| (iii) No current-breaking switches . . | 2(*b*), 2(*c*), 1(*b*) |
| (iv) Small operating power . . | 2(*a*) |
| *Range of application* | |
| (i) Wide range of voltage variation | 2(*b*), 2(*c*) |
| (ii) Suitable for high voltages . . | 2(*a*) |
| (iii) Suitable for heavy currents . . | 2(*b*), 2(*c*) |
| (iv) Suitable for large kVA ratings . . | 2(*a*) |
| *Technical features* | |
| (i) Low magnetizing current . . | 2(*a*) |
| (ii) Small load and no-load losses . . | 2(*a*) |
| (iii) Little noise or vibration . . | 2(*a*) |
| (iv) Low electromagnetic forces . . | 2(*a*) |
| (v) Smooth voltage variation . . | 2(*b*), 2(*c*), 1(*b*) |
| (vi) Minimum wave-form distortion | 2(*a*) |
| (vii) Small voltage phase-displacement . . . . . . . . | 2(*a*), 2(*b*) |
| (viii) Instantaneous operation . . | — |
| (ix) No windings in slots . . . . | 1(*a*), 1(*b*), 2(*a*) |
| (x) Ease of control . . . . . . | 2(*b*), 2(*c*) |

The principle of operation* of the new regulator described in this paper involves an isolated and short-circuited coil moving up and down the leg of a laminated iron core over fixed coils: hence the title "moving-coil voltage regulator." Smooth voltage variation over any desired range is obtained without switches, contacts, flexible leads, or windings in slots. The principle is illustrated diagrammatically in Fig. 1. The fixed coils *a* and *b* are connected in series across the supply mains and form the primary winding of the regulator. The moving coil *c*, which is short-circuited on itself and isolated electrically, is moved up and down the core leg between or over the fixed coils. The effective impedance of coil *a* depends upon the position of the moving coil *c* and will vary smoothly from a small value when *c* is adjacent to *a* to a large value when *c* is at the remote end of the core leg. The impedance of coil *b* changes conversely in a similar manner. The voltage across the individual coils will depend upon their relative impedances and will vary from a small proportion of the supply voltage to a large proportion, depending upon the position of the moving coil. Fig. 1 shows, for example, that with an impedance variation of 40 : 1 the voltage across coil *a* can be varied smoothly and uniformly from 2·5 to 97·5 volts by varying the position of the moving coil. In actual practice a range of 0 to 100 % is obtained, as will be explained in the next Section.

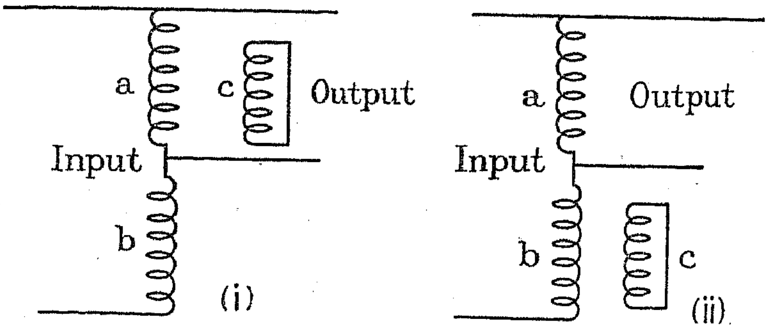

**Fig. 1.**—Fundamental circuit of moving-coil regulator.
Coils *a* and *c* are wound in the opposite direction from coils *b*.

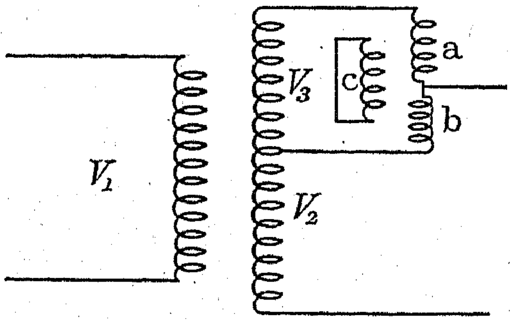

**Fig. 2.**—Type A connection.
All coils except *b* are wound in the same direction.

### (b) Alternative Circuits

The simple arrangement just described gives a fixed range of variations and definite voltages in relation to the primary voltage. Almost any desired values of voltage and variation can be obtained by means of auxiliary coils connected in series with the load and mounted on the regulator in close inductive relationship with one or both of the fixed coils *a* and *b*. These circuits are divided into five classes, each having its own particular field of application.

---

* British Patent Specification No. 341903—1929.

(A) The regulator having only the three essential windings *a*, *b*, and *c* is used in conjunction with a tapped transformer* as shown in Fig. 2. A voltage variation of $V_3$ is obtained, and this can be made any desired value by suitably choosing the transformer tappings. This connection is chiefly used where the regulator is installed in conjunction with a step-up or step-down transformer, the combination forming a self-contained unit.

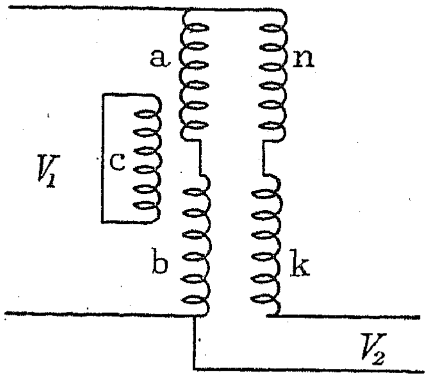

**Fig. 3.**—Type B connection.
All coils except *a* are wound in the same direction.

(B) The voltage variation obtained from the fundamental circuit (Fig. 1) is stepped down by means of auxiliary coils† *n* and *k* connected in the auto-transformer manner as shown in Fig. 3. The voltage variation is from $(1 + N)V_1$ to $(1 - K)V_1$, where $N$ and $K$ are respectively the turn ratios of coils $n/a$ and $k/b$. This connection is used where relatively small variations in the primary voltage are required, and is therefore employed in regulators for automatic voltage control in transmission and distribution circuits. Since $N$ and $K$ are independently determinable, any desired values of buck and boost are obtainable. If, for example, boost only were required, coil *k* would be omitted.

(C) Wider ranges of voltage variation (e.g. from zero to one or more times the input voltage) are given by an auxiliary coil *k* and an auto-transformer-connected coil *n*, as shown in Fig. 4. The voltage range is from $KV_1$ to $(1 + N)V_1$, the values of $K$ and $N$ being, as before, independently determinable. This connection‡ is used in most laboratory and industrial applications involving large voltage variations.

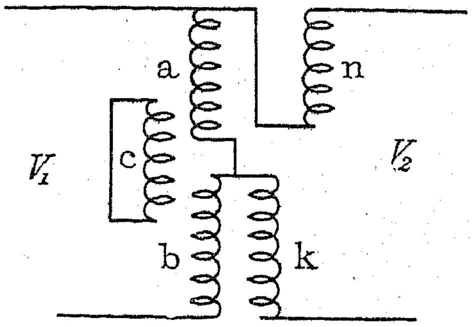

**Fig. 4.**—Type C connection.
Coils *b* and *k* are wound in the opposite direction from coils *a*, *c*, and *m*.

(D) Where a large transformation ratio is required and the auto-transformer connection would be uneconomical the auxiliary coils *n* and *k* are isolated from the regulator windings,* giving the effect of a double-wound transformer as in Fig. 5. The secondary voltage is variable from $NV_1$ to $KV_1$. Thus, in addition to giving the required smooth voltage variation, the regulator also performs the functions of a step-up or step-down transformer (or, in the case of Type C regulators, of an auto-transformer). This connection is employed where high-voltage or heavy-current outputs are required, and in all cases where electrical isolation of the secondary circuit is necessary.

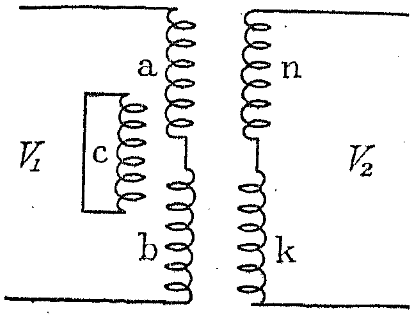

**Fig. 5.**—Type D connection.
Coils *b* and *k* are wound in the opposite direction from coils *a*, *c*, and *n*.

(E) The regulator described under (A) is used in a smaller form† instead of the choke coil or preventive auto-transformer in on-load tap-changing gear. Smooth voltage variation between tappings is given, and the rupturing duty of the tapping or transfer switches is greatly reduced as they do not have to break any load current. The life of the contacts and the time-interval for periodical inspection are correspondingly increased. This is of particular application in cases where the load current is relatively high and tap-changing is frequent (as in automatic operation).

(F) An alternative to Type D connection for heavy current outputs where isolation of the secondary circuit is not required, as in welding or electric-furnace work, is shown in Fig. 6. The auto-transformer connection of Type C or Type B is employed with a separate boosting or bucking unit.‡ The heavy-current windings are confined to the secondary circuit of this unit.

The various connections just outlined do not affect the essential windings *a*, *b*, and *c*, or the general structure of the regulator and its mechanical operation. The electrical isolation of the moving parts and the elimination of flexible leads and sliding contacts have been achieved by means of the short-circuited moving

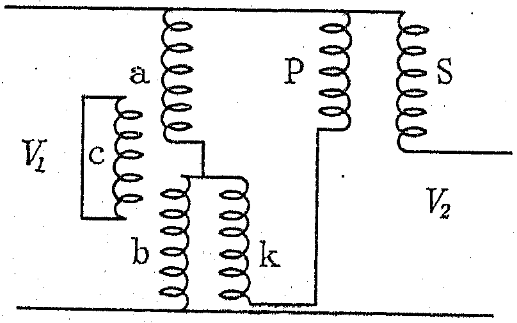

**Fig. 6.**—Type F connection.
Coils *a*, *c*, and *s* are wound in the opposite direction from *b*, *p*, and *k*.

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* British Patent Specification No. 368142—1931.
† British Patent Specifications Nos. 341903—1929 and 368592—1931.
‡ British Patent Specification No. 372493—1931.

* British Patent Specification No. 341903—1929.
† British Patent Specification No. 340538—1929.
‡ British Patent Specification No. 438791—1934.

coil and by the provision of separate fixed coils where buck and boost are required. In the ordinary induction regulator the same winding serves for both buck and boost through reversal of its induced e.m.f. by rotation of the rotor. In comparison, the additional coils in the moving-coil regulator are an extra cost and increase the relative proportions of copper to iron material. On the other hand, all the windings are single circular coils of the ordinary transformer type and are free from the labour cost and design limitations of windings in slots, e.g. magnetic tooth-locking, pole and phase pitch relations, and slot insulation.* Moreover, almost any desired values of boost and buck independently may be obtained by suitable choice of the auxiliary-coil ratios, permitting, in some cases, the elimination of step-up or step-down transformers.

In considering the principle of operation and its application, it should be borne in mind that the novelty of the new regulator lies not in an electromagnetic arrangement of cores and coils whereby movement of a core or coil (whether linear or angular) varies an induced voltage and thereby permits voltage regulation. Many such regulators have been devised in the past. The problem is to produce an economical and practical device which will be simple in construction and at the same time be reasonably efficient and meet satisfactorily as many as possible of the requirements of Table 2.

## (4) ANALYSIS OF TECHNICAL CHARACTERISTICS

### (a) Voltage Ratio and Range

For any given position of the moving coil the external technical performance of the regulator can be expressed in standard transformer terms, and the same vector diagram may be used. This analogy simplifies consideration of the regulator to engineers familiar with power-transformer performance.

The principal technical characteristics of the regulator are determined by the leakage impedances between the windings $a$, $b$, and $c$. Two of these values, $ac$ and $bc$, are dependent upon the position of the moving coil $c$.

For the simple connection shown in Fig. 1 the ratio of secondary voltage to primary voltage at no load is, from Appendix 1,

$$\frac{v_a}{v_1} = \frac{ac - bc + ab}{2ab} \quad . \quad . \quad . \quad . \quad (9)$$

If, for example, in the minimum-voltage position [Fig. 1($a$)] the impedance $ac$ is negligible in comparison with $ab$, and if $bc = ab$ (both assumptions being approximately correct), the secondary voltage $v_a$ will be zero. Similarly, with the moving coil in the maximum-voltage position [Fig. 1($b$)], if $bc$ is negligible and $ac$ is equal to $ab$ the secondary voltage $v_a$ will be equal to the primary voltage. The total voltage range of the regulator will then be approximately 100 %. In the case of the concentric-winding construction to be described in the next Section a voltage range of 100 % or more is actually obtainable, as the end-position conditions can be so designed that the relation

$$bc - ac \geq ab$$

holds for the minimum-voltage position and

$$ac - bc \geq ab$$

holds for the maximum-voltage position. Formula (9) shows that this will result in the voltage ratio ranging from zero or a negative value to 100 % or more. For intermediate positions of the moving coil the voltage ratio will be dependent upon the relative values of the impedances $ac$ and $bc$. The resulting no-load ratio calculated from (9) for a typical case is given in Fig. 7. Such curves show that the voltage varies practically linearly with the position of the moving coil.

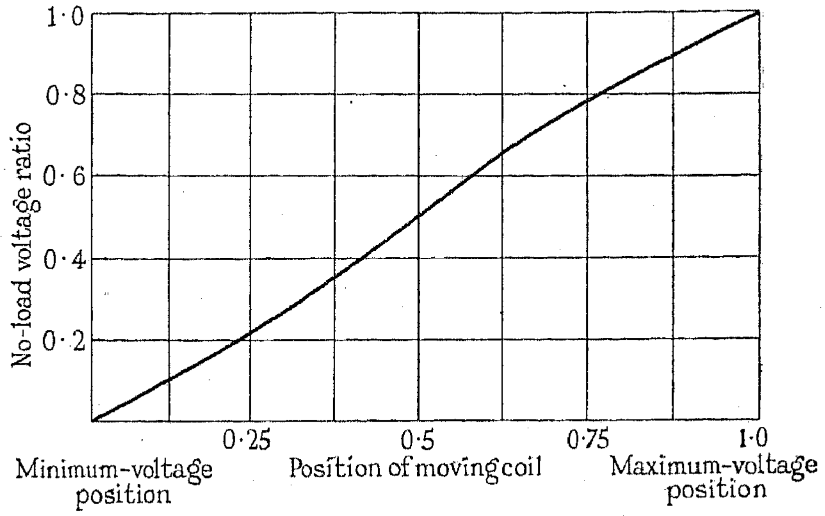

**Fig. 7.**—No-load voltage ratio of moving-coil regulator.

### (b) Technical Formulae

The magnetizing current is determined by the no-load impedance of windings $a$ and $b$. Formula (7) (see Appendix 1) shows that for equal turns in both coils this impedance is equal to $ab$. Since the impedance $ab$ is unaffected by the position of the moving coil, the magnetizing current of the regulator is constant for all positions and is determined solely by the fixed windings $a$ and $b$. The moving coil could in fact be open-circuited or removed without noticeable effect. The no-load current differs from the corresponding transformer exciting current in that its wave-form is practically sinusoidal because the greater part of the magnetic circuit is air path.

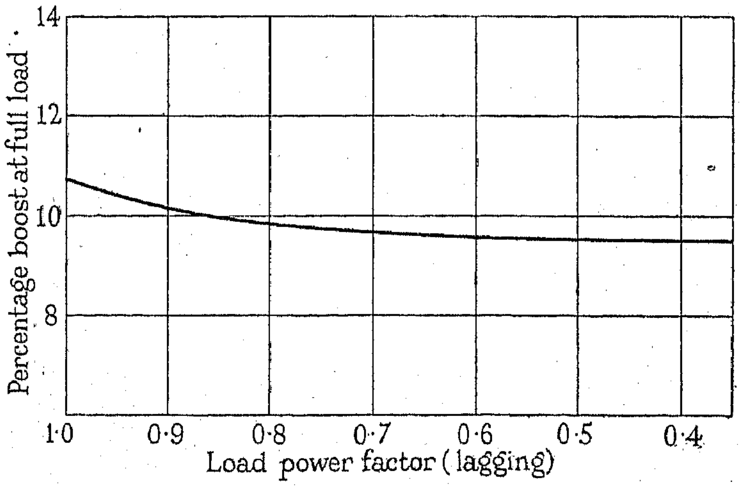

**Fig. 8.**—Effect of load power factor on regulator voltage boost.

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* E. F. GEHRKENS: "The Induction Voltage Regulator" (New York, 1923), p. 31.

The internal impedance of the regulator to load current is the leakage impedance of the auxiliary boosting and bucking coils plus the impedance of the main regulating windings, as given in formula (13) (see Appendix 2). The internal impedance value is not usually of importance, but in special cases, such as those referred to in Section (7), particularly high or low values of impedance may be obtained by suitable choice of coil arrangement and proportions in the same way as in ordinary static transformers. It is usual to design regulators so that the specified voltage-range or boost is given under full-load conditions, irrespective of the regulator impedance voltage-drop. A typical example of the effect of this impedance on the output voltage is shown in Fig. 8, for a Type B regulator of 10 % nominal boost.

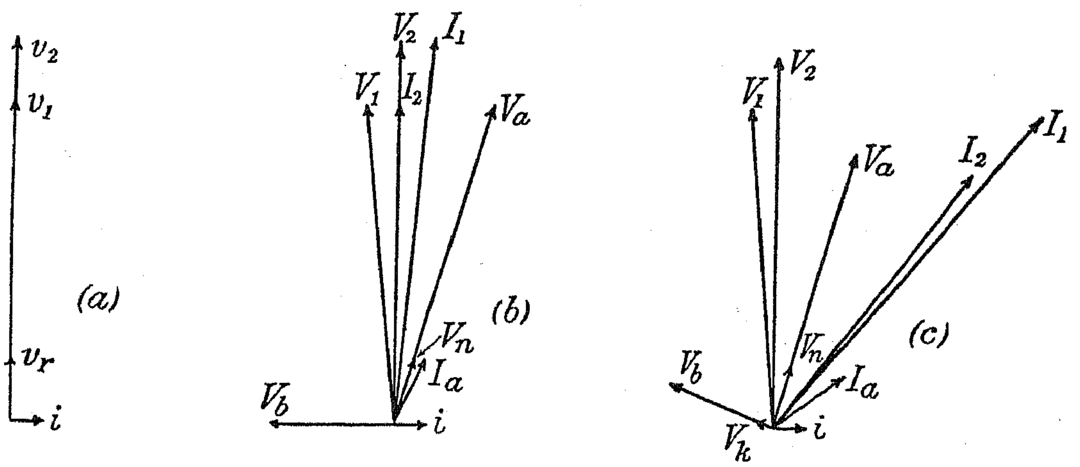

**Fig. 9.**—Regulator vector diagram: maximum-voltage position.
(*a*) No load. (*b*) Unity power factor load. (*c*) Load power factor 0·8 lagging.

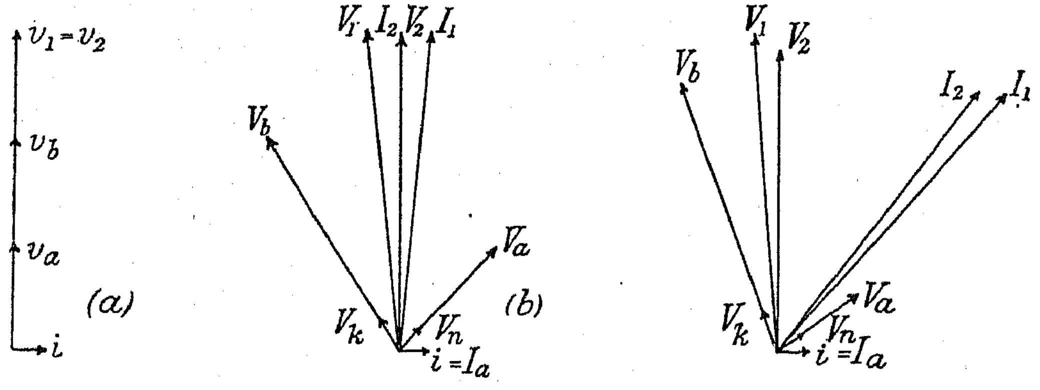

**Fig. 10.**—Regulator vector diagram: neutral position.
(*a*) No load. (*b*) Unity power factor load. (*c*) Load power factor 0·8 lagging.

Formulae giving the current and voltage values in any winding for any position of the moving coil and for any loading conditions are derived in Appendix 2. Experimental confirmation of the formulae is given in Appendix 3. For the Type B regulator with boosting and bucking coils $n$ and $k$, as in Fig. 3, the no-load output voltage is

$$\mathbf{v}_2 = \mathbf{v}_1 + N\mathbf{v}_a - K\mathbf{v}_b\text{*} \qquad (10)$$

where $N$ and $K$ are the turn ratios of coils $n/a$ and $k/b$ respectively.

The output voltage at a load current $\mathbf{I}_2$ is

$$\mathbf{V}_2 = \mathbf{v}_2 - \left[Z_s(N+K)^2 + na + kb\right]\mathbf{I}_2 \qquad (13)$$

where $na$ and $kb$ are the leakage impedances between coils $n/a$ and $k/b$ respectively.

The current in the primary windings $a$ and $b$ is

$$\mathbf{I}_a = \mathbf{i} + \frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{v}_1}\mathbf{I}_2 \quad . \quad . \quad . \quad . \qquad (14)$$

This is the same as for an ordinary auto-transformer of ratio $v_2/v_1$. The primary current is given by

$$\mathbf{I}_1 = \mathbf{i} + \frac{\mathbf{v}_2}{\mathbf{v}_1}\mathbf{I}_2 \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

which is again the same as for an ordinary auto-transformer. These equations show that for any position of the moving coil the regulator behaves like a static auto-transformer having a no-load voltage ratio corresponding to that particular position.

The current in the moving coil is, from formula (16) (see Appendix 2), $(N + K)\mathbf{I}_2$. Coils $a$ and $b$ are connected in series opposition so that their individual reactions on the moving coil are neutralized. As a result the current in the short-circuited coil for any regulator position is zero at no load and is proportional to the load current and in phase with it under all loading conditions. Short-circuited coils are frequently associated with heavy low-power-factor circulating currents which might be expected to result in a large regulator magnetizing current, or reactive drop. The formulae show that the moving coil does not have this effect.

Formulae for connections other than Type B are developed in a similar manner. Appendix 2 shows the modifications required.

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* Heavy-face type indicates vector quantities.

The effects of moving-coil position, load current, and load power-factor, on the various current and voltage values are shown clearly by vector diagrams. Typical diagrams are given in Figs. 9, 10, and 11 for the maximum, neutral, and minimum voltage-regulator positions respectively of a Type B regulator. In each case diagrams are drawn for the regulator (i) unloaded, (ii) on load at unity power factor, (iii) on load at 0·8 lagging power-factor. In order to show clearly the vector relation the magnetizing-current and impedance values have been exaggerated, so that the diagrams do not represent to scale the performance of an actual regulator. They do, however, illustrate qualitatively the effect of various loading conditions and moving-coil positions.

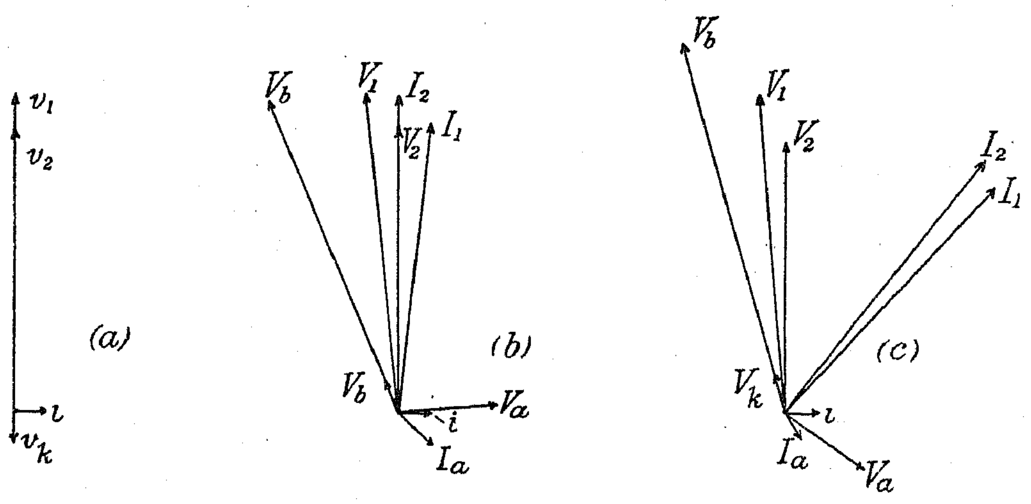

**Fig. 11.**—Regulator vector diagram: minimum-voltage position.
(a) No load. (b) Unity power factor load. (c) Load power factor 0·8 lagging.

### (c) Losses and Efficiency

The electrical efficiency of the regulator is determined by iron and copper losses in the same way as in an ordinary transformer. The major portion of the magnetic circuit is air, the iron serving chiefly as a surrounding yoke or container to form a return path for the magnetic flux and to control its direction. The flux-path diagram in Fig. 12 shows that only a small portion of the iron circuit is run at the full flux density. Since this maximum value should not reach saturation the average density in the iron is necessarily low.

Both load and no-load losses are dependent on current and flux densities and upon the weights of iron and copper. The losses can therefore be varied with cost exactly as in an ordinary transformer. In the earlier regulators, eddy currents both in iron and in copper due to stray flux caused increased losses, but these have been reduced by design experience and by the multi-yoke construction referred to later in the paper. The load losses are chiefly ohmic losses due to load currents in the windings. It should be noted, however, from the formulae in Appendix 2 that the windings do not all carry their equivalent full-load currents simultaneously for all positions of the moving coil. For example, formula (14) shows that in the neutral position, where $v_2 = v_1$, windings $a$ and $b$ carry no load current at all. For the most economical design the losses should be so chosen in relation to the weights of iron and copper employed that the total cost of the regulator plus the capitalized cost of the losses is a minimum. The conditions here are in general the same as for ordinary static transformers.

### (d) Noise and Vibration

The neutralization of the currents in the moving coil resulting from the series opposition connection of the coils $a$ and $b$ referred to earlier in the paper gives a corresponding reduction in the electromagnetic forces. Moreover, the mechanical force on a coil is proportional to the change in reactance with movement of the coil. Owing to the relatively large travel of the moving coil this rate of change of reactance, and therefore the resulting force, is small. (In regulators involving small air-gaps a very small movement increasing or decreasing this gap causes a large change in the gap magnitude and in the reactance, and hence a relatively large mechanical force.) The net forces on the moving coil are in effect zero in both end positions and also near the middle. For intermediate positions the forces may be in either direction. Fig. 13 shows a typical curve.

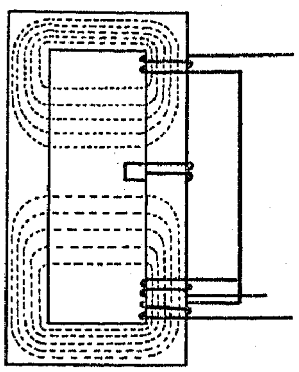

**Fig. 12.**—Diagram showing magnetic flux distribution.

It is difficult to express quantitatively and in a general manner the mechanical forces in the moving-coil regulator, but a practical comparison with other types of regulator is furnished by considering the motor power required for operation. This power must be sufficient to drive the regulator against the worst electromagnetic forces that may arise under any normal loading conditions. A later section of the paper shows that in all ordinary cases the moving-coil regulator is operated by motors of the induction-disc type, similar in principle to those used in driving gramophones.

Noise and vibration in iron-cored inductive apparatus are due either to magnetic forces in the core or to electromagnetic forces between coils. In the case of the regulator the former are usually negligible, owing to the necessarily low average flux density referred to earlier in this Section. The chief source of noise and vibration is the force on the moving coil. The previous paragraph showed how this force is largely neutralized, and the resultant noise and vibration will be correspondingly reduced. Also, since the forces on the moving coil are zero at no load and increase with load depending upon the coil position, the actual noise and vibration will follow the same course. Where regulators are installed in residential areas in transmission and distribution networks this characteristic is desirable, as during the night the load is usually small and noise emission will then be a minimum at a time when noise is most objectionable. This is in contradistinction to the ordinary transformer where in general the noise is mainly due to core vibration and therefore occurs at no load but is little affected by load. The regulator is not, however, noiseless, and, where operating conditions necessitate, special precautions must be taken.

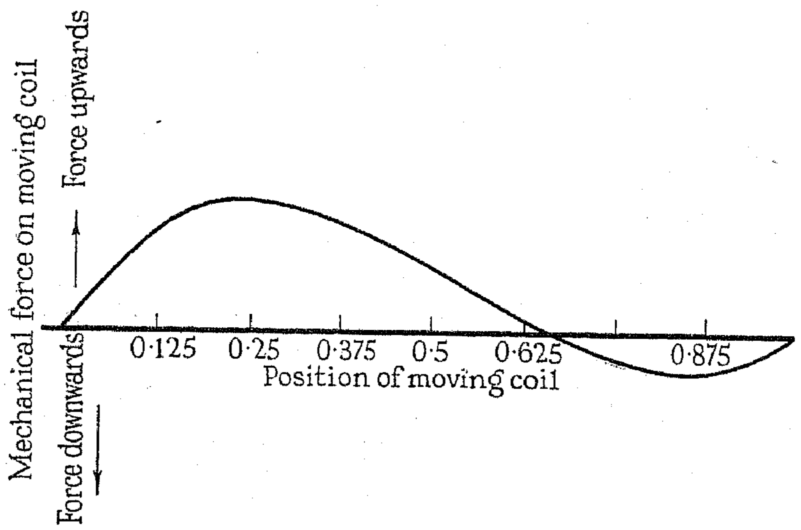

**Fig. 13.**—Mechanical forces on moving coil.

## (5) CONSTRUCTION

### (a) Winding Arrangements

The moving-coil regulator is built, in general, on the same lines as ordinary static transformers. The laminated iron core, coils, insulation, structural work, and general assembly are in accordance with standard transformer practice. In the original construction the core and coils were rectangular in shape and the coils were assembled in sandwich formation, as shown in Fig. 14(*a*). This arrangement was chosen because in the absence of manufacturing experience the technical performance could be calculated with greater certainty from formulae established in power-transformer design. However, the reactance relations between the various coils are relatively inefficient with this construction and, since the motion of the moving coil is at right angles to its largest dimensions, a great deal of free and otherwise waste space is required in the window of the core for this movement. These drawbacks were overcome by using circular coils assembled concentrically on the core,[*] as shown in Fig. 14(*b*). To permit comparison of the two methods, Figs. 14(*a*) and 14(*b*) are drawn to scale for the same regulator rating.

Incidentally, the use of circular coils simplifies the winding and insulation of the coils and results in greater mechanical strength. Moreover, the voltage range of the regulator can be increased as described in Section 4(*a*) (see also Appendix 1). Although this increase is small in comparison with the total voltage variation, it simplifies the design of the regulator and eliminates the need for a bucking coil to obtain zero voltage or zero boost.

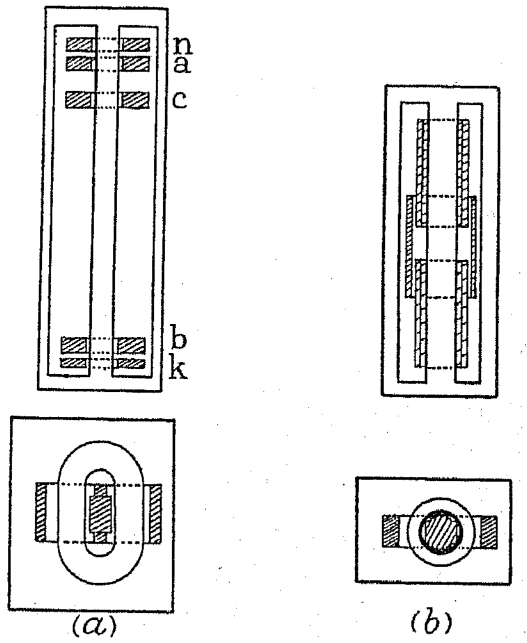

**Fig. 14.**—Sectional diagrams showing sandwich construction with rectangular coils and concentric construction with circular coils.

(*a*) Sandwich windings. (*b*) Concentric windings.

### (b) Single-phase and Three-phase Core Construction

The smaller single-phase regulators are built as shown in Fig. 14(*b*) [a simpler construction for very small units will be described in Section 7(*c*)]. For larger sizes the multi-yoke[†] construction shown in Fig. 15 is used to improve the magnetic flux distribution and reduce stray losses. The main core leg is divided into sections having the planes of their laminations at right angles. This, incidentally, gives a rigid mechanical structure even for relatively long legs, needing little clamping framework.

The same principle is applied to the 3-phase construction. Three single-phase units, modified as in Fig. 16, are grouped in triangular formation. The fourth yoke of each unit can be omitted, as the 3-phase fluxes cancel out. The resulting space is utilized for mechanical operation. A rigid construction of the main core leg is again obtained by sectionalization in two planes. The 3-phase assembly is shown in section in Fig. 17.

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[*] British Patent Specification No. 438791—1934.
[†] British Patent No. 438289—1934.

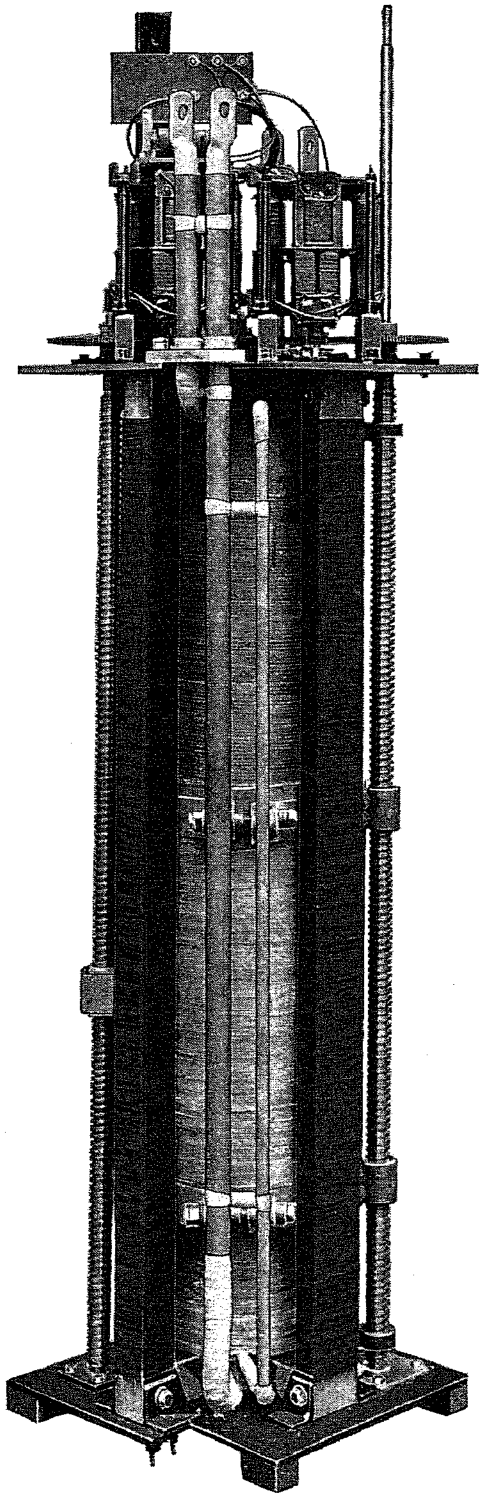

**Fig. 18.**—Typical single-phase moving-coil regulator, showing multi-yoke construction.

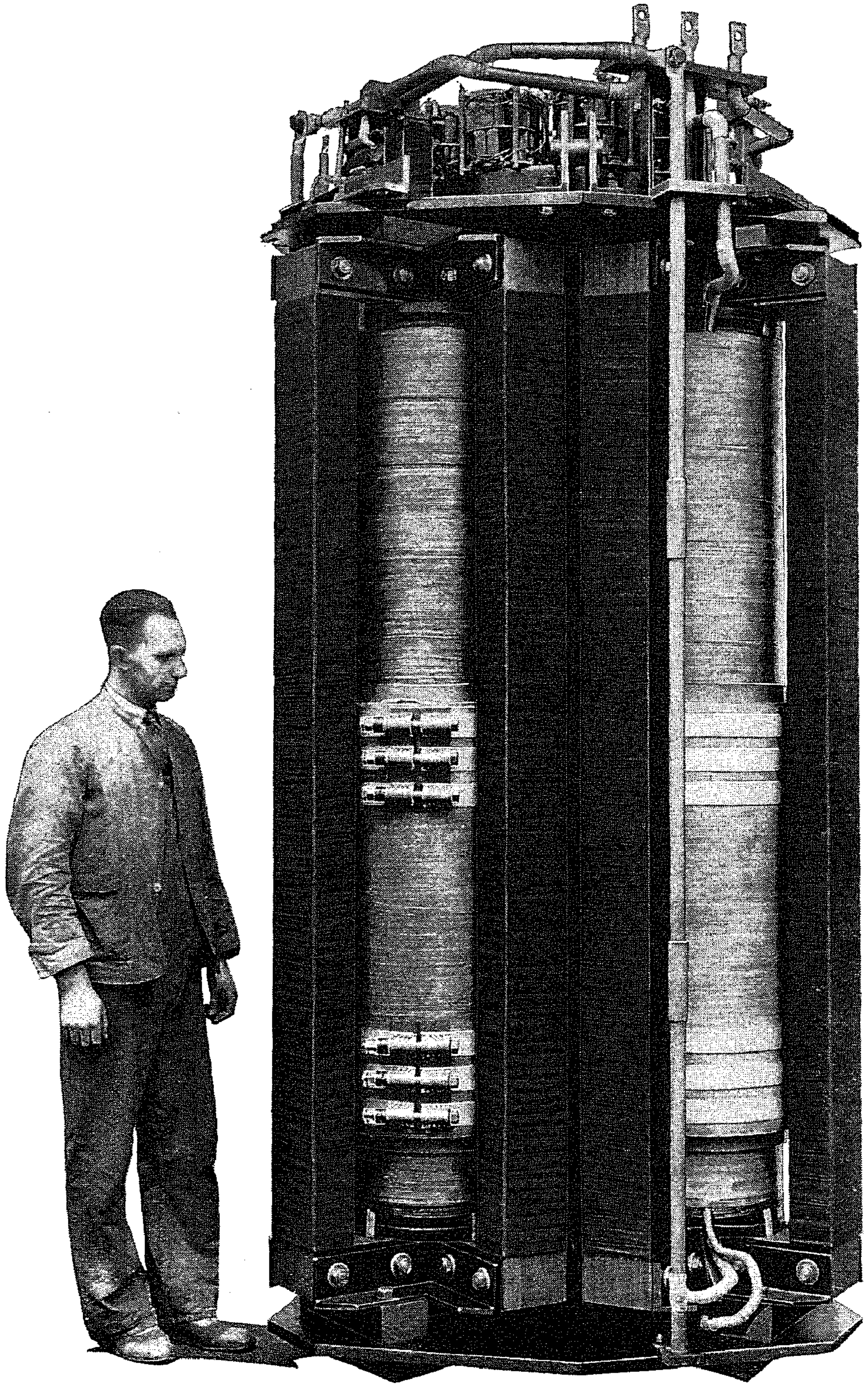

**Fig. 19.**—Typical 3-phase moving-coil regulator, showing assembly of modified single-phase units.

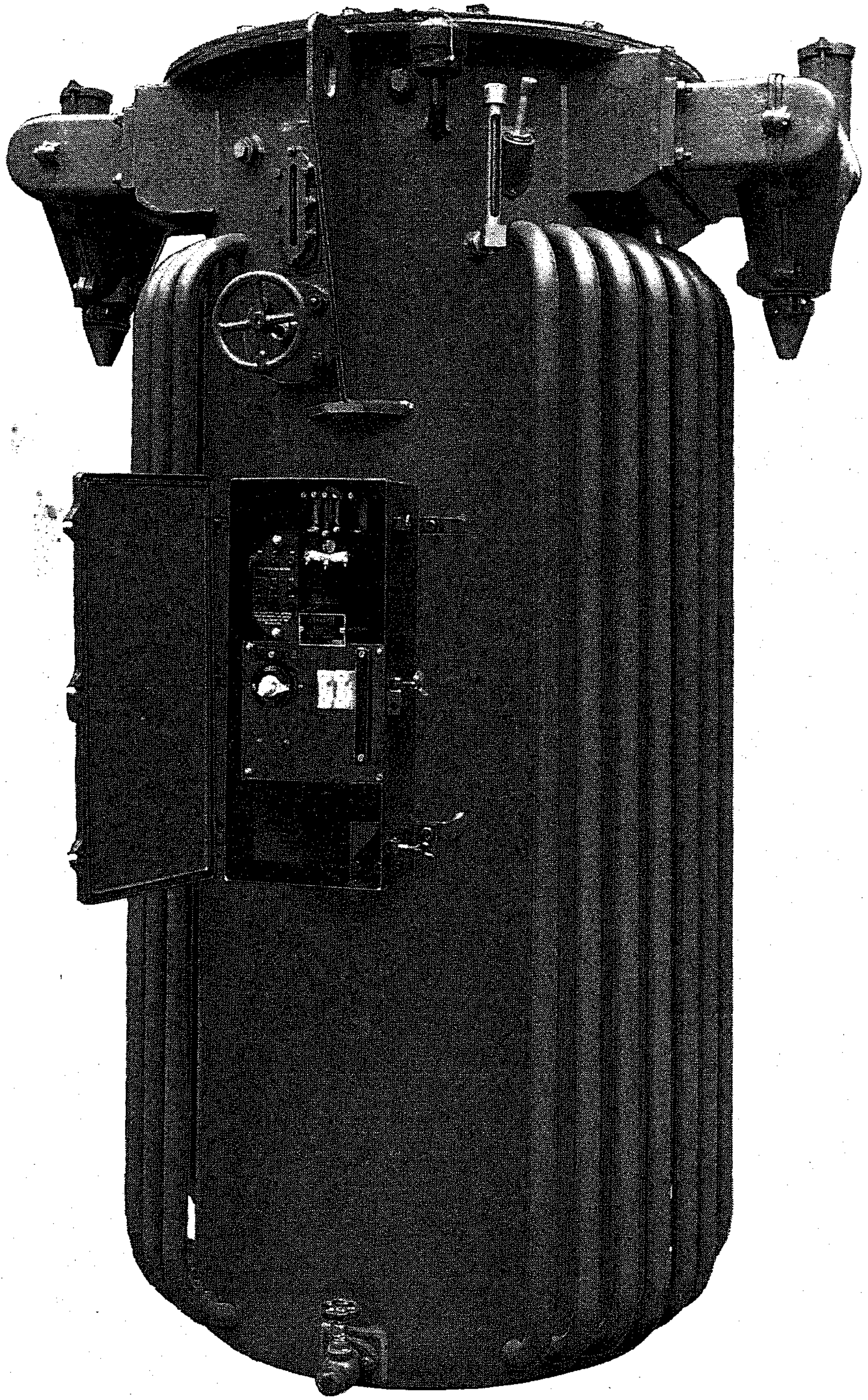

**Fig. 25.**—External view of typical medium-size automatic moving-coil voltage regulator.

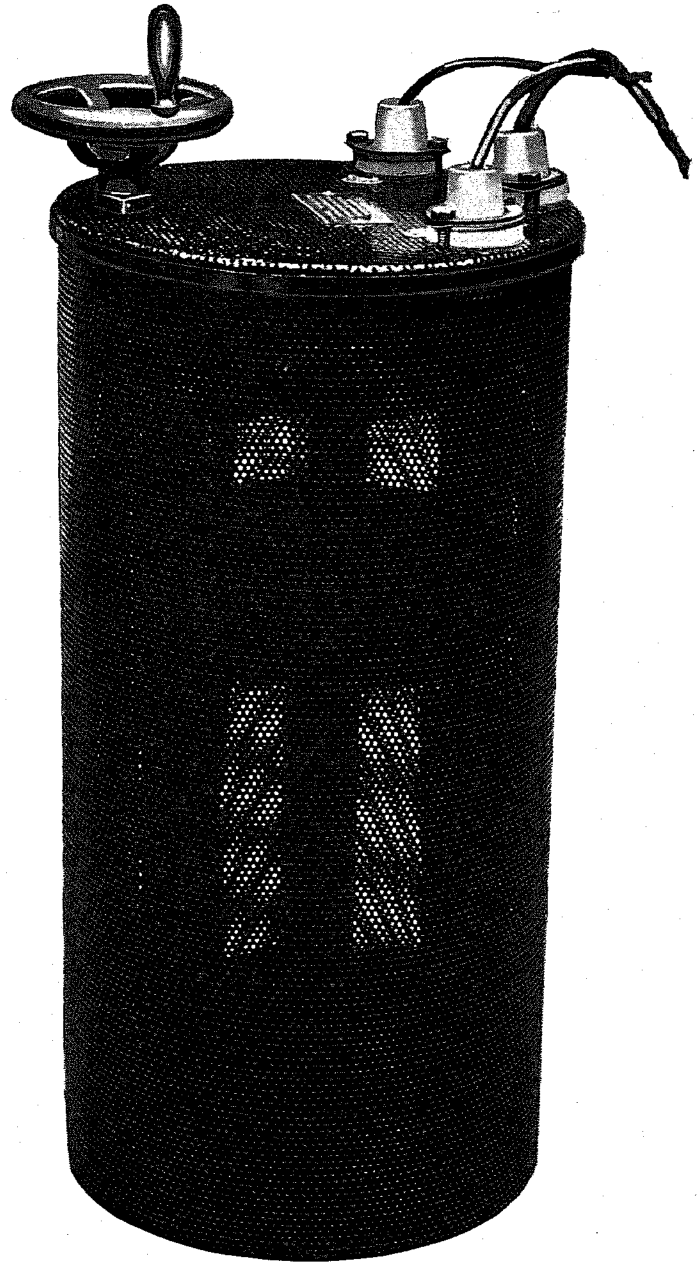

**Fig. 28.**—Typical small air-cooled hand-operated single-phase regulator, 230/0–100 volts.

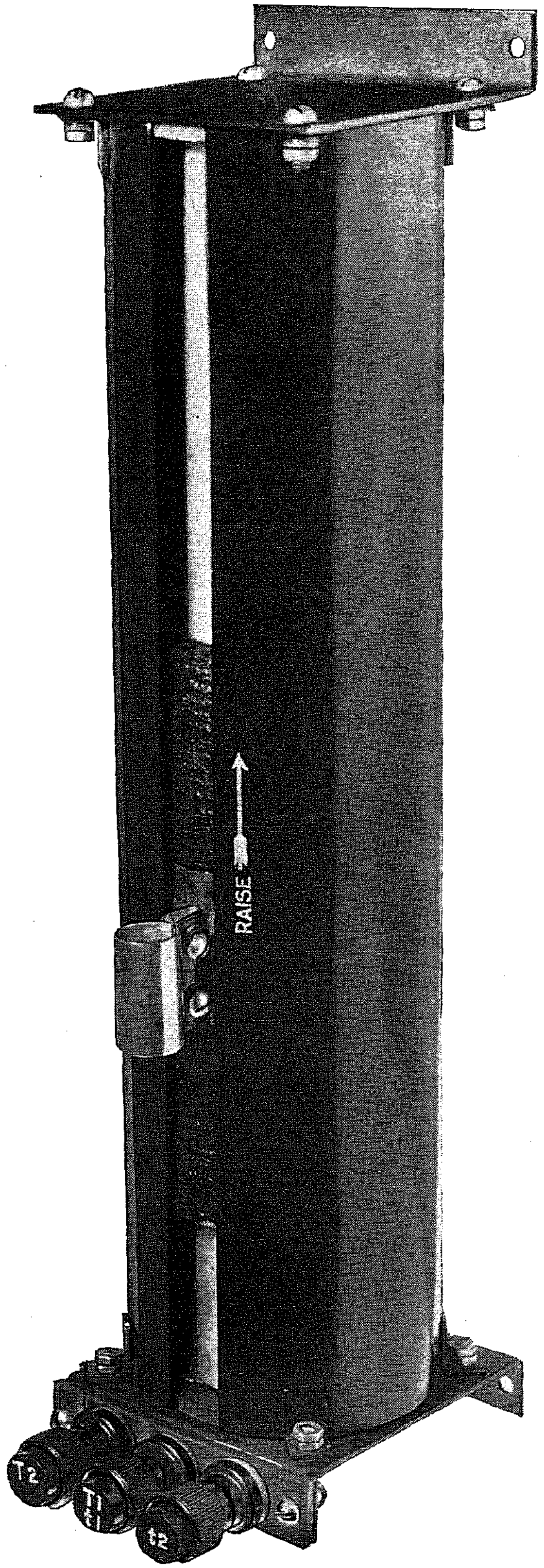

**Fig. 30.**—Typical small tubular-type regulator with sliding knob control.

In both single-phase and 3-phase regulators the coils are of the disc or the multi-layer type, depending upon the current and voltage. Coil supports, spacers, and oil cooling ducts are provided where necessary, both mechanical and thermal design being the same as for ordinary transformers. The bucking and boosting coils *n* and *k* are usually the innermost windings, with the primary coils *a* and *b* assembled immediately over them. The moving coil is earthed and mounted with mechanical and electrical clearances between it and the fixed coils and yoke sides.

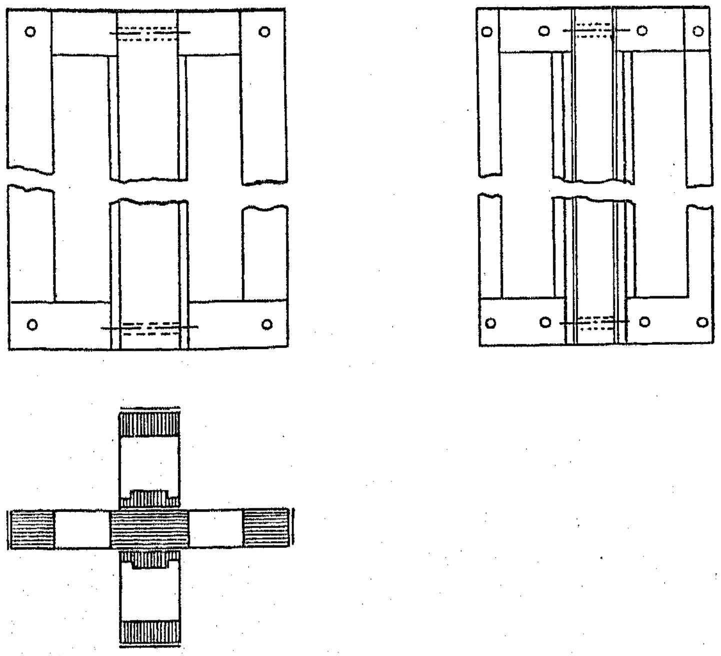

**Fig. 15.**—Typical multi-yoke single-phase core.

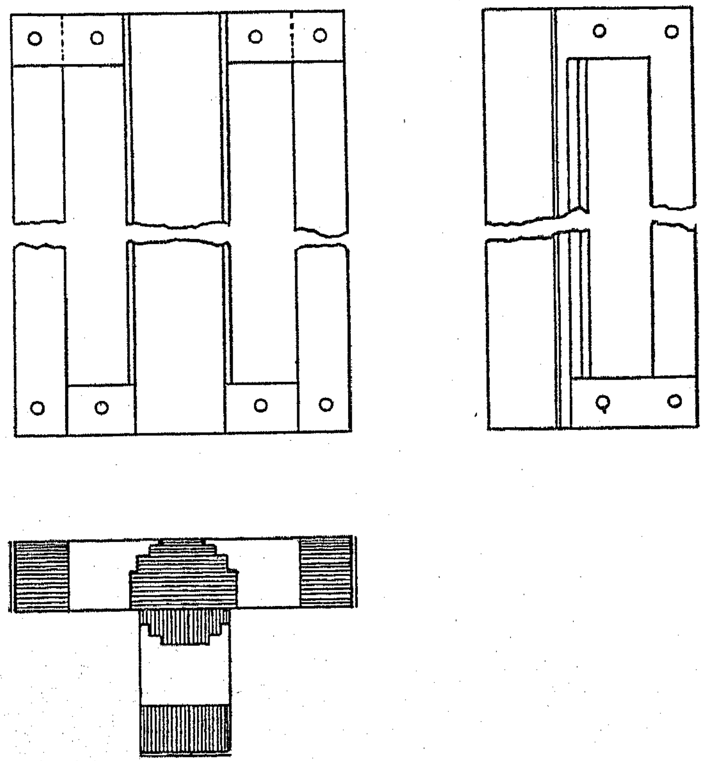

**Fig. 16.**—One-phase unit of multi-yoke 3-phase core.

Cooling follows the usual practice. Small sizes may be air-insulated and air-cooled, but all others are oil-immersed in plain, tubular, or radiator tanks, depending upon the size. In most cases the whole of the operating mechanism is oil-immersed with the regulator.

### (c) Mechanical Operation

The moving coil, clamped in two or more stainless-steel bands, is operated by a square-threaded shaft working on a nut attached to the coil clamps and driven through suitable gearing by hand or by motor. In single-phase units one, or in larger sizes two, of these shafts are employed. Three-phase regulators are driven by a central shaft operating all three coils. Figs. 18 and 19 (see Plate 1, facing page 8) show typical single-phase and 3-phase constructions respectively.

## (6) VOLTAGE CONTROL IN TRANSMISSION AND DISTRIBUTION CIRCUITS

### (a) Special Technical Characteristics

In general, the required range of voltage variation is limited to 10 % or 20 % of the declared voltage. The Type B connection is therefore the most economical and is generally used.* The actual values of buck and boost required will be different in each case. As explained in Section 3(*b*), the exact values can always be obtained by suitable choice of the auxiliary coil ratios. Usually boost only is required, and in these cases the moving-coil regulator is of approximately half the size of one giving equal buck and boost; whereas with the induction voltage regulator, for example, the sizes would be the same.

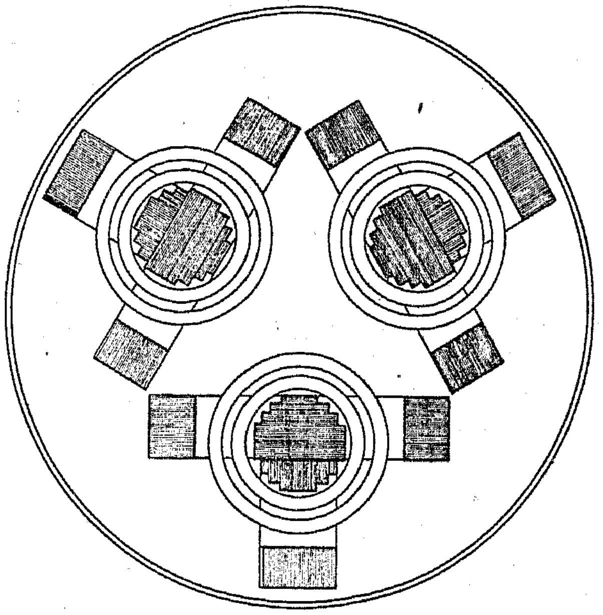

**Fig. 17.**—Sectional plan of 3-phase regulator, showing cores and coils.

Operating conditions in transmission and distribution circuits involve certain special features in addition to the technical characteristics discussed generally in Section (4). The most important of these is perhaps the ability of the regulator to withstand the mechanical and thermal stresses resulting from short-circuits on the supply system, particularly when the fault current is not safely limited by generating-plant capacity. Voltage regulators in general, being auto-transformer-connected devices with series windings, are subjected to the most severe stresses. The position has been summed up thus: "The feeder voltage regulator is more liable to injury both mechanically and electrically and more likely to cause damage to the station than any other apparatus. It is impracticable to design it with the same factor of safety as is feasible with other series devices. The short-circuit current must therefore be limited . . . by properly-designed current-limiting reactors."*

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* In cases where a regulator and transformer are installed together and it is convenient to combine them into a single unit, as described in Section 3(*b*), the Type A connection may, however, be preferable.

The electromagnetic forces in the moving-coil regulator are reduced by two factors discussed in Section (4). Calculated values show a mechanical strength sufficient to withstand severe short-circuits without the need for current-limiting reactors. In confirmation of these values full-scale tests were carried out on a standard distribution regulator† in which short-circuit currents up to 77 times full load were obtained by over-exciting the regulator. Fig. 20 shows a typical oscillogram. Such currents were more than could occur in practice in a power supply circuit of infinite generating-plant capacity. The mechanical stresses are relatively more for regulators of larger rating but are not, as yet, one of the limiting factors in design.

### (b) Interconnection and Parallel Operation

In installations involving interconnection, tie-lines between power stations, or parallel operation, voltage phase-displacement in the regulator is of importance. This displacement is the phase angle between the incoming and outgoing voltages, independent of load and usually due either to internal connections or to the inherent characteristics of the apparatus. Ordinary transformers connected delta/star, for example, have a voltage phase-displacement of 30°. A 3-phase induction regulator for ± 10 % voltage variation has a voltage phase-displacement of 6° in the neutral position. Formula (10) and the vector diagrams in Figs. 9(*a*), 10(*a*), and 11(*a*) show that neither single-phase nor 3-phase moving-coil regulators have any voltage phase-displacement in any regulator position. They are therefore suitable for use in interconnectors and for the control of voltage, load distribution, or power factor. Interconnectors up to 10 000 kVA capacity are being regulated in this manner.

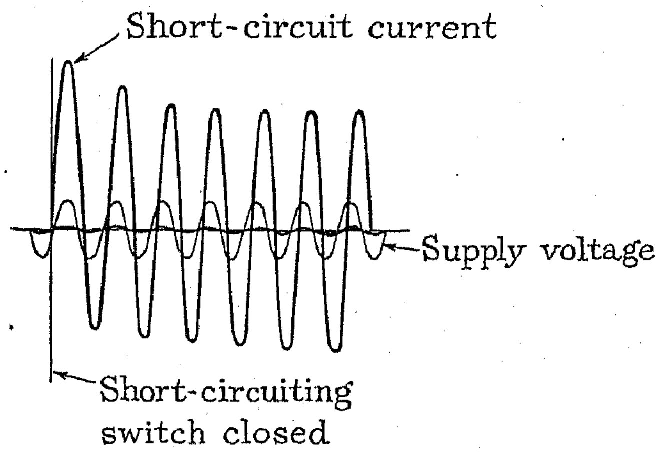

**Fig. 20.**—Oscillograms of current and voltage in short-circuit test.

The unmarked curve shows the normal full-load current to the same scale as the short-circuit current.

Where parallel operation is required the regulators are coupled either mechanically or, more often, by electrical control. The automatic gear described in Section (6)(*d*) is employed, with the addition of a coupling relay. Regulators connected in parallel must, of course, have appropriate internal impedance values in the same way as ordinary transformers. Similarly where regulated feeders are operated in parallel with unregulated feeders the combined impedance of regulator and feeder must be considered.

### (c) Three-phase Winding Connections

For use in 3-phase circuits either delta or star connection is permissible, or alternatively two single-phase regulators may be connected in open delta. Star connection is the most common in this country. Conditions determining the best connection for any given case are, in general, the same as for ordinary static auto-transformers or for single-phase and twin 3-phase induction regulators.*

The 3-phase star-connected regulator consists virtually of three single-phase units and, as regards its phase voltages, behaves like a shell-type transformer or a bank of single-phase auto-transformers. It is not, therefore, possible to use the neutral point of the regulator for loading purposes. In all cases where a 3-phase star-connected regulator is used in a 4-wire circuit and there is any possibility of unbalanced loading it is essential to connect the neutral point of the regulator to the system neutral and, if necessary, to provide interconnected-star static balancers to take care of unbalanced loads.

### (d) Remote and Automatic Control

All regulators controlling the voltage in distribution circuits should be automatically operated. Even where regulation is effected by on-load tap-changing gear on the substation transformers the changing of tappings by hand is impracticable. Manual operation was formerly employed on account of the complication and expense of automatic control gear. It is, indeed, this complication and expense which has seriously restricted the use of automatic regulation in distribution circuits, particularly among rural supply authorities with limited staffs and scattered networks. The widespread installation of regulators may be justified on both technical and economic grounds by increase in revenue† and by the reduction of capital expenditure on cables, lines, and substations;‡ but the automatic control gear must be reliable and reasonable in cost and, as an ideal, it should not require periodical maintenance and attention.

For the larger installations, including high-voltage transmission circuits, regulators of the on-load tap-changing-gear type, either fitted to the main step-up or step-down transformers, or as separate boosters, have commonly been employed with manual operation or remote electrical control. Modern practice, however, is tending towards automatic operation for even the largest sizes.§ The resulting greatly increased frequency of operation has emphasized the advantages of regulators giving a smooth voltage variation with no wear of switch contacts. Moreover, with the moving-coil voltage regulator both fully automatic and remote electrical control are simpler than with on-load tap-changing gear, as it is not necessary to ensure operation in specified steps.

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* E. F. Gehrkens: "The Induction Voltage Regulator," p. 439.
† *Electrician*, 1935, vol. 114, p. 528.

* W. E. M. Ayres: "The Application of the Induction Voltage Regulator," *Journal I.E.E.*, 1931, vol. 69, p. 1208.
† F. S. Naylor: "Loss of Revenue on Heating and Lighting Loads due to Poor Voltage Regulation," *ibid.*, 1936, vol. 79, p. 33.
‡ A. C. MacQueen: "Voltage Regulation on the L.T. A.C. Network," *Electrical Power Engineer*, 1936, vol. 18, p. 717.
§ W. Kidd and J. L. Carr: "Application of Automatic Voltage and Switch Control to Electrical Distribution Systems," *Journal I.E.E.*, 1934, vol. 74, p. 285.

Remote electrical operation of the moving-coil regulator requires relatively small power owing to the low mechanical forces referred to earlier in this Section. For all ordinary cases motors of the induction-disc split-phase type, similar in principle to many gramophone motors, are sufficiently powerful. The driving element consists of two iron-cored coils mounted on either side of a rotating copper disc. The phase displacement is obtained either by connecting a condenser in series with one of the coils or, in some 3-phase circuits, by connecting the coils between appropriate phases and neutral. Flux interactions in the rotor provide the driving torque. The motor runs at a low speed (about 150 r.p.m.), so that little mechanical reduction gearing is required. One or more of these motors are mounted with the shafts vertical and drive the square-thread operating shaft described in Section 5(*c*) through a single spur gear. The comparative inefficiency of this class of motor is not objectionable for such small ratings, particularly as the motor is run intermittently and for short periods. Moreover, to offset its inefficiency it has peculiar advantages for this type of work. The rotor is a plain copper disc, which can well be run under oil. As most regulators are oil-cooled the motor and the whole of the mechanical gearing are operated under oil, being thereby automatically lubricated and protected from dirt and atmospheric conditions. The starting current is practically the same as the full-load running current, thus eliminating special starting devices, and, as the motor can be stalled without damage, limit switches and overload devices are unnecessary. The starting torque is high, whilst the load torque rises almost linearly with decreasing speed, giving stable operation even under widely varying loads. Finally, the light weight and low speed of the rotor reduce the inertia of the mechanism so that electromagnetic brakes, automatically operated when the motor is switched off, are not needed to prevent over-running.

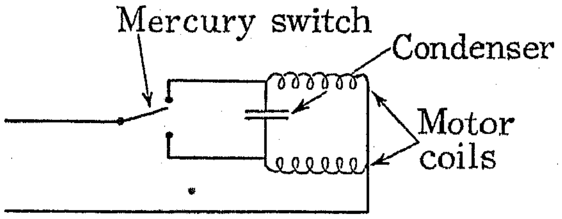

**Fig. 21.**—Induction-disc motor circuit and control.

For fully automatic operation the control gear is, in general, the same as has just been described for remote electrical control, with the substitution of an automatic voltage relay or contact-making voltmeter for the push-button or remote-control switch. It is, however, for the reasons given above, most important that the whole control gear shall be reliable and need little, if any, periodical maintenance. The complete control circuit for the automatic operation of voltage regulators in general has been described elsewhere.*

For the moving-coil regulator the simplifications permitted by the induction-disc motor are the same as for remote control, with the additional feature that since starting stresses are negligible there need be no limit to the frequency of operation of the regulator, and time-delay relays for this purpose are unnecessary. The voltage relay (which provides the automatic operation by closing the motor circuit whenever the voltage varies a prescribed amount, 1 % or less, from the normal value) is of the new astatic type.† The power/sensitivity ratio of this relay is sufficient to operate mercury switches direct instead of light open contacts, whilst there are no hold-on devices or control springs. These components of the ordinary voltage relay requiring maintenance and adjustment have thus been eliminated.

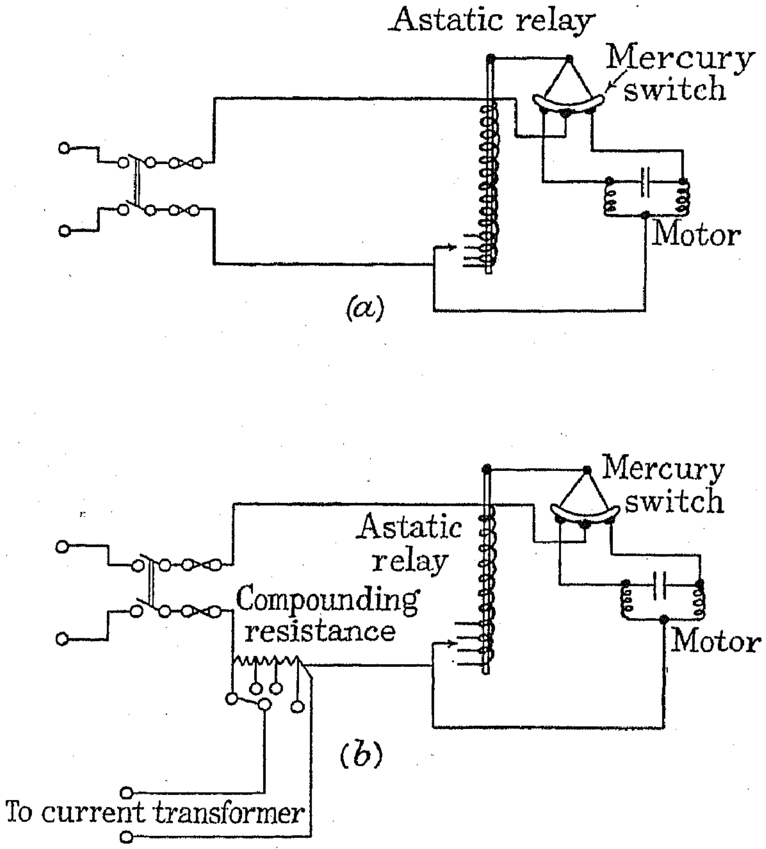

**Fig. 22.**—Typical automatic control gear circuits.

(*a*) Control circuit giving adjustable constant output voltage.
(*b*) Control circuit as (*a*), but with resistance compounding giving output voltage rising with load.

Reversal of rotation is obtained by reversing the coils in either split-phase circuit. This operation normally requires a double-pole double-throw switch, which must be inserted between the voltage relay and the motor. However, if the coils of each split phase are suitably designed, reversal of rotation can be obtained by changing the phase connection of the condenser. With this arrangement starting, stopping, and direction of rotation, can all be controlled by a single-pole 2-way switch, as shown in Fig. 21. For all except the largest regulators the voltage-relay mercury switch serves this purpose and thus controls the driving motor directly.

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* H. DIGGLE: "The Application and Construction of Transformer On-Load Tap-Changing Gear," *Journal I.E.E.*, 1937, vol. 81, p. 330, Fig. 22; also W. E. M. AYRES: "The Applications of the Induction Voltage Regulator," *ibid.*, 1931, vol. 69, p. 1209, Fig. 6A.

† E. T. NORRIS: "The Astatic Voltage Relay," *Electrician*, 1936, vol. 116, p. 442; British Patent Specification No. 465058—1935.

The combination of this relay with the control circuit just described and the moving-coil regulator results in an automatic voltage regulator requiring no periodical maintenance or adjustment. It is suggested that the regulator, so far as routine inspection and examination are concerned, should be treated exactly as an ordinary static transformer of similar voltage and rating. Typical control circuits with and without resistance line-drop compensation respectively are shown in Fig. 22, and comparative component diagrams in Fig. 23.

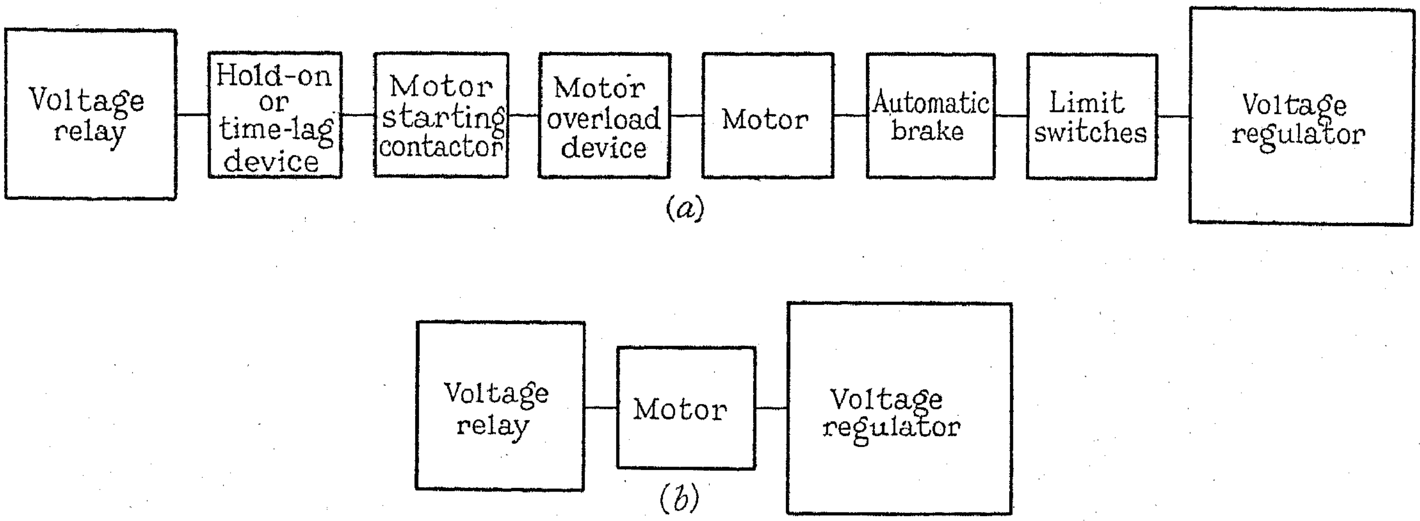

**Fig. 23.**—Automatic control gear components.

(*a*) Ordinary voltage relay and regulator or on-load tap-changing gear. (*b*) Astatic voltage relay and moving-coil voltage regulator.

### (e) High-Speed Operation

It is, in general, desirable that voltage variations shall be corrected by the regulator as soon as they occur. In severe cases where sudden changes in voltage occur due to intermittent loads, correction should be as rapid as possible; but since no voltage regulator as defined in Section (2) is instantaneous in operation it is impossible to remove flicks or momentary voltage variations due to abrupt load-changes. The operating speed can be controlled over a considerable range at small cost by suitable motor ratings and gear ratios. Values down to 20 sec. or less for travel over the full voltage range can be obtained in this way for small regulators. If induction motors and control gear of the usual type are employed, operating times down to 10 sec. or less are possible. These short times are not effective for high-speed operation, however, if they are obscured by the use of time-lag devices. The highest operating speeds have been obtained by hydraulically-operated servo-motors with balanced valve control.

Where the moving-coil regulator is controlled directly by the motor and astatic relay the delay in actuating the auxiliary components shown in Fig. 23 is eliminated and low operating times are possible. In one instance (the relay being fitted with a quick-resetting device) the time taken to correct a sudden change in voltage of 4 % was reduced to less than 1 sec.

High operating speeds are, in general, more expensive and in many cases are not worth while merely in order to reduce the duration of a voltage-change when its occurrence is not prevented. High speed is indeed often objectionable, apart from the increased frequency of operation usually involved. For example, suppose load conditions are such that sudden voltage-drops of short duration frequently occur. The higher-speed regulator does not affect the sudden drop in voltage, but corrects it more quickly. When the load causing this drop is removed the voltage will rise a similar amount above normal and the regulator will then have to correct this second variation. The effect of the high speed of operation is then not merely to reduce the duration of a voltage-drop due to the load but to create a voltage-rise due to the removal of load. In other words, a voltage-drop is converted into a similar voltage-drop of shorter duration plus a voltage-rise of the same duration. This conversion may well be more objectionable than the original momentary voltage-drop. The condition is illustrated in Fig. 24. The full-line voltage variation *abcd* is converted by a high-speed regulator into the dotted-line variation *abedfg*. It is desirable, therefore, to consider individually cases apparently requiring high-speed operation.

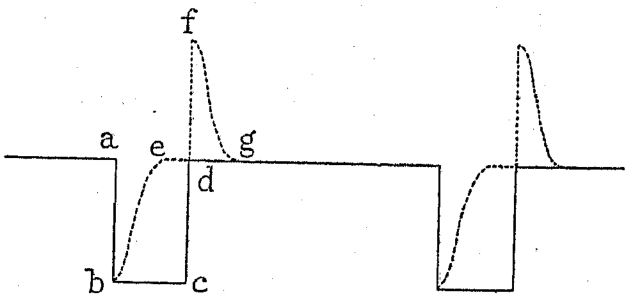

**Fig. 24.**—Voltage diagram showing effect of high-speed regulator operation.

*abcd* = voltage without regulator.
*abedfg* = voltage with regulator.

### (f) Special Structural and Housing Arrangements

The external construction of the automatic regulator follows ordinary transformer practice where ground installation indoors or outdoors is concerned. The control gear is mounted in a steel box on the side of the regulator tank, the whole forming a self-contained unit. Fig. 25 (see Plate 2) shows a typical 3-phase regulator for an 11 000-volt 500-kVA circuit. Regulators for pole or platform mounting in distribution systems also follow general transformer practice. The full and efficient use of regulators in distribution circuits, however, frequently demands installation in positions remote from the transformer substation. The general tendency, in fact, is for automatic regulation equipment to move steadily nearer to the load. This tendency follows a natural course as, in the limit, voltage variations are only important at the consumer's terminals. Voltage regulators installed at the substation can only handle l.t. voltage variations already within the limits of good service. Installation along the distributor permits loads and feeder lengths which would otherwise result in voltage-drops 4 to 5 times as great. Since the cost of regulating apparatus is small compared with the cost of distributors and substations, this increase in capacity is worth realizing. To meet these operating conditions a number of special constructions, which have been described in detail elsewhere, have been developed. They are briefly as follows:—

**(i) Feeder-pillar construction.***

For installation on street pavements or in any place accessible to the public without fencing or additional protection, the tank walls being extended on two parallel sides to form a second compartment to house the terminals of the regulator, cable boxes, tank fittings such as oil-level gauge and position indicator, and the control gear.

**(ii) Street-pit construction.†**

A combination of moving-coil regulator and astatic relay, protection from dirt and atmospheric conditions being obtained by mounting the whole operating gear, including the relay, under oil in the regulator tank.

**(iii) House-service construction.‡**

A simplified regulator assembly economically suitable for controlling small individual domestic supplies and isolated loads; the mechanism, control gear, and structural work are simplified by eliminating control-circuit switchgear and by mounting the relay under oil in the main tank, as in the street-pit construction.

## (7) INDUSTRIAL AND LABORATORY APPLICATIONS

### (a) Special Technical Characteristics

This Section covers regulators for such a wide range of purposes that no practical limits for voltage variation may be set. The growth, during recent years, of automatically-operated manufacturing processes and treatments has extended the application of voltage regulators for controlling indirectly power consumption, temperature values, and many electrochemical and electrolytic technical characteristics. In general, the range of voltage variation will be large, and the Types C and D regulator connections as described in Section 3(*b*) will be employed in preference to Type B. These connections are both suitable for voltage variations up to 100 % (i.e. from zero to full value) or more, and the choice will be determined mainly by the transformation ratio required (i.e. the ratio of the maximum output voltage to the input voltage). Type C, being auto-transformer-connected, is only economically suitable for transformation ratios not greatly exceeding 2, as for ordinary static auto-transformers. The essential function of the regulator is to give the required range of voltage variation and not to act as a static transformer. In many cases, however, the necessary transformation ratio may economically be included in the regulator in order to eliminate a separate step-up or step-down transformer. For example, if a voltage range 230/0–500 volts is required, it will be cheaper to employ a regulator with Type C connection for this ratio rather than a regulator having a ratio 230/0–230 and an ordinary transformer of ratio 230/500 volts. If, however, the required voltage range is 230/0–20 volts, the Type C connection will be uneconomical, and the alternatives will be either Type D or a Type C regulator with a separate transformer of ratio 230/20 volts. The choice may be made on economic grounds, apart from the advantage in the first case of a single unit, and will depend upon the kVA rating and the relative costs of transformers and regulators. In all cases, however, where electrical isolation of the output is necessary, the Type D connection or a separate double-wound transformer must be employed.

Many industrial and laboratory applications involve certain special features in addition to the technical characteristics discussed generally in Section (4). In most cases where the regulators are used for testing or experimental work it is important, not only that the regulator itself shall withstand deliberate or accidental short-circuits on the output side, even when these are of frequent or routine occurrence, but also that the short-circuit current shall be limited to a low value to prevent disturbance to the supply system or damage to the connected apparatus. The internal impedance of the regulator can be increased to give almost any desired short-circuit current limitation by suitable winding proportions.

In many applications it is important that the output voltage wave-form of the regulator shall be as nearly as possible sinusoidal. Since the major portion of the regulator magnetic circuit is in air, whilst in the iron portion the average flux density is necessarily low, as explained in Section 4(*c*), the magnetizing-current harmonics are relatively small compared with those for the case of an ordinary transformer, and voltage wave-form distortion inherent in the regulator is usually negligible. The output voltage will then be sinusoidal provided that the load is a linear circuit. In some cases, however, the load is of such a nature that the load current even with sinusoidal voltage is badly distorted (mainly by third and fifth harmonics). The most common instances are loads involving closed iron magnetic circuits. The internal or leakage impedance of the regulator acts as a linear impedance in series with the load. There will therefore be a non-sinusoidal voltage-drop across this impedance, due to the load current. This distorted voltage-drop subtracted from the sinusoidal input voltage gives a corresponding distorted output voltage. This kind of distortion is then directly dependent upon the load-current harmonics and the regulator impedance. The latter can not only be reduced to a low value but since it can be accurately calculated the wave-form distortion can be predetermined for any given loading conditions. The most difficult cases arise where, for other reasons, it is not permissible to reduce the regulator impedance, e.g. where a high impedance is required for short-circuit current limitation. These instances must be considered individually and a compromise adopted. In exceptional cases where the expense is justified the necessary impedance may be retained and the load-current harmonics eliminated by a series of resonant shunts.

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* *Electrical Times*, 1936, vol. 90, p. 681.
† *Electrical Industries*, 1937, vol. 37, p. 385.
‡ *Electrician*, 1937, vol. 118, p. 417.

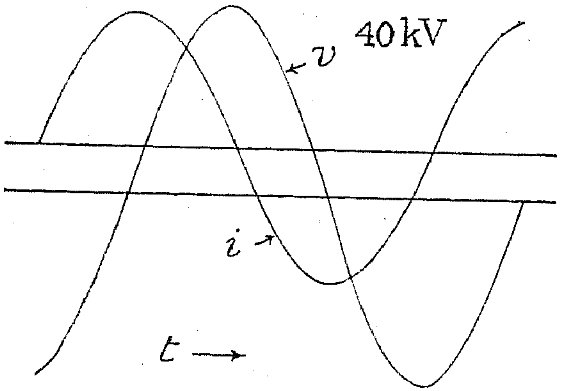

**Fig. 26.**—Current and voltage wave-forms of cable-testing equipment.

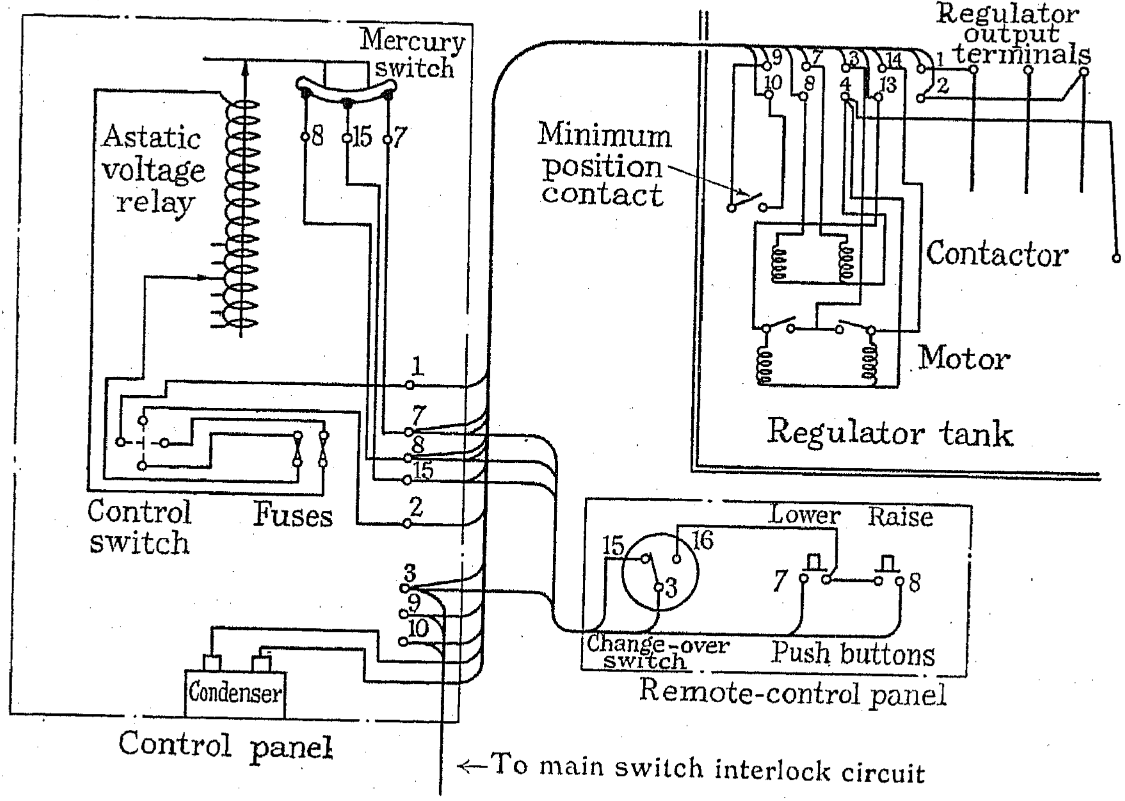

**Fig. 27.**—Diagram of connections for preset zero-return automatic and remote electrical control of moving-coil regulator.

One of the most important regulator applications where purity of voltage wave-form is essential is in the control of high-voltage testing equipments, particularly for cable testing and research work. A typical installation of the moving-coil regulator for this purpose has been described by Dr. P. Dunsheath.* Fig. 26 is a reproduction, taken from his remarks at a discussion before The Institution, of an oscillogram showing the wave-form at 40 kV of a testing transformer controlled by a 150-kVA moving-coil regulator when charging a length of 60-kV cable.

The load on high-voltage testing transformers, again particularly in cable testing and research work, is frequently of low leading power-factor, requiring a large reactive-kVA input. Since the magnetizing current of the regulator is practically sinusoidal and can be controlled over a wide range by suitable winding proportions [see formula (7) of Appendix 1], it is possible to increase it and utilize it to neutralize the capacitance component of the load current, thus reducing the kVA input required and improving the power factor on the supply side. In one case of this kind a 250-kVA alternator is used to supply up to 600 kVA of connected testing-transformer and cable load. Larger ratios of load and supply ratings are practicable.

### (b) Forms of Manual and Automatic Control

The speed of operation of the regulator is controlled as described in Section (6) by suitable motor ratings and gear ratios. In certain cases where it is desired to obtain any specified voltage value by hand or remote electrical control as quickly as possible, multi-speed control is employed. Usually two speeds are sufficient (giving coarse and fine adjustment), and a 2-speed motor is satisfactory. In other cases a variable-speed gearbox must be used.

Methods of operating the regulator include those described in the last Section, namely manual and automatic operation and remote electrical control. There are, however, a number of applications where more complicated methods of operation are justified. A common case is where a variable but specified voltage is required using unskilled operation, as in some installations of medical and industrial apparatus. By means of a presetting device a pointer is turned to the required voltage. When the switch is closed the regulator raises the voltage to that value. In certain cases automatic voltage control is added, so that the voltage is automatically raised to the preset value and maintained constant at that value regardless of supply-voltage or load variations.

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* *Journal I.E.E.*, 1934, vol. 74, p. 397.

In some applications, particularly in many heat-treatment processes and in the control of thermionic-valve filaments and anode voltages, it is essential that the applied voltage shall be low initially and be increased gradually to the final value. Means are provided automatically to return the regulator to the minimum-voltage position if and when the load is removed, and to ensure that it can only be switched on in that position. A control diagram of connections typical of these applications is shown in Fig. 27. Alternative automatic and remote electrical control using push buttons are provided, with a minimum-voltage interlock in each case. When the regulator has raised the voltage from zero to the specified value the automatic control maintains it constant at that value irrespective of supply-voltage variations.

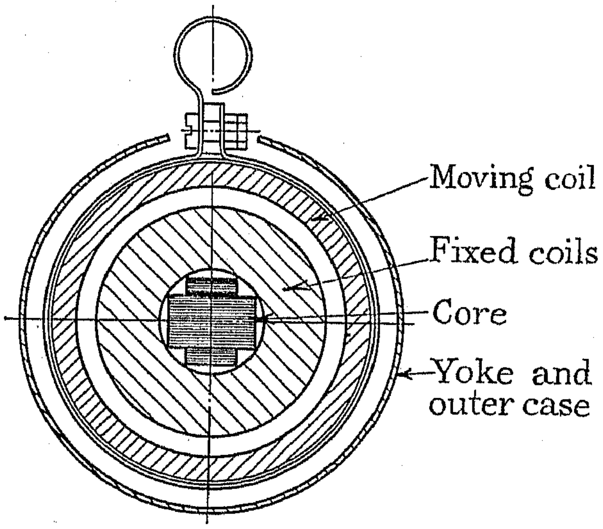

**Fig. 29.**—Cross-section of small tubular-type moving-coil regulator.

### (c) Types of Construction

The external construction of the regulator follows ordinary transformer practice. Most installations are fitted indoors, and in the smaller sizes natural air cooling and insulation are employed. As with the equipment described in the previous Section, where the regulators are oil-immersed, the operating mechanism is in general also placed under oil and the construction is generally similar to that shown in Figs. 18 and 19 (Plate 1) and Fig. 25 (Plate 2). A typical illustration of a small air-cooled hand-operated regulator suitable for general testing and laboratory work is shown in Fig. 28 (Plate 2).

The circular windings of the regulator permit its economical construction in very small sizes. Regulators of 15-VA output have been built. In these ratings there are many applications where a regulator giving a smooth and uniform voltage variation is a convenience or luxury rather than a necessity, and is only permissible where economically attractive. A construction especially developed for these small ratings is shown in cross-section in Fig. 29. The multi-yoke construction as described in Section 5(*b*) is replaced by a cylindrical yoke of sheet steel, which also forms the outer case of the unit. The regulator is therefore self-contained and similar in shape to ordinary wire-wound tubular-resistance rheostats. Fig. 30 (Plate 2) illustrates a typical regulator of this construction giving 18 % voltage variation in a 200-VA 230-V circuit.

## (8) CONCLUSION

The technical and constructional descriptions given in the paper show that whilst the moving-coil voltage regulator is similar in some respects to existing apparatus its major characteristics are distinctive both in theory and in construction.

It resembles the ordinary induction regulator in operation, in that it varies the voltage smoothly and not in steps; it can be left in any position and involves no switches. On the other hand, the moving parts are isolated electrically. Moreover, in design and construction it follows ordinary transformer rather than induction-motor practice. Since there are no windings in slots its limitations in regard to high voltages, heavy currents, and size, correspond to those for transformers of similar rating. Mechanical forces are low and mechanical clearances inherently large. This combination facilitates construction and permits simple automatic control apparatus.

Operating experience has been satisfactory and indicates that in comparison with apparatus hitherto available the new regulator has a number of peculiarly advantageous characteristics and none that is seriously objectionable.

## (9) ACKNOWLEDGMENTS

The author wishes to thank his colleagues for their co-operation in the development of the regulator, and, in particular, Mr. A. J. Glover for assistance in the electrical design and experimental work, and Mr. A. W. G. Tucker for supervision of the mechanical design and construction.

Thanks are also due to Messrs. Ferranti, Ltd., for permission to publish the paper.

## APPENDIX 1

### Theoretical Analysis

Consider the fundamental circuit of the moving-coil regulator shown in Fig. 1. The equivalent reactance of the windings $a$ and $b$ in series in the presence of the short-circuited winding $c$ may be written sufficiently accurately for the present purpose as

$$a + b = ac + bc \pm 2M_{ab}$$

where $ac$* and $bc$ are the leakage reactances between the windings $a$–$c$ and $b$–$c$ respectively and $M_{ab}$ is the mutual reactance between windings $a$–$b$ in the presence of the short-circuited winding $c$. If windings $a$–$b$ are connected in series opposition,

$$a + b = ac + bc - 2M_{ab} \quad . \quad . \quad . \quad (1)$$

$$a = ac - M_{ab} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$b = bc - M_{ab} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

---

* Note that $ac$, $bc$, etc., are symbols and not the products $a \times b$, $b \times c$, etc.

The value given by formula (1) is equal (again sufficiently closely for the present purpose) to the leakage reactance $ab$ between the windings $a$ and $b$. Hence

$$ab = ac + bc - 2M_{ab}$$

whence

$$M_{ab} = 0\cdot 5\,(ac + bc - ab) \quad . \quad . \quad . \quad (4)$$

Combining (4) with (2) and (3) gives

$$a = 0\cdot 5\,(ac - bc + ab) \quad . \quad . \quad . \quad . \quad (5)$$

$$b = 0\cdot 5\,(bc - ac + ab) \quad . \quad . \quad . \quad . \quad (6)$$

These equations are, incidentally, similar in form to the formulae for the individual-winding load reactances in multi-winding transformers. The actual values of the leakage reactances $ac$, $bc$, etc., will depend upon the position of the moving coil $c$, but if these are known for any given position the complete performance of the regulator can be calculated.

The formulae, and all others given in this paper, assume the same number of turns in each of the windings $a$, $b$, and $c$. If the turn ratio of windings $b/a$ is $N_1$ then the effective reactance of winding $b$ is $N_1^2$ times the value given by formula (6). The following formulae assume for simplicity that $N_1 = 1$, but the correction can easily be made where necessary. The number of turns in the short-circuited winding $c$ does not affect the calculations, but merely determines the current flowing in the moving coil for a given value of ampere-turns. If necessary, resistance can be included in the expressions for $a$ and $b$, and these values then become vector quantities. In general, however, resistance may be neglected without serious error.

With regard to the magnetizing current, the no-load impedance of the regulator is that of windings $a$ and $b$ in series opposition, and is obtained from formula (1) in the form:—

$$\text{No-load impedance} = a + b = ab \quad . \quad . \quad (7)$$

As regards the no-load voltage ratio, the simple diagrams in Fig. 1 show, in effect, auto-transformer connection of windings $a$ and $b$. The ratio of secondary to primary voltages is, therefore,

$$\text{No-load voltage ratio} = \frac{a}{a+b} \quad . \quad . \quad (8)$$

$$= \frac{ac - bc + ab}{2ab}, \text{ from (5) and (6)} \quad . \quad . \quad (9)$$

For the sandwich-winding construction shown in Fig. 14($a$) the reactance $ab$ is approximately equal to $(ac + bc)$ for all positions of the moving coil. Formula (9) then becomes

$$\text{No-load voltage ratio} = \frac{ac}{ab} \quad . \quad . \quad . \quad (9a)$$

Since $ac$ must be finite and less than $ab$, the minimum voltage ratio is necessarily greater than zero and the maximum less than unity.

In the concentric-winding construction shown in Fig. 14($b$) the reactances $ac$ and $bc$ are not bound by the reactance $ab$, and these limitations no longer hold. If the design is such that

$$bc \geq (ab + ac)$$

for the minimum-voltage position, the no-load ratio from formula (9) is zero or negative. Similarly if

$$ac \geq (ab + bc)$$

for the maximum-voltage position, the ratio is unity or more. A total voltage variation of 100 % or over is thus obtained.

## APPENDIX 2

### Calculation of Current and Voltage Values

In the following derivation of formulae giving the current and voltage values in any winding for any position of the moving coil and for any loading condition, small letters represent no-load values and capital letters the corresponding values on load. The symbols for the leakage impedances between the various windings have the same significance as in Appendix 1. It is assumed that the regulator is giving a variable output voltage over the desired range, the primary voltage remaining constant.

For the Type B regulator with boosting and bucking coils $n$ and $k$ as shown in Fig. 3, let $N$ and $K$ be the ratios of turns in coils $n$ and $a$ and in coils $k$ and $b$ respectively. Then

$$\mathbf{v}_n = N\mathbf{v}_a, \quad \mathbf{V}_n = N\mathbf{V}_a - na\mathbf{I}_2$$

$$\mathbf{v}_k = K\mathbf{v}_b, \quad \mathbf{V}_k = K\mathbf{V}_b + kb\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_n - \mathbf{V}_k = \mathbf{V}_1 + N\mathbf{V}_a - K\mathbf{V}_b - (na + kb)\mathbf{I}_2$$

$$\mathbf{v}_2 = \mathbf{v}_1 + N\mathbf{v}_a - K\mathbf{v}_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

whence

$$\mathbf{v}_2 - \mathbf{V}_2 = N(\mathbf{v}_a - \mathbf{V}_a) - K(\mathbf{v}_b - \mathbf{V}_b) + (na + kb)\mathbf{I}_2$$

But

$$\mathbf{V}_1 = \mathbf{v}_1 = \mathbf{v}_a + \mathbf{v}_b = \mathbf{V}_a + \mathbf{V}_b$$

Therefore

$$\mathbf{v}_a - \mathbf{V}_a = \mathbf{V}_b - \mathbf{v}_b$$

and

$$\mathbf{v}_2 - \mathbf{V}_2 = (N + K)\,(\mathbf{v}_a - \mathbf{V}_a) + (na + kb)\,\mathbf{I}_2 \quad . \quad (11)$$

The voltage drop $(\mathbf{v}_a - \mathbf{V}_a)$ is due to the internal impedance $\mathbf{Z}_s$ of the windings $a$ and $b$ to the load current. The equivalent load current is $(N + K)\mathbf{I}_2$ due to the auto-transformer effect of windings $n$ and $k$. Hence

$$\mathbf{v}_a - \mathbf{V}_a = \mathbf{Z}_s\,(N + K)\,\mathbf{I}_2 \quad . \quad . \quad . \quad (12)$$

Combining formulae (11) and (12) gives

$$\mathbf{v}_2 - \mathbf{V}_2 = \left[\mathbf{Z}_s\,(N + K)^2 + na + kb\right]\mathbf{I}_2 = \mathbf{Z}\mathbf{I}_2 \quad (13)$$

whence

$$\mathbf{V}_2 = \mathbf{v}_2 - \mathbf{Z}\mathbf{I}_2.$$

The arithmetic voltage-drop for any load $\mathbf{I}_2$ of power factor cos $\phi$ may be calculated from the usual transformer-regulation formulae, where $\mathbf{Z} = R + jX$ and the regulation is approximately $(R \cos \phi + X \sin \phi)I_2$.*

The load currents in the windings are derived as follows:—

$$\mathbf{I}_a = \mathbf{i} + \frac{\mathbf{v}_a}{\mathbf{v}_1}N\mathbf{I}_2 - \frac{\mathbf{v}_b}{\mathbf{v}_1}K\mathbf{I}_2$$

---

* The complete practical formulae are given in Appendix E of B.S.S. No. 171—1936.

where $\mathbf{i}$ is the no-load current in windings $a$ and $b$. From formula (7), $\mathbf{i} = \mathbf{v}_1/(ab)$. From (10),

$$\mathbf{I}_a = \mathbf{i} + \frac{\mathbf{v}_2 - \mathbf{v}_1}{\mathbf{v}_1}\mathbf{I}_2 \quad . \quad . \quad . \quad . \quad (14)$$

This is the same as for an ordinary static auto-transformer of ratio $v_2/v_1$. The primary current $\mathbf{I}_1$ is the vectorial sum of $\mathbf{I}_a$ and $\mathbf{I}_2$, i.e.

$$\mathbf{I}_1 = \mathbf{i} + \frac{\mathbf{v}_2}{\mathbf{v}_1}\mathbf{I}_2 \quad . \quad . \quad . \quad . \quad . \quad (15)$$

which is again the same as for an ordinary auto-transformer of ratio $v_2/v_1$.

The current in the short-circuited moving coil, assuming the same number of turns in coil $c$ as in coils $a$ and $b$, is

$$\mathbf{I}_c = \mathbf{I}_b - \mathbf{I}_a + N\mathbf{I}_2 + K\mathbf{I}_2 = (N + K)\mathbf{I}_2 \quad . \quad (16)$$

Since there is no imaginary term the moving-coil current is in phase with the load current, is directly proportional to it for all positions of the moving coil, and is practically zero at no load.

The load voltages across coils $a$ and $b$ are

$$\mathbf{V}_a = \mathbf{v}_a - \mathbf{Z}_s(N + K)\mathbf{I}_2, \text{ from formula (4)} \quad . \quad (17)$$

$$\mathbf{V}_b = \mathbf{V}_1 - \mathbf{V}_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The corresponding no-load values $v_a$ and $v_b$ may be obtained from formula (9), and the no-load value $v_2$ from (10).

Formulae for other winding arrangements are developed in a similar manner. It will be found* that formulae (11) to (18), for example, will be correct as they stand for the Type D connection (shown in Fig. 5), whilst for the Type C connection (Fig. 4) the term $(N + K)$ in the formulae should be replaced by $(1 + N - K)$.

## APPENDIX 3

## Experimental Confirmation of Regulator Theory

(*Received 20th October*, 1937.)

The theory and formulae developed in Appendices 1 and 2 though not rigorously correct are sufficiently accurate for all practical purposes. A comparison of calculated and measured values is given in the following tests, carried out on a small experimental Type B regulator. All of the coil connections were brought out to separate terminals so that measurements could be made of the voltage and current in each winding.

The no-load range of the regulator was 230/230–259 volts (i.e. a variable boost up to 12·5 %). The coil arrangement is shown in Fig. 3, except that coil $k$ is omitted as the voltage range involves boost only.

The leakage impedances between the various windings which determine the voltage ratio (see Appendix 1) are calculated from the coil dimensions, using ordinary transformer-reactance formulae. These values, together with the corresponding measured values, are given in Table 3. The equivalent impedances of each individual winding are calculated from formulae (5) and (6). For the maximum-boost position,

$$a = 0\cdot5\,(123 - 2\cdot96 + 120) = 120\cdot02 \quad . \quad . \quad (5)$$

$$b = 0\cdot5\,(2\cdot96 - 123 + 120) = -\,0\cdot02 \quad . \quad . \quad (6)$$

For the minimum-voltage or no-boost position the figures are reversed, giving $b = 120\cdot02$ and $a = -\,0\cdot02$.

The no-load voltage ratio $v_a/v_1$ from formula (8) is $120\cdot02/120 \simeq 1\cdot0$ for the maximum boost and $-\,0\cdot02/120 \simeq 0$ for the no-boost position. The impedance values $ab$, $ac$, and $bc$ were, in fact, so designed as to give zero and unity respectively for these ratios.

The ratio $N$ of the turns in coils $n$ and $a$ is $0\cdot125$, so that the calculated voltage ratio of the regulator from formula (10) is:—

$$v_2 = 230 + (0\cdot125 \times 230) = 258\cdot8 \text{ (maximum value)}$$

and

$$v_2 = 230 + (0\cdot125 \times 0) = 230 \text{ (minimum value)}$$

From formula (7) the no-load impedance is $(a + b) = 120$ ohms, giving a calculated magnetizing current of $230/120 = 1\cdot92$ amp. for any position of the moving coil. A no-load test on the regulator with 230 volts applied to the primary winding gave voltages of 230 to 259 with a magnetizing current of 1·93 amp. in each case.

In order to confirm the formulae for the load values of voltage and current derived in Appendix 2, the regulator was energized at 230 volts in the maximum-boost position and loaded on the secondary or output side with a practically non-inductive resistance load of 7·10 ohms, giving a load current of 36 amp.

The resulting load currents in the windings are calculated as follows:—

$$\text{From formula (14), } I_a = -\,1\cdot92j + \left(\frac{28\cdot8}{230} \times 36\right)$$

$$= -\,1\cdot92j + 4\cdot52 = 4\cdot91 \text{ amp.}$$

$$\text{From formula (15), } I_1 = -\,1\cdot92j + \left(\frac{258\cdot8}{230} \times 36\right)$$

$$= -\,1\cdot92j + 40\cdot4 = 40\cdot45 \text{ amp.}$$

The measured values were 4·95 and 40·8 amp. respectively.

The current in the short-circuited moving coil, calculated from formula (16), is $0\cdot125 \times 36 = 4\cdot5$ amp., assuming the same number of turns in coil $c$ as in coils $a$ and $b$. The actual numbers of turns in coils $c$ and $a$ are 350 and 560 respectively, so that the actual current in coil $c$ will be calculated at $(4\cdot5 \times 560)/350 = 7\cdot2$ amp. The measured value is 6·90 amp.

The voltages across the two fixed coils $a$ and $b$ are given by formulae (17) and (18). The impedance $Z_s$ is approximately the same as the impedance $bc$. Both reactance and resistance components must be considered, and these are calculated at $(2\cdot08 + 2\cdot01j) = 2\cdot89$ ohms. Hence, assuming unity power-factor load,

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* Formula (14) for $\mathbf{I}_a$ will, however, be replaced by formula (15) as $\mathbf{I}_a$ is the same as $\mathbf{I}_1$ for this connection.

$$V_a = 230 - (2\cdot08 + 2\cdot01j) \times 0\cdot125 \times 36 = 220\cdot64 - 9\cdot05j = 220\cdot7$$

$$V_b = 230 - 220\cdot64 + 9\cdot05j = 9\cdot36 + 9\cdot05j = 13\cdot0$$

The measured values were 221 and 13 volts respectively.

The drop in output voltage due to the load is calculated from formula (13). For unity power-factor load $na$ is the equivalent resistance of coils $n$ and $a$, calculated at 0·023 ohm. Thus

$$V_2 = 258\cdot8 - (2\cdot08 \times 0\cdot125^2 + 0\cdot023) \times 36 = 258\cdot8 - 2\cdot02 = 256\cdot80$$

The actual measured value was 257 volts.

The calculations show that the effects of load on the output voltage $V_2$ and of excitation on the primary winding currents $I_a$ and $I_1$ are small. The design values of load and no-load impedances were in fact chosen relatively high and low respectively in order to exaggerate these influences, so that the measured values would show more clearly the component terms of formulae (13) to (15) as well as the total values.

### Table 3

LEAKAGE IMPEDANCE BETWEEN WINDINGS OF EXPERIMENTAL REGULATOR: MAXIMUM-BOOST POSITION

| Windings | Impedance (ohms) | |
|---|---|---|
| | Calculated | Measured |
| *a–b* | 118 | 120 |
| *a–c* | 121 | 123 |
| *b–c* | 2·89 | 2·96 |
| *a–n* | 1·99 | 2·08 |

Calculations of the current and voltage values for other positions of the moving coil are carried out in a similar manner using the appropriate values of coil impedances.

## APPENDIX 4

### Critical Comparison with Some Other Types of Voltage Regulator

(*Received* 20*th October*, 1937.)

The principal methods of voltage regulation employed hitherto have been discussed briefly in Section (2) of the paper. The best-known types are classified in Table 1, whilst the principal operating and technical characteristics of voltage regulators in general have been summarized in Table 2. It was stated that for general application most regulators were of the induced-voltage type in which regulation is obtained by variation either of the magnetic flux or of the coil turns in an electromagnetic circuit. The outstanding examples of this type are respectively induction voltage-regulators and tapped transformers fitted with on-load tap-changing gear, i.e. variable-ratio transformers. A brief comparison of both these types of regulator with the moving-coil regulator described in the paper is attempted here in regard to the more important operating and design characteristics. Familiarity with the general theory and construction of all three types is assumed.

The moving-coil regulator resembles the ordinary induction regulator in operation in that it varies the voltage smoothly and not in steps; it can be left in any position, and involves no switches.

The induction regulator gives, inherently, values of buck and boost of any desired but equal magnitude by means of a reversible primary winding on the rotor supplied through flexible connections or slip-rings. The auxiliary coils in the moving-coil regulator permit equal or different values of buck and of boost. Section (3) of the paper shows that almost any desired range of voltage variation can be obtained. Moreover, the moving parts are isolated electrically as the moving coil is short-circuited on itself.

There are, in effect, separate primary and secondary windings for buck and boost, and these additional coils, including the moving coil, are an extra cost and in themselves tend to increase the relative proportions of copper to iron material. On the other hand, all the windings are single circular coils of the transformer type, free from the design limitations and labour cost of multiple windings in slots. For this reason also the practical and economic range of voltage and current ratings is greater than in the induction regulator, and is in fact the same as for the ordinary static transformer. There would seem to be no fundamental difficulty in building the moving-coil regulator for working voltages of 66 or 132 kV.

The use of interleaved cores of plain strip laminations instead of slotted stampings requiring dies, and the elimination of the rotor and stator construction with small air-gaps and mechanical clearances, are manufacturing and economic advantages. The relatively small mechanical forces and large travel referred to in Section (4)(*d*) have permitted considerable simplification in the control gear normally necessary for automatic operation. This is described in detail in Section (6)(*d*), but the comparison is shown graphically in Fig. 23. The combination of moving-coil regulator and astatic relay results in a fully automatic regulator requiring no periodical maintenance or adjustment. Since there are no open switches or contacts the whole of the control gear including the relay can be mounted under oil in the regulator, as in the street-pit and house-service constructions mentioned in Section (6)(*f*).

The foregoing considerations indicate the relative advantages of the two types of regulator, but for the reasons given in Section (2) a general comparison of costs is not practicable. This applies also to the electrical efficiency, since cost and losses are inter-related. In individual cases where efficiency is important it should be considered with price on a capitalized basis, as described in Section (4)(*c*).

In comparison with transformers having on-load tap-changing gear the most obvious distinction is the elimination of both the current-breaking series or transfer switches and the off-load tapping or selector switches. These switches, unless of the mercury-in-glass type, require periodical attention and renewal. This is not, in itself, either difficult or costly; but for regulators working in transmission and distribution systems maintenance, and frequently even simple inspection, involves isolation of the regulator or transformer with, in some cases, interruption to the supply. The combination of on-load tap-changing gear and Type E moving-coil regulator described in Section (3)(*b*) partly overcomes these objections by relieving the switches of the duty of rupturing any load current.

The smooth voltage variation given by the moving-coil regulator is not usually an important advantage, as tapping steps are seldom large enough to be objectionable; but the necessity for operating tap-changing gear in definite steps and ensuring that stopping in an intermediate position is impossible complicates the automatic control mechanism. The comparison in this respect is otherwise the same as for induction regulators, and is also shown typically by the diagram in Fig. 23.*

Comparisons of electrical efficiency depend upon whether the on-load tap-changing gear is fitted to the main step-up or step-down transformers or to separate booster or regulating transformers. In the former case the additional losses due to voltage regulation are relatively small and, in general, are lower than the corresponding losses in the moving-coil regulator. In the latter case there are load and no-load losses in the regulating transformer and sometimes also in the preventive auto-transformer or choke coil. The comparison is then more even but still indefinite, as in some instances a simple tapped auto-transformer booster suffices whilst in others shunt and series transformers are required, and the losses vary accordingly. Moreover, the inter-relation between cost and efficiency of both types of regulator makes comparisons impracticable except on a capitalized basis, as in the previous case. The total price thus obtained must be considered in conjunction with the operating characteristics just summarized.

## DISCUSSION BEFORE THE INSTITUTION, 3RD FEBRUARY, 1938

**Mr. A. G. Ellis:** The practice of moving coils relative to one another for the purpose of varying voltage and reactance is about as old as the art of transformer design itself, and the title of the paper should rather be "*A* moving coil regulator." One such form of variable reactor, with fixed line coils and moving short-circuited coils, was described by J. L. Thompson and myself in a paper† read before The Institution in 1918, and some years later there were described in the United States‡ a number of ways of varying the voltage. There was a windlass type where the windings were wound off from one drum to another, and also a type of regulator somewhat similar to that which the author has described, but differing in that it had two fixed coils and a moving coil connected in series with the line. It is better, of course, not to make the line coil the moving coil, and the author is quite right in making the short-circuited coil the moving one, as in the case of the reactors to which I have referred.

With regard to the principles governing the design of this particular type of regulator, it will be noticed from Fig. 12 that the flux has to pass a very wide air-space between the core legs. These large gaps are not usually sought after by designers; they make for a rather poor magnetic circuit and are not generally conducive to economy in design. Further, stray flux is a source of additional loss which adds to the input current and increases the load loss. The author does not give any loss figures in the paper, but presumably these losses are of a higher value than in the corresponding regulating transformer with tappings. Perhaps he would give us some figures in regard to this point. The magnetizing current would be excessively high unless the gap was made as small as possible, and such a gap means a very tall type of design (see Fig. 18); tall, thin coils are not the best as regards mechanical rigidity. Designers would, of course, like to find some kind of circuit which would obviate the introduction of these features, and they have not yet given up the quest.

With regard to the construction of the author's regulator, I should like to ask two questions regarding the forces on the coils. Firstly, regarding Fig. 13, why is the force curve not symmetrical about the centre? Secondly, referring to Fig. 17, what is the magnitude of the forces between the coils of the three phases under short-circuit conditions?

So far as power-distribution systems are concerned, the application of the regulator would, I presume, be limited to existing feeders of not too small capacity, where voltage regulation is subsequently required. I should think that where new transformers are being put down at a substation it would be more economical in cost and better as regards efficiency to provide on-load tap-changers on the transformers themselves, in preference to a non-tapped transformer with any kind of external regulator.

The problem of designing an economical and small automatic regulator for 5 to 10 kVA for house service is still unsolved. In the case of existing feeders of small capacity it is questionable whether a moving-coil regulator is a more economical proposition than a tapped auto-transformer with one of the simpler forms of tap-changing arrangement, e.g. mercury switches. The mercury-switch type of changer is not such a fearsome thing as Fig. 23(*a*) might lead one to suppose, nor is the arrangement shown in Fig. 23(*b*) so simple as it appears.

Operating experience with on-load tap-changers, now extending over many years, has demonstrated that maintenance is not a very serious factor; and the question of renewal of switch contacts is more imaginary than real, except in cases of very heavy duty.

I would ask whether the author contemplates using moving-coil regulators for such conditions as the 132-kV substations of the grid, and, if he considers them practicable, whether they would be more economic.

It is regrettable that the question of cost is rather glossed over in the paper. Such studies are the everyday work of the engineer; we have frequently had to make them when comparing on-load tap-changers with induction regulators, for example, and it should be practicable to compare a few typical cases as regards cost price, efficiency, and noise. This comparison, on the basis of cost, efficiency, and noise, could and should be made for the tapped transformer with tap-changer, the moving-coil regulator, and the induction regulator. Generally speaking, the induction regulator has a higher cost price and worse efficiency than an auto-regulating transformer, and this is one of the reasons why it has not been so popular in this country as in America, where very small induction regulators are made in quantities and used to a much larger extent.

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* Certain forms of small-capacity on-load tap-changing gear for use in distribution circuits have simplified automatic control—but the diagram in Fig. 23 is representative of general practice.

† *Journal I.E.E.*, 1919, vol. 57, p. 547.

‡ *Electric Journal*, 1928, vol. 25, p. 443.

In conclusion, it is my general experience that transformer apparatus with a large leakage flux and with any moving parts is more vibratory and more noisy than a normal transformer of the same size. In suburban installations even the modest hum of a standard transformer is sometimes made the cause of complaint, and it is therefore fortunate (see page 8) that the noise emitted by a moving-coil regulator is least in the night, when it would be most objectionable.

**Mr. J. K. Webb:** I should like to refer briefly to the behaviour of the moving-coil regulator on capacitive loads. The problem arises in applying life tests to lengths of power cable, where the large capacitive kVA involved is a source of embarrassment. In 1934 it was decided to replace an existing rotary set with a regulator. Trouble had already been experienced with the alternator of this set, owing to its tendency to self-excitation on loads of leading power-factor, and it was thought to be a good idea to specify a regulator with an abnormally large magnetizing current, so that some of the leading kVA of the capacitive load might be partly neutralized by the lagging kVA of the regulator, the net result being a small leading input current. Such an arrangement is known to work quite well in the case of an ordinary transformer. Conditions in the case of the regulator, however, soon turned out to be quite different; under all conditions of load a heavy lagging input current still resulted. The greater proportion of the load leading-kVA appeared to be absorbed by the secondary series leakage-reactance, and also the positive regulation was so great that the coil had only to be moved a small fraction of its total travel from zero to give full voltage output. Although the regulator was designed for 415 volts input, in many instances a great reduction of input kVA resulted from applying only 240 volts, while full output was still obtained on account of the positive regulation. I appreciate that with the new type of concentric-coil construction the leakage reactance can be considerably reduced, but I should like to ask the author whether such a regulator could now be designed with a large magnetizing current without necessarily increasing the leakage reactance, so providing an effective means of lagging- and leading-kVA neutralization. If this were not practicable, I believe that a variable choke could be designed, using the moving-coil principle, which might be connected in parallel with the capacitive load. Perhaps the author can suggest the best means of overcoming the difficulties which I have enumerated.

**Dr. W. Wilson:** It seems to me that the most important field for voltage regulators is in connection with power supply, and there the regulator has to compete with the on-load tap-changer to which Mr. Ellis has referred. It is true that the latter when first produced in this country was complicated, because the condition was laid down that once a tap-change had been started it must be finished, even if the power supply failed half-way through. The result was that flywheels, springs, or falling weights had to be provided to store up the necessary energy to enable the tap-change to be completed. Experience has shown, however, that this condition is not necessary, and that if the preventive choke can be designed with a continuous rating the flywheel or other storage device can be omitted. This simplifies matters very much. It removes the condition mentioned in the Appendix, which is really responsible for most of the complication; and the choice is then between on the one hand a piece of apparatus with a motor and a moving coil, and on the other hand one with a motor and a moving contact. At first sight, at any rate, one would prefer the moving contact. Further, if the dimensions of the two pieces of apparatus are compared, the on-load tap-changing transformer is of normal size, whereas the moving-coil apparatus has to be of larger proportions and rather inconvenient shape. There is no doubt that the on-load tap-changer has proved itself in the comparatively few years during which it has been in the field.

The author suggests that it is an advantage that the gear is immersed in oil, since it is certain to be lubricated. I am not quite certain about that, at any rate so far as transformer oil is concerned. When I made some tests on a somewhat similar piece of equipment, the friction turned out to be unexpectedly high. A special apparatus was designed to test the lubricating qualities of the oil, and it was found that whereas when cold a transformer oil undoubtedly had a certain small lubricating power, when it was hot it acted as an anti-lubricant, i.e. there was more friction than there would have been if it had been absent. Various additions to the oil were experimented with, in the endeavour to make it a lubricant at all points in the range of temperature, but without any particular success. Perhaps the author can give some information with regard to the oil used.

Fig. 12, which shows the flux crossing the air space, would seem to imply that the magnetizing current was fairly high, but I should like to know the value of the transient switching current. One of the chief difficulties in the use of balanced protective gear such as the McColl type for a transformer of this description is the possibility of there being a heavy switching-in current, which produces the same effect on the relays as an internal short-circuit, and in the past has been rather apt to trip the apparatus unnecessarily.

Finally, I should like to know how the remote signalling as to the voltage ratio is carried out with the author's moving-coil regulator.

**Mr. W. Burton:** In Section (6)(*a*) the author says that in general the required range of voltage variation is limited to 10 % or 20 % of the declared voltage. I should like to point out that such a regulator cannot deal with more than about 10 % of the original drop, because immediately it is put into service the regulator itself, being a kind of auto-transformer, increases the current on the input side, and, according to the particular value of step-up being dealt with, the initial drop is increased by the increased current. Therefore for an initial drop of 10 % to 12 % a regulator having a range of nearly 20 % is necessary; thus it is undesirable to adopt a regulator if the drop is already over 10 %. The 20 % range is also needed because it is impossible to say exactly what is the centre point of the range being dealt with; for instance, on one circuit we may need to deal with 11 000 volts ± 7½ %, and on another circuit with a mid-point voltage of 11 200 volts. In addition, we may at times wish to raise the voltage to more than the stated value; thus, if the stated value is 11 000 volts, we may want that 11 000 volts not at the regulator output position but ½ mile farther down the distributor.

Some very reassuring figures are given in the last paragraph of Section (6)(*a*). It is astounding that about 77 times full load may be carried by a regulator without damage; I do not think that the overload capacity of any "on-load" tap-changing transformer switchgear could compare with this. If such a short-circuit happened to occur just at the time the tap-changer was operating, I am afraid that a good deal of damage would be done.

We have in service a number of regulators of the type described in the paper; they have been in service for less than a year, and have been very satisfactory. I must admit that the losses are higher than those of other regulators, but against this must be offset the absence of maintenance charges. So far we have not had to carry out any maintenance work, and we shall probably not need to; there is nothing to maintain in the equipment except a relay and a few very simple motors.

I should like to join issue with the author when he says that high operating speeds are not necessary. One case in which a high speed would be very helpful is that where a heavily loaded feeder trips out while the boost is at a high value. In these circumstances it is important to bring down the boost rapidly to such a figure as will enable the feeder which has tripped out to parallel again with the other feeders, which in turn, as the result of taking over the load from the first feeder, have probably had their voltage reduced.

I am not in favour of the use of small regulators such as have been suggested for installation in houses. I think that we should regulate at key points only, and that the field for the use of small regulators in electric supply work is very limited. A foreign maker—I believe it is Ganz—has suggested that we need correction on lighting circuits only, and he proposes to install lighting regulators in houses and not to bother about the heating and cooking load. Voltage regulation is perhaps less important so far as heating is concerned, but I am sure that it is necessary to have some regulation of cooking circuits if we are to compete with the gas interests.

The author is in favour of installing voltage regulators along the distributor; but I should like to point out that each regulator installed increases the current on the input side, and therefore the voltage drop. If several regulators are put along a feeder, a stage will probably be reached where the drop is so great that the system becomes unstable; it is easy to show that in the case of an overhead 0·06-sq. in. feeder, 1 mile long, carrying only about 70 amp., that stage is reached very quickly. It may be said that for the standard case this does not really matter, but a few years ago many suppliers had a trying time—from the engineering point of view—when every consumer switched on all the heaters he could find, because the weather was so cold. We must cater not only for the average load of to-day but also for the emergency case. I suggest that it is much better to have high-voltage supplies rather than a multitude of voltage regulators to cater for the doubling or even trebling of the load, which frequently happens in a cold snap or when a big works is working overtime.

**Mr. J. B. Morgan:** Transformer tap-changing gear is most reliable and fulfils its function perfectly on a transformer which is feeding on to busbars supplying a network, but it is not so satisfactory as the voltage regulator in districts which are partly urban and partly rural and where the situation often exists of a substation feeding three or four long distributors not in any way interconnected, with the bulk of their load at the far end and carried on only one or two of the distributors. I have had in use for some years several voltage regulators of the type described by the author, and they have worked marvels so far as the engineer and his relations with the consumer are concerned.

Where such a regulator is used on a distributor a balancer is necessary. I have in mind one distributor, 0·75 mile long, fed at one end, with a load of only 30 % of its 1 000-amp. current rating. It was satisfactory while it was feeding lighting circuits only, but the advent of a cooker campaign speedily brought the voltage down from about 230 to about 210 volts, with the result that complaints were received from the consumers. The cure was applied in the shape of a balancing transformer at one end and a voltage regulator at a prescribed distance along the distributor; now, under any or all conditions of load we get less than 2 volts variation.

Some distribution engineers do not fully realize the value of a supply voltage which remains perfectly steady at the declared figure. I carried out some interesting experiments a short time ago on the tap-changing gear of our grid transformers, and I found that raising or lowering the primary voltage by 1¼ % made a difference of 4½ % in the total output. This result suggests that the closer we keep the voltage to the declared value the better it will be, not only for our pockets but also for our peace of mind when we are dealing with the consumer.

Regarding the question of noise, my voltage regulators are mounted on the roadside and it is impossible to hear any noise from them at any time of the day or night. They have worked perfectly night and day for 2–3 years without any attention whatever. In the case of long distributors where the load is not large enough to justify running a high-voltage cable and putting in a transformer, the use of such a regulator as those described in the paper is a valuable stop-gap.

I have connected a balancing transformer across every one of my mains, whether equipped with an l.t. voltage regulator or not, because voltage curves taken on one main without a balancing transformer showed that I was getting a 19-volt drop. A balancing transformer brought this down to about 10 volts under all conditions of unbalanced load on cookers, and a voltage regulator reduced it to less than 2 volts.

**Mr. H. Diggle:** It is of interest to note that a regulator based on the principle of varying the inductances of two coils, connected in series across the supply voltage, was described in a Continental technical periodical in 1908.* This regulator (Fig. A) reproduces the conditions of the author's Fig. 3 with the coil *k* omitted as it is designed to give boost only, but the method of variation of the relative inductances differs from that of the author.

The author's regulator should, I think, be included in Class 1(*b*) of Table 1, as in its simplest form it behaves exactly like an inductive potentiometer in which the moving coil takes the place of a sliding contact. It has the usual disadvantages of potentiometers in that there is no direct transformation effect and the no-load current must necessarily be chosen at a high value to avoid high voltage-drop on load. For example, in the regulator referred to in Appendix 3 the no-load current is 1·92 amp. compared with a winding full-load current of 4·52 amp., i.e. 42 % as compared with approximately 5 % in a standard transformer regulator. When the regulator is used in auto-transformer connection to give 10 % or 15 % boost, this is not important, but it would be of account in the double-wound arrangement of Fig. 5 and should be considered in assessing the virtues of simplicity of operation of the regulator.

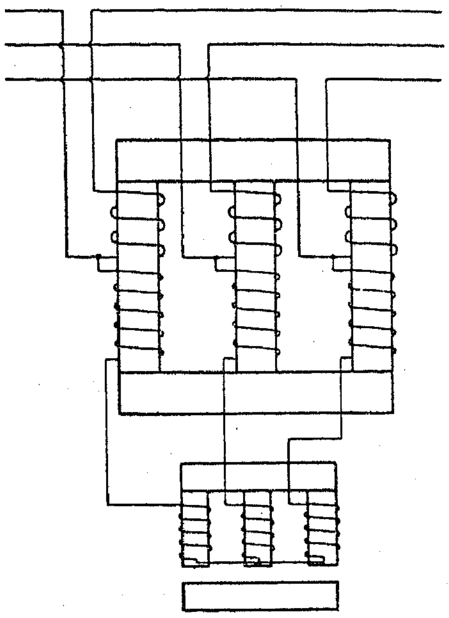

Fig. A

On page 3, reference is made to the fact that the reactance ratio of the coils *a* and *b* in the extreme positions of the moving coil is 40 : 1. In one of the author's slides a voltage variation of 95 in one coil to 5 in the other was shown, however, and earlier published data indicate ratios of about 20 : 1. I should be obliged if he would indicate whether the higher figure applies to the new arrangement of concentric coils.

In some of the patent specifications for the regulators described in the paper, reference is made to compensating coils in the sides of the middle limb of the core. No mention is made of these in the paper, and I should like to know whether they have been found unnecessary.

The inherent regulation of the author's regulator would appear to be rather poor except in the maximum-voltage position, and I should like to ask whether the regulation curve of Fig. 8 with varying lagging power factors of load will not droop very much more in, say, the position of half boost.

According to the published reference, the oscillogram of the short-circuit test shown in Fig. 20 applies to tests made only on a small single-phase regulator of about 6 kVA physical frame size. I should like to know whether any similar tests have been made on a large 3-phase regulator of the type shown in Fig. 17, and whether the pull between the three rather long centre legs has caused any damage.

In Appendix 4 the author mentions that this type of regulator could be made for 132-kV circuits. What is the highest voltage class and largest regulator kVA—i.e. physical frame size—for which this type of regulator has been made?

Despite its inefficiency, the simplicity of the induction-disc motor warrants its use on small units, but I think it is carrying the idea to an extreme to put 8 or 10 such motors round the periphery of a large gearwheel. The conventional induction motor with gearing would appear to be the sounder engineering job, and the latter type of motor has to be used when rapid movement is required. When a large number of small motors is used, a contactor has to be interposed between the voltage relay and the motors —as witness Fig. 27—so that the comparison of Fig. 23 is somewhat exaggerated, as the author's regulator will then have another link in the chain. The footnote to Appendix 4 shows that some small tap-changers also need only have three or four links in the chain.

Referring to Mr. Burton's remarks on the desirability of quick operation of a regulator to compensate for sudden changes in voltage-drop due to changes in load, any such advantage has to be balanced against the extra wear and tear on the apparatus. In the discussion of my recent paper on "Transformer On-Load Tap-Changing Gear"* reference was made to some tests in America which had proved fairly conclusively that for normal power systems no practical advantage results from very rapid response of the regulator.

While agreeing with the author that there is some tendency for the regulator to be put nearer to the load, I think this is chiefly in the case of long distributors. It is still generally rare in this country to have the regulating means nearer to the load than the distribution transformer. Tap-changing directly on the transformer has the advantages of higher efficiency and of no floor space being required for the regulator. The tap-changer admittedly involves contact maintenance, but this, as the author points out, is not a serious matter on modern equipment. A method giving a smooth voltage-change is rarely a practical advantage over tap-changing if the steps are made sufficiently small.

**Mr. R. C. Mildner:** The moving-coil regulator has certainly effected a revolution in the design of cable-testing sets, particularly for use with accelerated ageing tests, where the set is in continuous operation and where maintenance is a very serious item.

We have used regulators of the type described in the paper since soon after they were introduced, and have had them in operation for several years under quite heavy loading; we have had no more trouble with them than one expects from a transformer. The losses with the later type of regulator, and particularly in the case of the larger units, are quite reasonable. The older type of construction did not permit good neutralization of the rather large lagging current, but the later concentric designs operate very satisfactorily; and, furthermore, the positive voltage regulation on capacitive load is reduced to a reasonable amount.

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* E. Siedek: *Elektrotechnik und Maschinenbau*, 1908, vol. 26, p. 981.

* *Journal I.E.E.*, 1937, vol. 81, p. 330.

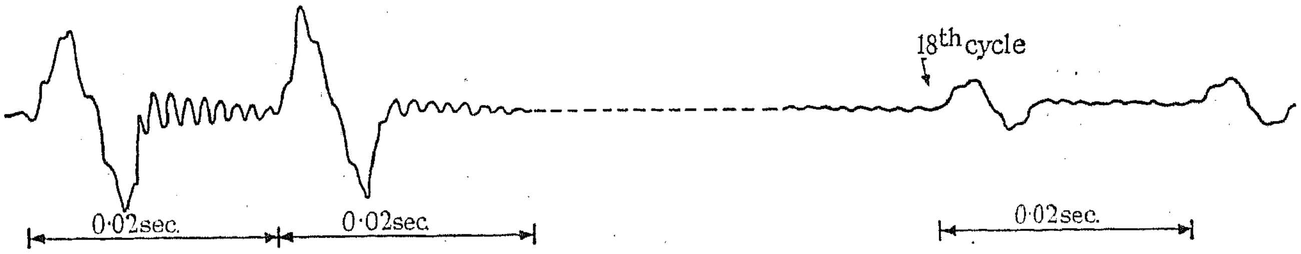

**Fig. B.**—No-load conditions: initial cycle.

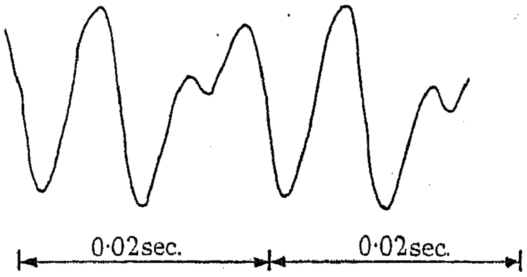

**Fig. C.**—Capacitance load, $0 \cdot 046\ \mu F$ on secondary.

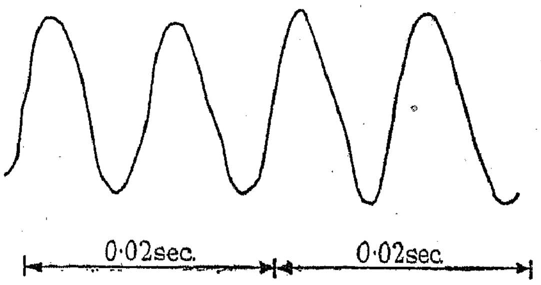

**Fig. D.**—Capacitance load, $0 \cdot 071\ \mu F$ on secondary.

One objectionable feature of the author's regulator which we have detected relates to the switching surges which are impressed upon the load, even when the regulators are in a position to give zero voltage on steady load. The amplitude of these surges depends upon the point of the impressed voltage-wave at which contact is made, and also upon the nature of the load. The surges consist of a train of waves of continually decreasing amplitude, and persist for about 100 cycles (or 2 sec.).

I should like to refer to some tests carried out on a 500-kVA moving-coil regulator which is connected to the 6-kV mains and feeds a transformer (Test-set B) which steps up from 6 kV to 150 kV. Fig. B shows the initial wave-form and also the wave-form of a later cycle. On no-load the switching surge consists of a double-frequency wave in which the alternate wave is suppressed, and superimposed on it is a higher-frequency oscillation of about 1 000 cycles per sec. As the surge progresses, the initial double-frequency wave is apparently reduced in periodic time. What is the explanation of this? Figs. C and D show the effect of increasing the capacitive load.

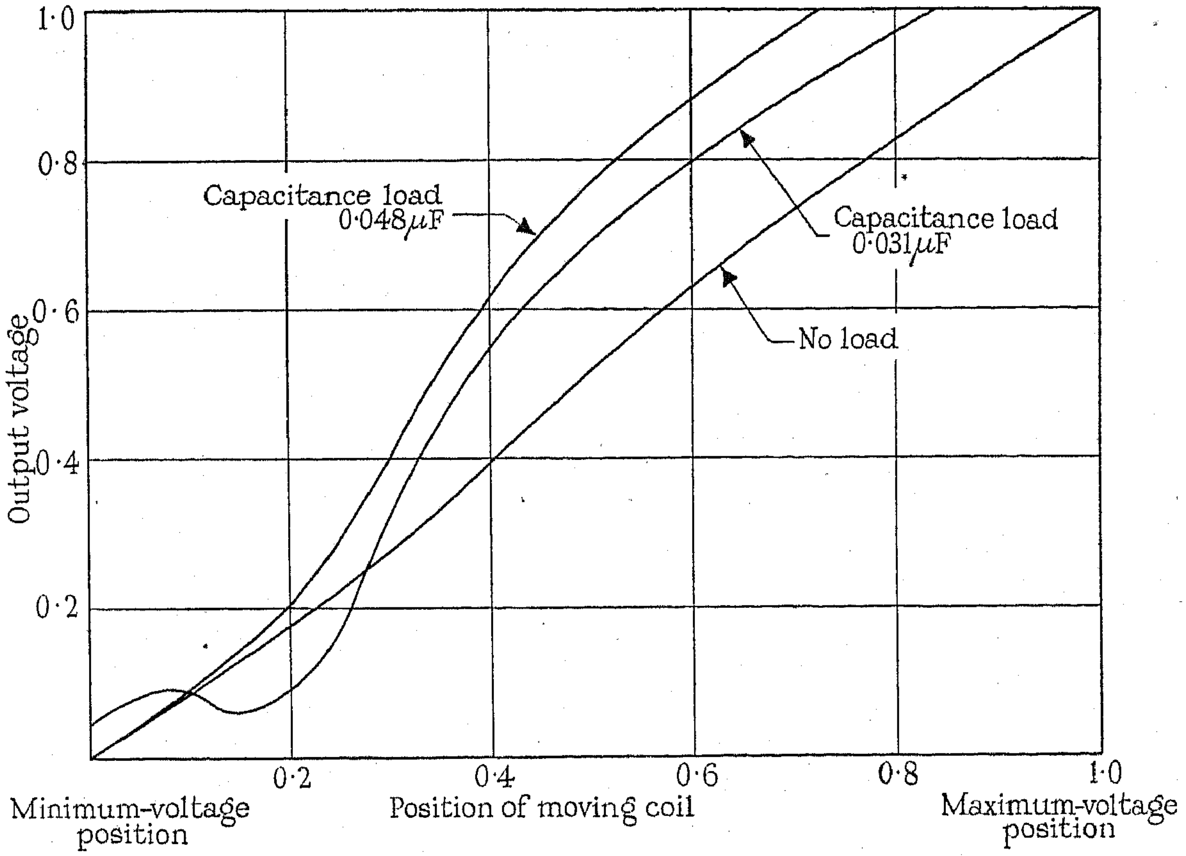

**Fig. E.**—Voltage regulation of cable test-set.

On full load the higher-frequency harmonic disappears, and we are left with approximately a double-frequency wave which extends for about 2 sec. in all. The amplitude of this surge on open-circuit may amount in a severe case to about 25 % of the impressed voltage, and on a capacitance load we have obtained records showing a peak voltage of rather more than 50 % of the impressed voltage.

In conclusion, I should like to refer to Fig. 7, in which the author shows a linear variation of voltage with change of position of the coil. We can confirm his results, but it should be pointed out that the effect of capacitance loading may change the distribution a good deal (Fig. E). With capacitance loading there is a very curious effect, which occurs at approximately half load; perhaps the author can offer an explanation for the phenomenon. There is an initial rise of voltage followed by a recession, and of course the full voltage output of the regulator is obtained without reaching the top position of the coil.

**Mr. N. E. G. Hill:** I also am interested in moving-coil regulators from the point of view of cable testing, and I have had a good deal of experience of a cable-testing plant using four 150-kVA moving-coil regulators.

These regulators were designed to have a very high no-load current, of substantially zero lagging power-factor, with the object of neutralizing part of the leading current taken by a cable under test, and thus reducing the out-of-phase current load on the supply. It was expected that the input current to the regulators would be sometimes lagging, sometimes nearly zero, and sometimes leading, depending upon the capacitance of the cable under test. These expectations were not fulfilled, for in practice it was found that the input current was lagging under all conditions of load, and that with a very heavy cable load the input current was increased above the no-load value. In addition, the voltage regulation was very bad, and it was no uncommon experience to get full voltage on the output side when the moving coil was as little as 20 % from its zero position. The control of voltage was thus made difficult and even dangerous on heavy load.

I found, by mathematical analysis of the circuit, that the leakage reactance of the regulators, which is effectively in series with the load, could form a series resonant circuit with the cable under test. The leakage reactance of the regulators was so high that resonance at the fundamental frequency occurred within the working range of cable loads. This resonant condition due to high leakage-reactance affords a complete explanation of the input-current and voltage-control phenomena already mentioned. The practical effect was that not more than half of the total kVA rating of the h.t. transformers could be used without the danger of a resonant rise of voltage on the cable.

This was a serious difficulty. My own solution was to use the moving-coil regulators as fixed inductances in parallel with the primaries of the transformers. The moving coils were adjusted so that the reactance of the regulators was such that their lagging kVA exactly neutralized the leading kVA taken by the cable. Thus the effective load was a pure resistance taking sufficient current to make up the very small losses in the circuit. The voltage was controlled by a small auxiliary induction-regulator on the input side. This arrangement worked extremely well. With the full 600-kVA cable load, the input to the induction regulator was about 60 kVA at 0·9 lagging power-factor.

It is very desirable, however, to use the moving-coil regulators as direct means of controlling the voltage rather than as chokes adjusted to suit the load. It is evident that resonance will occur in any cable-testing plant under some conditions, but the point of resonance must lie outside the working range. This would have happened in the present case if the leakage reactance of the regulators had not been extremely high. I should like to ask the author whether it is possible to design a moving-coil regulator, suitable for cable testing, with a very small leakage-reactance and at the same time a fairly large no-load current to neutralize the capacitance current of the cable.

There is a further objection to high leakage reactance. If there is any impurity in the input voltage wave, harmonic resonance may occur and, if the leakage reactance is high, the voltage wave will then be badly distorted. This is an important matter in cable testing.

If the high leakage reactance can be reduced to a low value the moving-coil regulator will be very attractive for cable testing because of its smooth, uniform, and continuous control of voltage.

**Mr. W. R. Cooper:** I should like to ask the author about the use of the moving-coil voltage regulator for regulating low voltages and very heavy currents. There is a great deal of difficulty in regulating the voltage of the heavy currents which are used in electric furnaces in steel-works for smelting high-carbon steels and special steels, as the current varies so much—from zero on open-circuit to a dead short-circuit, when thousands of amperes will pass. Some types of voltage regulator have been used, but not with very much success, as there is great difficulty in operating rapidly to follow the changes in current. I think that in the case of the 3-phase electric furnace the function of the regulator would be to balance as far as possible the currents in the three phases. Similar remarks apply to a smaller degree in electric arc-welding.

**Mr. W. E. M. Ayres:** My remarks will be directed to some of the statements made in the paper, and to some of the features of the particular form of differential flux-regulator described.

Referring to Section (4)(*c*), it is easy to see why the losses in this type of regulator are relatively high, quite apart from stray losses, if one examines the theory of operation as shown in Fig. F. Diagram (*a*) represents the full-boost position, and the right-hand half of the combination is a complete transformer for full kVA. Instead of the copper losses being due to this active portion, and proportional to $2I_n$, they total $6I_n$. Diagram (*b*) represents the neutral position, and here the losses are proportional to $4I_n$. It is worth noting that in all positions of the moving coil the ampere-turns in this short-circuited member are $2I_n$.

I would point out that some of the statements in Section (4)(*d*) are very misleading. In the induction regulator the small air-gap is not varied and does not come into the calculation of operating forces. A regulator of 18 in. rotor diameter has a travel of 180° or 28 in., which is comparable with that of the moving-coil type. It is also quite meaningless to compare the forces required by the motor power provided, without taking into account gear-ratio and time-to-travel. The moving-coil regulators described are very slow in operation (I have timed some to 12 minutes for complete travel) and hence may be worked by a disc motor. The mechanical forces on the moving coil are given by the product (Ampere-turns × Flux), and the author mentions that the flux path is mainly air; therefore the saturation effect does not enter into the problem, which means that the stresses are proportional to the current squared (the quantity always used for calculating mechanical stresses on normal transformers). The author states elsewhere that in certain positions the coil weight is counterbalanced by the upward forces at normal load. A regulator of about 10 kVA which has a moving coil of about 20 lb. weight may therefore suffer a hammer blow of $20 \times 25^2 = 12\,500$ lb. due to a short-circuit current of 25 times full load; such a current is not abnormal. An induction regulator of 10 kVA with an 8-in. diameter rotor has a maximum torque of 23·5 lb.-ft. at full load, corresponding to a force of 70 lb. at the periphery. Owing to the effect of saturation, however, this peripheral force only increases to 2 800 lb. with a short-circuit current of 25 times full load, and the coils are firmly embedded in slots. The author's attempted comparison is therefore very inaccurate, and unfavourable to the regulator described.

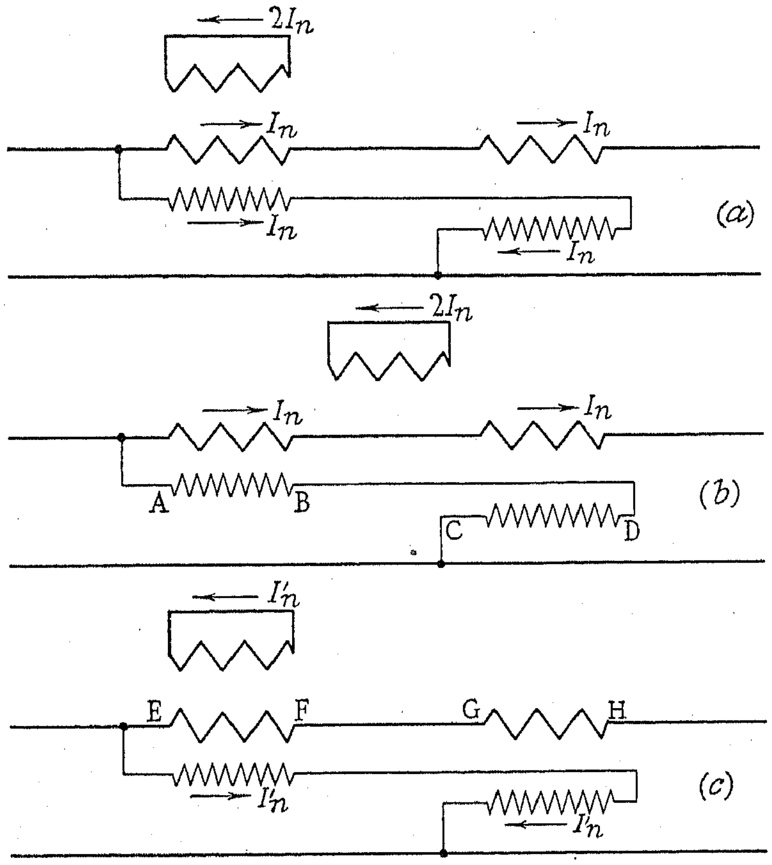

**Fig. F**

(*a*) Full boost, on load. (*b*) Neutral, on load.
(*c*) Full boost, no load.
$I_n$ = ampere-turns in one winding.
$I'_n$ = magnetizing ampere-turns.
No current flows in AB, CD, EF, or GH.

In Section (6)(*b*) he states that voltage phase-displacement is of importance, but then proceeds to limit his definition of this term. The only phase displacement which matters is the angle between the incoming and outgoing voltages, however produced. Electrical networks are unable to discriminate between a phase displacement due to connections or inherent characteristics and a phase displacement brought about by reactance in the connected apparatus. The concentrically wound moving-coil regulator is a very great improvement on the original sandwich-coil type, but still has a very appreciable angle at full load; moreover, this angle all represents reduced power factor on the system, whereas with induction regulators the voltage and current angles are shifted simultaneously without the system power-factor being affected.

In formula (16) the author shows that the current in the moving coil is zero at no load. Referring to Fig. F(*c*), showing the no-load full-boost conditions, it is seen that ampere-turns equivalent to the magnetizing current must flow in the moving coil, and as, according to Appendix 3, this is 42 % of full-load current, I cannot consider it negligible. Similarly, the difference between the calculated value (7·2 amp.) and the measured value (6·9 amp.) of the current in the moving coil is not a slight discrepancy, and is certainly not due to the impedance of the ammeter circuit, which could only put up the voltage across the moving coil without altering the ampere-turn equilibrium on the regulator limb. This difference is due to a reactive component of $\sqrt{(7{\cdot}2^2 - 6{\cdot}9^2)} = 2{\cdot}04$ amp. When the whole analysis is based on an approximation of an approximation (as admitted in the beginning of Appendix 1) one may obtain very simple formulae and eliminate inconvenient quantities, but no accurate solution of the performance is obtained.

**Mr. E. V. Clark** (Australia) (*communicated*): The advantages of the moving-coil regulator described by the author are many; but it is a pity that the questions of cost, weight, and efficiency, should have been dismissed so cursorily. Typical figures for one or two regulators of specified capacity would be of interest, since it seems that in weight and in efficiency it will compare unfavourably with regulators of other types. For as the regulator acts as a transformer with wide air-gap, it must be bulky for the amount of flux carried; and as regulation is effected by neutralizing the magnetic effects of current in one coil by that induced in another, and in Types B, C, and D (Figs. 4, 5, and 6) we have coils *n* and *k* which at all times carry full output current but are never both fully operative in generating e.m.f., it is evident that both the amount of copper and the copper losses are on the high side. It certainly appears, therefore, that the moving-coil regulator will find its greatest field of use in the smaller sizes, where simplicity of operation and low maintenance costs may outweigh internal losses, and where the elimination of costly accessories may make up for the greater cost of iron and copper.

In contrasting the original type, shown in Fig. 14(*a*), with the newer type, shown in Fig. 14(*b*), it is of interest to note that the original type of regulator should work reasonably well if the moving coil is a solid ring of copper without insulation, though doubtless eddy currents therein would make such construction inadvisable except in small sizes. With the newer type, on the other hand, very poor results might be anticipated if the moving coil consisted of a solid sleeve of copper or aluminium; a wire-wound moving coil seems essential.

Two modifications of the present design suggest themselves. Firstly, just as the multi-yoke construction has been found superior to the double-yoke shell type, so one might expect that a design upon the lines of the Berry transformer would be superior to the 4- and 3-yoke forms adopted. Such a design should appreciably increase the equivalent cross-section of air-gap and assist in keeping the lines of flux in the planes of the laminations, with reduction in stray eddy-current losses. Space for the operating mechanism of the moving coil could be provided without elimination of much of the iron from the Berry design. Secondly, there seems no reason, other than difficulties in construction, why the stationary coils *a*, *b*, *n*, and *k* should not be wound in slots on the central core, so that the length of air-gap would be merely that necessary to accommodate the moving coil. Manufacturing difficulties would no doubt be considerable; but with Types B, C, and D connections it should be possible to reduce the length of air-gap to less than half its present figure; and in regulators of large size the consequent saving in materials might justify the greater complexity of manufacture.

The statement on page 8 that in the 3-phase type ' The fourth yoke of each unit can be omitted, as the 3-phase fluxes cancel out " seems hardly warranted by the diagram shown in Fig. 16, since the top and bottom end-pieces of the missing yoke are also omitted, and no iron is shown in the central core in position to carry flux that such a yoke would convey to it. Any appreciable quantity of flux returning in this manner, at right angles to the laminations, would surely add materially to the eddy-current losses in the core. However, if the Berry type of construction were adopted it would probably be advantageous, technically if not commercially, to cross-connect the three cores, top and bottom, with laminations to permit of the use of this imaginary central core. Unless the three units could be brought closer together than in Fig. 17, however, it would probably be found desirable to provide a stack of horizontal laminations in the central space to assist the flux back from the core of one phase to that of another.

[The author's reply to this discussion will be found on page 30.]

## SCOTTISH CENTRE, AT GLASGOW, 8TH FEBRUARY, 1938*

**Prof. S. Parker Smith:** I should like to show in actual operation two moving-coil regulators, of the type described by the author, which have been purchased for the laboratories of the Royal Technical College.

The extensive terminal boards of these regulators are naturally specially designed for testing purposes. The general feature to note about these regulators is their lightness in comparison with the induction regulator. It must be remembered, however, that much of the weight of the induction regulator is due to its cooling device; of course, it would not be possible to build an induction regulator like the moving-coil regulator, which is comparatively a skeleton.

The object of installing these regulators in the laboratories here is to enable students to control the supply voltage in accordance with the principle that every group of students must be able to conduct its experiment without interruption. Consequently, throughout these laboratories there is a large number of motor-generators and voltage regulators of all descriptions.

**Mr. W. Sutcliffe:** More than 40 years ago the late Dr. Ferranti conceived the idea of a moving-coil constant-current regulator; this was applied to his rotating synchronous rectifier, employed principally for the rectification of alternating currents for series d.c. arc-lighting circuits, and was then regarded as a definite improvement on the old Brush arc-lighters which previously had enjoyed a measure of popularity. It is pleasing to find that Dr. Ferranti's old idea of a moving coil for regulating purposes has not been entirely forgotten; on the contrary, it seems to have been actively followed up by the author's company and applied in a different and much improved form to the more modern problem of voltage regulation.

The importance of close voltage-regulation is now appreciated by supply engineers, not only from the point of view of avoiding complaints from consumers, but also from the equally important one of revenue; it is not always realized that consumption in terms of kilowatt-hours falls as the square of the voltage, and that a drop of, say, 5 % at the consumer's terminals means a reduction of nearly 10 % in the units registered.

Turning to the author's design of voltage regulator, the first reaction to the suggestion of a moving coil on the limb of a transformer may be one of surprise, and possibly concern, especially to engineers who, not unmindful of short-circuit stresses, emphasize the importance of robust coil-supports and bracings in transformers. It is true that these requirements refer to fixed primary and secondary windings, whereas the author's moving coil in the ordinary sense is neither, being isolated and at earth potential; none the less, under short-circuit conditions the coil must be subjected to mechanical stresses of considerable magnitude, and these must necessarily be borne by the rather slender-looking screwed supporting shafts.

Fig. 13 indicates that maximum stress occurs when the coil is in the 0·2 position, but unfortunately no values are given to the ordinates of the curve, and therefore no comparisons can be made; it would be interesting if the author could state an approximate figure for the mechanical stress under short-circuit in the moving coil of a 250-kVA regulator, assuming the coil in its worst position, and a plant capacity of 50 000 kVA with total impedance of 20 %.

On page 7 the author refers to the difficulty of expressing quantitatively the mechanical forces in the moving-coil regulator, but he implies that, since the latter can be driven under normal loading conditions by one or more fractional-horse-power motors, the forces must be of a low order; this is rather misleading, because the motor power required to move the coil is governed not only by the resisting force but also by the rate of travel. In any case, the power required to operate the regulator has little bearing on the mechanical stresses imposed during short-circuit conditions.

The author occasionally makes comparisons between the moving-coil regulator and the induction regulator, rather in favour of the former; but it should be remembered that hundreds of the latter are in successful operation all over the country, with no more noise or vibration than is associated with a normal transformer.

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* The meeting was held at the Royal Technical College, Glasgow, where, by the kindness of Prof. S. Parker Smith, various types of moving-coil regulators were on view.

Turning to page 10, the phase shift of 6° introduced by a single induction regulator for ± 10 % voltage variation when in the mid-position can be neutralized by adopting the twin design, as is invariably done for interconnectors, or other cases where phase displacement is undesirable. Is the author quite sure that phase shift is entirely absent in the moving-coil voltage regulator under all conditions of loading?

The value of the paper would have been enhanced had the author given the relative costs and internal losses of the moving-coil voltage regulator and the induction regulator.

**Mr. A. P. Robertson:** One of the advantages which the author claims (on page 8) for the moving-coil regulator is the almost complete absence of noise. The demonstration model is not quite noiseless but is very nearly so. I do not support his argument that so far as residential areas are concerned it is desirable that the noise should increase with load "as during the night the load is usually small and noise emission will then be a minimum at a time when noise is most objectionable." In residential districts the load as a rule is heavier in the evening, when people are at home, and those who live next door to a substation will be inconvenienced. If, however, the regulator is installed in a works instead of adjacent to a residential district, then the noise will be less in the evenings and will not be likely to trouble residents.

I should like to know whether the noise emitted is more or less when the regulator is contained in a tank. The author says that in certain circumstances special precautions are taken to minimize noise. I should like to know what these precautions are. A low flux-density is conducive to silence.

On page 10 it is stated that it is not possible to use the neutral point of the regulator for loading purposes on an unbalanced load, but that static balancers are required (separate from the regulator, I presume). My experience of static balancers has not been happy; but if it is necessary to install a static balancer what should be its magnitude, and can it be incorporated in the regulator itself, inside the tank or adjacent to it?

I should like to know what are the losses of the moving-coil regulator when it is in the maximum position.

Referring to the regulator for laboratory work which was demonstrated by the author, does the long travel of the coil affect its ability to deal with quick fluctuations of voltage?

What is the time-lag on the author's initial relay system? I should also be interested to have details of the sizes, losses, and prices, of moving-coil regulators. I am particularly interested in a regulator of 50 kVA rating, giving 20 % regulation.

**Prof. M. G. Say:** The moving-coil voltage regulator seems to have all the essential features of a true invention: it is a simple and effective application of known electromagnetic principles. Possibly in respect of losses, magnetizing current, and efficiency, the device is not quite so good as the induction regulator (but I should like the author's views on this point): on the other hand, it has obvious advantage in regard to construction and operation.

The mercury switch is employed in the control of the regulator because it can be operated by the relay directly. Would it not be possible to employ for this purpose switches of the micro-gap type? These work with quite small forces, and are capable of breaking 15 amp. at 230 volts, 50 cycles per sec., with a gap of only a few mils. They are used in thousands on cheap and simple thermostats.

I should be interested to know the order of the maximum flux-density used in the cores. It has to be low, presumably, because much of the magnetic-flux path is in non-magnetic material. Is there any objection to the use of a plain copper sleeve for the moving coil?

The moving-coil regulator in miniature sizes should prove very useful in the laboratory for breaking down constant supply-voltages. The objections to the usual resistance potential-divider are heat, loss, and poor regulation. I suppose that the regulator would have a much smaller loss and would in consequence develop less heat, but would the regulation be any better? Would the output voltage for a given setting of the moving coil remain constant within a few per cent between no-load and full-load output?

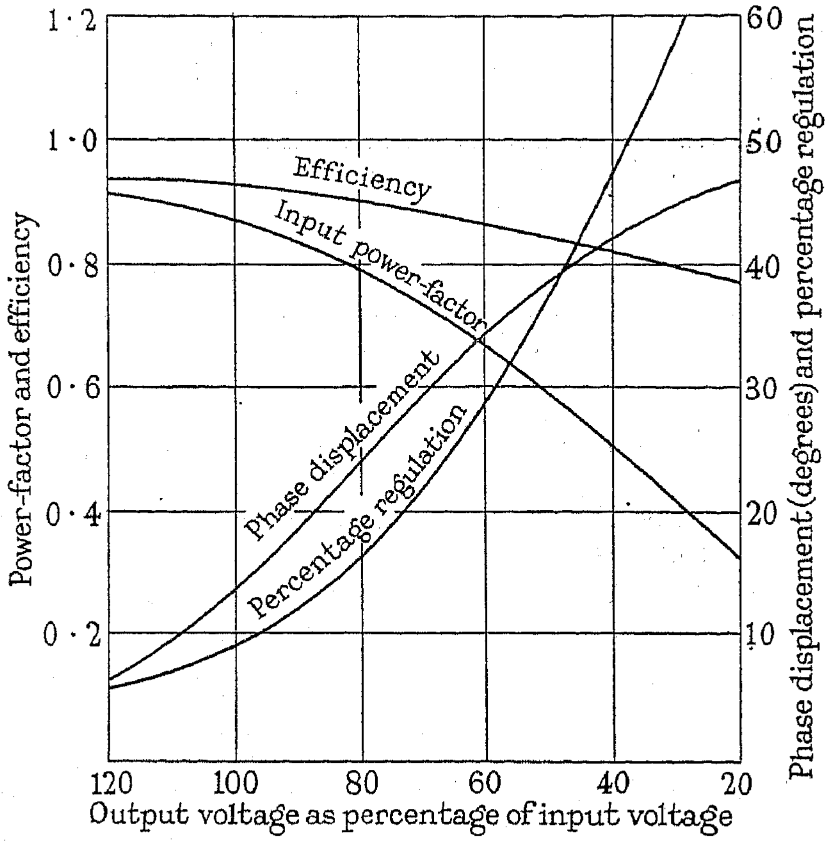

Fig. G

**Mr. E. S. Fairley:** In view of the number of inquiries which have been made about the loss and power factor of regulators, the following test results which I recently obtained on a 3-phase moving-coil regulator may prove of interest.

The regulator tested was rated at 10 kVA and the output-voltage range was from zero to 120 % of the input voltage. Fig. G shows the results obtained by varying the output voltage over a wide range while maintaining the output current constant at full-load value. The load power-factor was unity. The losses occurring in the regulator comprise iron losses and copper losses, the values of which depend on the position of the short-circuited coil. The former vary by about 30 % and the latter by about 15 % over the full travel of the short-circuited coil. The full-load copper losses are about $2\frac{1}{2}$ times the iron losses. The efficiency decreases with reduction in output voltage because the losses, which are approximately constant, become an increasing proportion of the decreasing output.

The phase displacement, percentage regulation, and input power-factor, depend on the leakage reactance of the regulator, which from these results would appear to be considerable. It is difficult to see how a high leakage reactance could be avoided, as the coupling between all the coils appears to be as close as is mechanically possible. Perhaps the author can explain how the design could be modified so as to reduce the leakage reactance while maintaining the same range of output voltage.

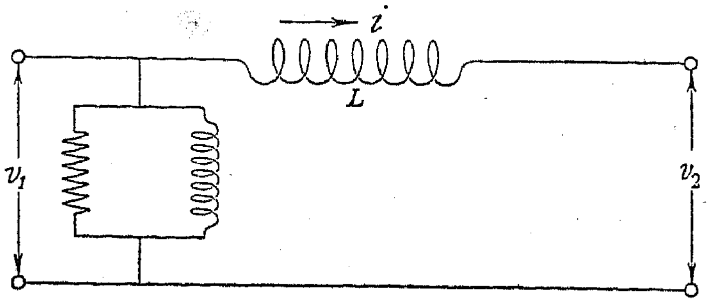

**Fig. H**

I should like to amplify the author's remarks regarding the distortion of the output voltage which occurs when the current is not sinusoidal. Referring to the circuit shown in Fig. H, which may be taken as the equivalent circuit of the regulator, $v_2$, the instantaneous value of the output voltage, will always be given by

$$v_2 = v_1 - L\frac{di}{dt}$$

where $v_1$ is the input voltage, $L$ the equivalent inductance, and $i$ the current. Let us assume that the wave-form of $i$ is as shown in Fig. J($a$) and that $v_1$ is sinusoidal; then from the above equation the wave-form of $v_2$ may be deduced. The result is shown in Fig. J($b$).

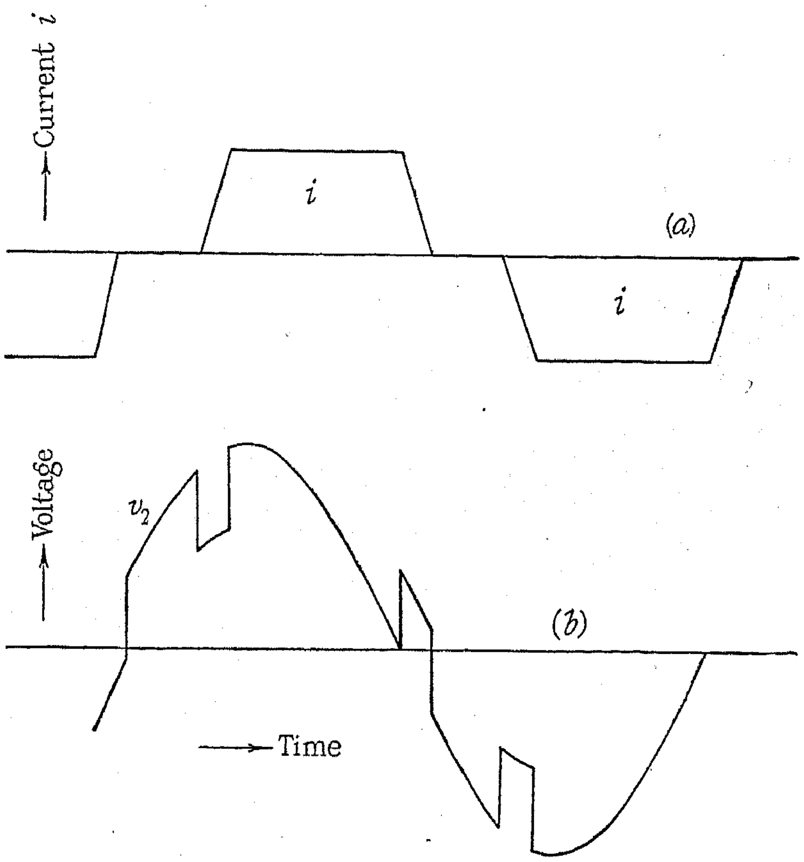

**Fig. J**

The case instanced above may occur in practice if a large percentage of the total load on a system consists of rectifier plant.

[The author's reply to this discussion will be found on page 30.]

## NORTH MIDLAND CENTRE, AT LEEDS, 1ST MARCH, 1938

**Mr. Douglas C. Field:** To use long overhead transmission lines economically it is necessary to allow reasonable voltage-drops, and these have to be corrected for at some point on the system. The use of transformer on-load tap-changing gear for this purpose is the more common method, although separate regulator equipments are sometimes employed. On low-voltage systems heavy loads are frequently met with at the end of the system, and often voltage regulators are the most economical method of dealing with the situation. All these methods of regulation, however, necessarily entail movement and consequently continuous maintenance. It is not surprising if supply engineers are slow in installing regulators in large numbers when they know that these, unlike transformers, will probably require a good deal of attention.

It is significant that the two characteristics included in Table 2 which are not met by any existing form of regulator, namely "no moving parts" and "instantaneous operation," are probably the most desirable, and one still looks forward to the day when some static form of regulator will be available which will solve both these problems.

The author will probably agree that his particular form of regulator is not superior to others in respect of cost and of losses. I have been comparing some figures relating to one of the early moving-coil regulators with those for a modern type and I note that the author has been able to reduce the losses considerably, but still further improvement in this direction would be welcomed.

On page 6 the author says that the internal impedance value is not usually of importance, but so far as regulators used on distribution and transmission systems are concerned I am not altogether in agreement. A voltage regulator really only maintains a steady average voltage, and it cannot correct sudden kicks; peaky loads and heavy loads suddenly applied often give rise to complaints, and the insertion of a voltage regulator increases rather than lessens the fluctuations since it means adding to the impedance of the circuit. Therefore the lower the internal impedance of the regulator can be kept the better.

I should like to inquire what is the minimum starting voltage of the induction-disc type of motor. A regulator may be called upon to deal with a sudden big drop in voltage, and it is important that the motor should be able to start under these conditions.

At the bottom of page 11 the author says "For all except the largest regulators the voltage-relay mercury switch serves this purpose and thus controls the driving motor directly." Does he mean that mercury switches are always used for controlling disc-type motors and that contactors are required for other types of motors, or that in the case of large regulators employing several disc-type motors contactors are required? If the latter is the case I should have thought that the difficulty could easily have been overcome by having more than one mercury switch operated by the relay.

The Type D connection is interesting, and I hope that it may be developed for use on distribution transformers in place of on-load tap-changing gear.

Advantage may be taken of the internal impedance of the moving-coil regulator by using it for protecting switchgear, at the same time overcoming the drawback of the usual type of reactor by correcting for the voltage-drop under normal load conditions.

**Mr. H. Peace:** I should like to mention some experience which I have had of the operation of one of these regulators of 2 000 kVA capacity. The only portion which has given trouble is the contacting gear, and this began to give rise to a great deal of noise which no amount of adjustment was successful in silencing. Since the contactors were changed, however, no further trouble has been experienced. I suggest that some further investigation is desirable with a view to eliminating contacting gear on even the larger regulators. The noise I have mentioned could be very objectionable in a residential area, especially if the gear were installed in a sheet-steel kiosk. Regarding the actual operation of the regulator, I have found the smoothness of variation to be very marked, in both near and remote parts of the network under regulation. A short-circuit which occurred in a high-voltage isolating link box at a substation about $\frac{1}{2}$ mile beyond the outgoing side of the regulator had no ill effect on the latter, although our attendant heard a noise which might have been caused by the regulator being lifted up bodily and dropped.

I notice from one of the author's slides that these regulators are being applied to X-ray work, and I should like to ask what results they have achieved in this sphere. I am rather doubtful whether the action is quick enough to follow the variations in voltage which occur, for instance, on a supply driven by blast-furnace gas engines, during the time an X-ray exposure is being made. Whilst an X-ray exposure is only a matter of a very small fraction of a second, it is, nevertheless, necessary to preset the regulator before setting up the X-ray apparatus to take the exposure, and during this latter period the incoming voltage, on such a supply as I mention, may have varied considerably.

**Dr. E. C. Walton:** The author states that the magnetic circuit of his regulator is chiefly air, and this being so it would seem that the magnetizing current must be very large unless the means taken to reduce it consist in suitably increasing the number of turns on the coil. The induced e.m.f. in one of the coils is proportional to the product of flux and turns, so that one means of keeping the flux down is to increase the number of turns. This also has the effect of further reducing the magnetizing current for a given number of ampere-turns. It would seem from this point of view that the concentric-coil construction shown in one of the author's slides is an advantage, because not only is the length of air-gap reduced but one single magnetic circuit is converted into two or three in parallel. I imagine that the change from the single-core to the multi-yoke type of construction goes a long way towards reducing the magnetizing current.

In the paper the author uses the not very common term "leakage impedance"; I suppose this means a combination of "leakage reactance" with resistance, although in Appendix 1 he makes the statement that the resistance can usually be neglected.

The regulator is a most useful piece of apparatus for the laboratory. It will fulfil two distinct functions, one being the provision of a variable-voltage supply for testing and experimental work and the other the provision of a variable impedance, which is so often required. The author refers to the use of this regulator in meter-testing panels; one can see that it will also prove very useful in the calibration of a.c. indicating instruments.

**Mr. R. D. Ball:** The customary induction regulator has a magnetic circuit which includes an air-gap of about $\frac{1}{16}$ in., and I understand that the transformer windows form the air-gap in the moving-coil regulator, probably a few inches in dimensions. This fact would seem to raise the magnetizing current of the moving-coil regulator considerably above that of the normal induction regulator, for a given amount of active material.

When the moving-coil regulator is used as a potentiometer device, are the component voltages dephased with reference to the line voltage?

Assuming a sine-wave voltage on the mains supplying the regulator, what wave-form could be expected on the secondary side of the regulator?

[The author's reply to this discussion will be found on page 30.]

## TEES-SIDE SUB-CENTRE, AT MIDDLESBROUGH, 2ND MARCH, 1938

**Mr. H. V. Field:** The moving-coil voltage regulator is an extremely ingenious and interesting application of eddy-current screening-effects. One of the earliest applications of voltage control of an allied character is found on small induction or "shocking" coils, where the eddy currents in a moving copper or brass tube sliding over the primary winding control the flux linkages of the secondary, and hence the secondary voltage.

In Figs. 1–6 the need for mentioning the winding direction of coil C does not appear to be necessary, as its action is independent of the winding direction.

It is suggested that in the interest of completeness this new type of regulator should be added to Tables 1 and 2. The design requirements of this type of regulator would appear to be somewhat conflicting, in that the opposition of windings increases the copper necessary, so that on this score a design with high flux and low ampere-turns would appear desirable. As, however, the flux paths are largely in air, this favours a low flux and high ampere-turns. Actual designs would appear to require a large amount of both copper and iron, the former owing to the low flux, and the latter owing to the long coils which would be necessary to keep down the magnetic current reluctance. Even so, it is considered that the magnetizing current would be high as compared with that in transformers of normal design.

In regard to Fig. 2, it is stated on page 4, col. 1, that a "voltage variation of $V_3$ is obtained." This seems to be incorrect, as $V_3$ is not varied, but only the distribution of this voltage across coils "a" and "b," thus controlling the fraction of $V_3$ which is added to $V_2$. As coils "a" and "b" are in opposition, it would appear necessary for their ampere-turns to be approximately equal, as otherwise one would dominate the other and send a flux round the closed iron circuit. No indication of the relative ampere-turns is given in the paper. Can the author say how the current in the short-circuited coil varies (*a*) with the position at no-load, and (*b*) with the load for a fixed position?

On page 8, col. 2, referring to the 3-phase construction, it is stated that "the fourth yoke of each unit can be omitted, as the 3-phase fluxes cancel out." Top and bottom yoke portions would, however, appear to be necessary in order to redistribute the phase components of this flux. In addition, accepting the above statement, it would appear logical to carry this further and treat the remaining three yokes of each phase in the same way, thereby eliminating all the yokes. The outer yokes are, however, presumably necessary for magnetic screening.

Can the author say what are the probable size limits of this type of regulator? With large transformers the regulator would appear to be a very useful adjunct to give smooth and progressive voltage variation between taps in tap-changing, or as an alternative to taps by covering the full desired range of variation.

The application of small sizes fitted with automatic voltage control for remote situations such as isolated farms and country houses will appeal to power companies, owing to the considerable saving in line cost by the removal of the restriction of line drop.

## THE AUTHOR'S REPLY TO THE DISCUSSIONS AT LONDON, GLASGOW, LEEDS, AND MIDDLESBROUGH

**Mr. E. T. Norris** (*in reply*): The discussion on the paper has been very comprehensive and has included practically every aspect of design and performance as well as the more practical operating features. Two matters in particular have been raised in different forms by a number of speakers, and it is proposed to deal with these before considering individual comments.

The more important subject is the cost and efficiency of the moving-coil regulator in comparison with other types of voltage regulator—a relation of obviously prime importance in practical engineering application. It was found impossible to give in the paper comparative figures of cost and efficiency, even of a general nature, for the reasons given on page 2 and in Appendix 4, and price was omitted from the comparison of different types of voltage regulator in Table 2. The difficulty is due to the fact that cost is not solely dependent on the regulator design and construction, but also to a large extent on factors having no common basis and varying widely among different manufacturers and for different products of the same manufacturer. Typical examples are the scale of production, manufacturing facilities and efficiency, and the distribution of overhead charges. The efficiency, also, has not a fixed value for any given regulator, but can be varied in relation not only to loading conditions but also to other technical characteristics—the inter-relation between cost and efficiency alone would prevent effective comparison. It is, moreover, a natural feature of any new development that both efficiency and production are steadily being improved as manufacturing and operating experience is gained. Figures given, even when this paper was written, would be obsolete and unfair to the present design.

It must, however, be emphasized that the difficulties just described in giving comparative figures are entirely in the abstract and do not mean that price and data comparisons are impracticable or unobtainable. In any particular case firm prices and all technical data can be stated with the load and no-load losses specified, and guaranteed with or without tolerance limits. Exact comparison can then be made with any other type of regulator, or any alternative solution such as additional high-tension lines and substations, or parallel feeders. Interesting examples of the latter alternatives have been studied by MacQueen.*

A further general view of the position is given by the demand for the new regulator; nearly 2 000 units have been manufactured and all of them have been sold in the open market. It is, therefore, reasonable to assume that the comparison has been favourable to the moving-coil regulator, as it is unlikely that a new and untried apparatus would be adopted unless it was relatively attractive. Fig. K shows the growth in production from the start and may be taken as a practical indication of the relative technical performance and operating characteristics of the apparatus.

Apart from efficiency and cost, other factors enter into the comparison. Automatic voltage regulators normally involve complicated control gear requiring periodical adjustment and maintenance. This has hitherto restricted the use of automatic regulation in distribution circuits, particularly among rural supply authorities with limited staffs and scattered networks. The simplification of this gear, as described on page 11 and shown diagrammatically in Fig. 23, has resulted in an automatic voltage regulator requiring no periodical maintenance or adjustment. The complete automatic control and mechanical operation of a voltage regulator involving only a voltage relay and driving motor, as shown in Fig. 23(*b*), would appear to be nearing the limit in simplicity and reliability. A number of speakers have testified to this unique feature. Mr. Burton, for example, referring (among others) to the regulator shown in Fig. 19, speaks of the absence of maintenance charges; whilst Mr. Morgan refers to regulators working "perfectly night and day for 2–3 years without any attention whatever."

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* "Voltage Regulation on the L.T. A.C. Network," *Electrical Power Engineer*, 1936, vol. 18, p. 717.

The foregoing remarks reply to inquiries by Dr. Say and Messrs. Ellis, Sutcliffe, Robertson, and Clark.

It is shown on page 7 that the major portion of the magnetic circuit is air path. This has minor advantages, in that the magnetizing current is practically sinusoidal, whilst the magnetic losses and heating in air are, of course, zero. The principal advantage fundamental to the regulator is that the mechanical forces are greatly reduced. Electromagnetic force is proportional to $dX/dS$, where $X$ is the reactance and $S$ the air-gap. In the induction voltage regulator this gap must necessarily be very small (merely mechanical clearance for the rotor), and the mechanical stresses are therefore relatively high. In the moving-coil regulator this gap or window is the major portion of the magnetic circuit, and the relatively large value of $S$ results in very low values of $dX/dS$ and hence of the mechanical stresses. This relation partly accounts for the silence in operation and for the short-circuit strength described on page 10.

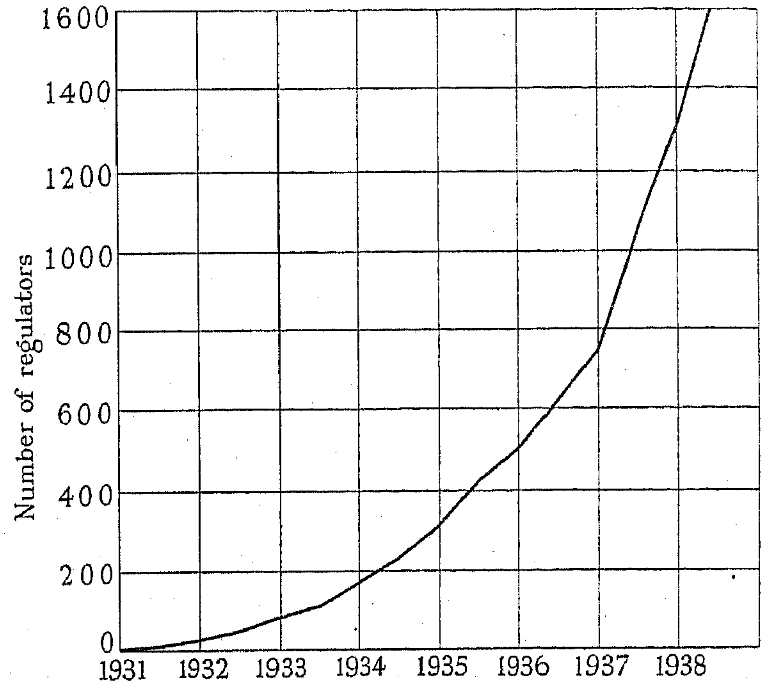

**Fig. K**

Electromagnetic apparatus in general (e.g. generators, motors, induction regulators, and meters) requires an iron-path magnetic circuit. Air-gaps, where essential, are made as small as possible. It is, therefore, natural that the introduction of a new piece of apparatus having mainly air magnetic path should give rise to comment. The most obvious suggestion is that, since the permeability of air is only unity whereas it is several thousands for iron, the magnetizing current will be relatively high. A number of speakers, including Drs. Walton and Wilson and Messrs. Ellis, H. V. Field, Ball, and Clark, have made comments of this nature. The permeability relation is, of course, fixed, but other factors more readily controllable are also involved. The magnetizing current is inversely proportional to $AT^2$, where $T$ is the number of turns and $A$ the area of the magnetic path. Both these factors can be chosen to reduce the magnetizing current. The area $A$ is, in fact, inherently relatively large in the standard construction shown in Fig. 14(*b*). In practice, the magnetizing current compares favourably with representative values in other electromagnetic apparatus. The ease of control of this current makes it possible to obtain abnormal values when required, as described on page 14.

Mr. Ellis and Mr. Diggle have referred to early—and obsolete—types of voltage regulator. A study of the Patent Office records will reveal the very large number of attempts which have been made to produce a good voltage regulator during the last 40 years, and thereby emphasize the potential demand for a satisfactory solution. Illustrations in the articles quoted by these speakers show some that have actually been made, but they have all been abandoned many years ago as impracticable.

It is not suggested in the paper that variation of reactance by means of a movable short-circuited coil is new. The most familiar example is probably the toy shocking coil mentioned by Mr. H. V. Field. The position is, in effect, summarized in the second paragraph on page 5 of the paper. The windlass type referred to by Mr. Ellis has obvious practical difficulties, whilst the second type patented in 1904 involves heavy forces and flexible connections from the moving coil carrying full-load line current. The regulator described by Mr. Diggle and shown in Fig. A is apparently simply a variable reactor in series with an auto-transformer as classified in group 1(*b*) of Table 1. The voltage variation obtained is determined not only by the particular value of the series reactance but equally by the load current. Such devices cannot be considered true voltage regulators, as their operation is not independently determinable but involves an external characteristic (the load current or impedance).

In reply to Mr. Ellis's and Mr. Diggle's comments on short-circuit stresses, the coils, both fixed and moving, are of the single helical or multi-layer type, which is naturally strong mechanically. Any possible decrease in mechanical strength due to their axial length is more than offset by the accompanying decrease in force, so that the factor of safety is increased. The mechanical stresses are inherently low, due partly to the air magnetic path and partly to the neutralization of currents in the fixed coils, as explained in Section 4(*d*). The short-circuit tests of 77 times full load described on page 10 were carried out chiefly to confirm calculated figures and to determine the mechanical rigidity of the windings. A number of cases have occurred of severe short-circuit on large regulators in service, with satisfactory results. One such instance, involving a large regulator of 2 000 kVA capacity, is described by Mr. Peace. The force between the coils of the three phases is negligible. The force curve in Fig. 13 relates to a Type B all-boost regulator. The curve would be symmetrical about the centre for equal buck and boost, but otherwise of the same general shape.

The installation of on-load tap-changing gear in the substation transformer, in comparison with the moving-coil regulator, is not merely a question of cost and efficiency. In most cases the best place for a voltage regulator in distribution circuits is not in the substation but out on the line. Voltage regulation in the substation can do little more than handle high-tension voltage variations, whereas installation along the feeder permits loads and feeder lengths which would otherwise result in voltage-drops four or five times as great. The type of regulator described in Section 6(*f*) is intended for house-service use. It is designed for installation indoors on the load side of the main switch and fuses in low-voltage lighting and power circuits. There would seem to be no fundamental difficulty in building the regulator for working voltages of 132 kV.

In reply to Mr. Ellis's last question, the moving-coil regulator is in general much quieter than an ordinary static transformer of the same equivalent size. Noise is probably most objectionable in quiet suburban areas, and Mr. Ellis's query is best answered by Mr. Morgan, who states in respect of his feeder-pillar type voltage regulators that " It is impossible to hear any noise from them at any time of the day or night."

In high-capacitance cable testing equipment it is often desirable that the voltage regulator should have low leakage reactance and high magnetizing current to neutralize the capacitance current of the cable and so reduce the kVA input required and improve its power factor. In the original design of regulator having sandwich windings with rectangular coils, as shown in Fig. 14(*a*), the minimum reactance obtainable in some cases prevented compensation as described by Messrs. Webb and Hill. In Mr. Hill's case the conditions were accentuated by a change in the voltage ratio, involving re-connection of the coils after installation. The concentric-winding construction shown in Fig. 14(*b*) gives inherently much lower values of internal reactance and permits almost any combination of reactance and magnetizing current. This is confirmed by Mr. Mildner, who has had experience of both types and finds that the concentric designs " operate very satisfactorily in this respect." The reduction in leakage reactance also improves the voltage regulation which, owing to the capacitance loading, is positive.

In reply to Dr. Wilson, energy-storage mechanism is essential with all on-load tap-changing gear, either in the switches themselves or in the operating mechanism. It must definitely be made impossible for the gear to stop with a switch in an intermediate position. For normal operation the tap-change cycle must be completed, so that the choice is not as simple as Dr. Wilson states—apart from the wear and tear of current-breaking switches. Even if energy storage could be eliminated the control gear would not be greatly simplified. Typical automatic control apparatus for on-load tap-changing gear was described recently by Mr. Diggle[*] and is in accordance with Fig. 23(*a*) of this paper.

Ordinary transformer oil in accordance with B.S.S. 148—1933 is used and, although some regulators have been examined after 7 years' service, no sign of wear was apparent. It should be noted that the mechanism is lightly rated and that there are no fast-running parts. The highest speed is that of the induction disc rotor, viz. 140 r.p.m. Routine tests of speed show an average increase of 10 % or more when the regulator is oil-immersed, notwithstanding the damping effect of the oil on the rotor disc. This would seem to indicate that the oil has an effective lubricating value both when hot and when cold.

Transient switching-in currents of the regulator are much lower than in an ordinary transformer, as the magnetic path is mainly air and saturation effects do not arise.

Remote indication of the voltage ratio is usually provided by the " Ferranti repeater."[*]

The limitation in voltage range described by Mr. Burton only occurs when the whole of the voltage-drop is on the input side of the regulator and is due entirely to the load through that particular regulator. The theoretical limit in this extreme case is 20 % voltage-drop, but even then the regulator will effect some improvement for larger drops. If $D$ is the per-unit drop without the regulator in circuit, then the improvement in voltage with a suitable regulator will be

$$D - \left[1 - 0{\cdot}5\sqrt{\left\{1/D - 1\right\}}\right]$$

With two regulators installed at intervals along the line much higher voltage-drops can be controlled, even under these extreme conditions. There are several installations of this kind giving satisfactory service where the low-tension distributor is 3 to 4 miles long, and the installation of two regulators with static balancers has saved the much higher cost of high-tension lines and substations.

I agree with Mr. Burton that in certain cases high operating speed may be essential; but as it is invariably more expensive, and in many cases unnecessary, it is desirable, as stated in the paper, " to consider individually cases apparently requiring high-speed operation." The increase in line current due to the transformation ratio of the voltage regulator does not increase the total load on the system; the maximum demand will be just the same with high-voltage supplies if the voltage regulation is similarly maintained within the limits of good service. Mr. Burton does not advocate bad voltage regulation in order to reduce the maximum demand during a cold snap, and Mr. F. S. Naylor has shown[†] that maintenance of voltage gives increased net revenue notwithstanding higher maximum demand.

The results of operating experience given by Mr. Morgan are extremely interesting and emphasize the advantages of good voltage regulation. The pronounced effect of small changes in the supply voltage on output is of great practical importance. It is seldom that actual figures derived directly from operating experience, such as those quoted by Mr. Morgan, are available.

It seems to me that the moving-coil regulator is a combination of Classes 1(*b*) and 2(*a*) and not Class 1(*b*) only, as suggested by Mr. Diggle. There is a transformation effect due to the auxiliary coils—" n " and " k "—which gives, in effect, a variable-turn boost and buck. The no-load current in the regulator tested in Appendix 3 is not 42 %, as the test load did not represent the rating of the regulator. Testing conditions were in fact so chosen as to exaggerate the internal load impedances in order to demonstrate the accuracy of the formulae.

The reactance ratio of the coils " a " and " b " in the extreme positions of moving coil varies from 15 to 50, depending upon the design and application of the regulator. The inherent regulation of the regulator is a function of the equivalent impedances of the windings "a" and "b" in parallel. The effect of moving-coil position on this regulation depends on the design and on the coil proportions. Either higher or lower mid-position values are possible with the concentric winding design. The reactance was inherently higher in the middle position for the ordinary sandwich design—Fig. 14(*a*). The patent specifications referred to by Mr. Diggle described compensating coils for reducing this reactance. These have been found unnecessary in the concentric-winding type and have not actually been employed.

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[*] " The Application and Construction of Transformer On-load Tap-changing Gear," *Journal I.E.E.*, 1937, vol. 81, p. 330.

[*] *Engineer*, 1936, vol. 162, p. 525.

[†] " Loss of Revenue on Heating and Lighting Loads, due to Poor Voltage Regulation," *Journal I.E.E.*, 1936, vol. 79, p. 33.

The largest regulator as yet manufactured in physical frame size is similar to Fig. 19 and can give 10 % voltage variation in a 15 000-kVA circuit. Larger sizes, however, are in contemplation. The use of 8 motors is exceptional, although these are of the simple low-speed induction-disc type with plain copper-disc rotors. In the majority of cases one motor only is sufficient with two or three motors for the larger sizes. In the very large sizes, where more than three motors are employed, a contactor is necessary, as stated by Mr. Diggle; but, even so, the additional gearing and other complications shown in Fig. 23(*a*) are still eliminated. The advantages of installing the regulating apparatus nearer to the load than the distribution transformer have been described above and are emphasized in Mr. Morgan's remarks. The operating objection to on-load tap-changing gear in distribution circuits, unless mercury switches are employed, is not the actual cost of contact maintenance but the necessity for periodical inspection, which involves isolation of the transformer and frequently interruption to the supply.

With reference to Mr. Mildner's remarks on switching surges, it is a characteristic of alternating-current electromagnetic apparatus that starting conditions are different from running conditions and are therefore likely to involve transient operating effects. These are frequently accentuated by saturation of the iron magnetic circuit, but this effect is relatively small in the moving-coil regulator, due to its magnetic path being mainly air. The moving-coil regulator, in common with all other types, gives voltage variation by means of transformation and differential reactance effects based upon sustained 50-cycle alternating current. This sustained frequency condition does not of course exist at the instant of switching-in, and the regulated voltage will have corresponding transient values until steady conditions obtain. In general these transient effects are only of importance in regulators having a large variable voltage range (such as those used for testing purposes) when in the minimum voltage position, i.e. when the 50-cycle regulating effect is at its greatest, and when at the same time relatively heavily loaded. One solution is to excite the regulator before loading, and another is to switch on with the regulator in the zero voltage position and short-circuited.

The no-load voltage-ratio curve shown in Fig. E agrees closely with Fig. 7 of the paper. The increase in secondary voltage on capacitance load is due to the positive voltage regulation on leading power factor referred to earlier in this reply. The kink in the curve at $0 \cdot 031\ \mu$F loading would seem to be due to local resonance effects.

Mr. Hill's remarks have already been dealt with.

In reply to Mr. Cooper, the regulator is suitable for low voltages and heavy currents in approximately the same degree as an ordinary static transformer, since the windings are of the same type and construction. The absence of switches and contacts permits continual operation unrestricted by time-lag devices, whilst the speed of operation is largely a matter of mechanical gear ratio and motor power.

The effect of the auxiliary bucking and boosting coils "n" and "k," and of the moving coil, on cost and losses is described on page 5 and the fourth paragraph of Appendix 4. This effect is referred to by Mr. Ayres and illustrated in his Figs. F(*a*) and F(*b*) for regulators having equal buck and boost. In the more usual case of all-boost the bucking coil and the corresponding ampere-turns in the moving coil are eliminated. Due to the inter-relation between cost and efficiency referred to at the beginning of this reply these additional coils do not necessarily involve additional losses, especially as all the windings are single circular coils of the transformer type free from the design limitations and labour cost of multiple windings in slots.

Mr. Ayres's statement that "in the induction regulator the small air-gap is not varied and does not come into the calculation of operating forces" is literally correct. It is not, however, a criticism of Section 4(*d*), as he states, as this Section is entitled "Noise and Vibration" and deals with the relevant forces. These include other important forces besides those involved in operation. In particular the electromagnetic force due to the small air-gap in the induction regulator is a radial force causing vibration of the rotor and is relatively large due to the small value of the air-gap $S$ in the function $dX/dS$. This force is almost zero in the moving-coil regulator, as the change in reactance with lateral movement of the short-circuited coil is negligible. For this reason no structural bracing of the coil laterally is necessary, and the relatively small-diameter operating shafts shown in Fig. 18 are satisfactory.

The speed of operation of the regulator is determined by the operating forces, the mechanical gear ratio, and the motor power. The last two of these are independent of the regulator characteristics, so that operating speed is not an inherent function of the type of regulator but may have any desired value. The time of 12 minutes mentioned by Mr. Ayres is exceptionally long and would only be permissible in certain cases. On the other hand, times as low as 1 second for correcting the maximum voltage variation have been obtained using the induction disc motor, but, as stated in Section 6(*e*), such high speeds are seldom necessary.

The internal reactance of any regulator is objectionable under normal loading conditions but useful on short-circuit in limiting the fault current and the resulting mechanical stresses. The saturation effect in the induction regulator mentioned by Mr. Ayres results in the reduction of the reactance on short-circuit, when it would be an effective protection. Largely because of the relatively small saturation effect the moving-coil regulator can be made self-protecting under short-circuit conditions, as indicated in Section 6(*a*).

The statements in Section 6(*b*) are correct. It is this voltage phase-displacement which prevents operation in ring mains and interconnectors, as it occurs under no-load as well as load conditions, and would result in continuous circulating currents and losses. It is directly responsible for the double or twin 3-phase construction necessary for the induction regulator but not for the moving-coil regulator. In stating that in the induction regulator the system power-factor is not affected, Mr. Ayres is apparently ignoring both the internal reactance and the magnetizing current, neither of which is negligible.

With regard to magnetizing current and the current in the moving coil, Mr. Ayres's statement that the ampere-turns equivalent to the magnetizing current must flow in the moving coil is apparently based on Fig. F(*c*), which is incorrect. The moving-coil current is substantially zero at no-load for all regulator positions, as shown by formula (16). In tests made recently, where, in accordance with Mr. Ayres's suggestion, the moving-coil current would be 15·5 amp., the actual measured value was less than 0·4 amp. I agree with Mr. Ayres that the difference between the calculated value (7·2 amp.) and the measured value (6·9 amp.) of the moving-coil current on load is not due, as was suggested in the paper, to impedance in the ammeter circuit. Neither is it a reactance component, as this would cause an increase in current and not the decrease calculated by Mr. Ayres. It was in this particular case due to the exaggerated conditions under which these tests were deliberately made, for the reasons stated in Appendix 3. Tests made recently on a standard regulator to check this point gave the following values:

| | Moving-coil current | |
|---|---|---|
| | Measured | Calculated |
| Half load .. .. .. | 7·78 | 7·72 |
| Full load .. .. .. | 15·44 | 15·40 |

It will be seen that under normal operating conditions the measured values check closely with formula (16). These results, together with the measurements given in Appendix 3, show that the formulae are all sufficiently accurate for practical purposes notwithstanding the last remark of Mr. Ayres.

The modifications suggested by Mr. Clark are all technically sound; in particular, the Berry construction certainly gives the improvements he describes. However, all these modifications involve increased manufacturing costs. For reasons given earlier in this reply it is considered that the advantages of the air-gap more than outweigh any disadvantages, so that the extra labour cost of windings in slots in order to reduce the air-gap is not considered desirable. Mr. H. V. Field and Mr. Clark are correct in stating that although the centre yoke can be omitted in the 3-phase regulator the top and bottom yoke portions are still necessary. In practice, however, the omission of these yoke pieces has been found to be economically justified.

In reply to Mr. Sutcliffe, the major mechanical stresses are borne by the screwed supporting shafts as he suggests, and these are designed accordingly to give sufficient strength under short-circuit conditions. The mechanical stress under short-circuit in the example he gives would be approximately 500 lb. per sq. in. The remaining questions of Mr. Sutcliffe have been dealt with in the reply to Mr. Ayres.

Mr. Robertson's inquiries regarding noise are answered in effect by Mr. Morgan. Special precautions, when necessary, would include special sound insulation of the regulator and its tank, with possibly external screening. However, these precautions have not as yet been found necessary. As explained in Section 4(*c*), the average value of flux density is relatively low, and this contributes to relative silence. I would recommend that full use should be made of static balancers, as they are cheap and should be quite reliable and satisfactory in operation. The size of balancer depends upon the expected out-of-balance current and the voltage control desired. It can be incorporated in the regulator itself, but in general this is not the most effective arrangement, as the best place for the balancer is on the side of the load remote from the substation, whilst the best place for the regulator is in an intermediate position.* In general it is important that the neutral points of the balancer, the voltage regulator, the load, and the substation transformer, should all be connected solid.

The ability of the regulator to deal with quick fluctuations of voltage, as stated in the reply to Mr. Ayres, is merely a question of motor power and mechanical gear ratio. The inherent time-lag on the astatic relay is that due to the inertia of the armature—no artificial time-delay devices are incorporated.

It is possible that the micro-gap switch suggested by Prof. Say would be as satisfactory as the mercury switch actually employed in the relay. The maximum flux density is similar to that adopted in ordinary static transformers, but the average value is much lower, as explained in Section 4(*c*) and illustrated in Fig. 12. One objection to the use of a plain copper sleeve for the moving coil would be increased eddy-current losses. The output voltage for a given setting of the moving coil remains constant within a few per cent, as suggested by Prof. Say.

The regulator examined by Mr. Fairley was one of the multi-range type referred to by Prof. Parker Smith, designed specially for experimental work in universities and technical colleges. Numerous coil connections are possible, and a high internal reactance is incorporated for safety purposes. These conditions, together with the subordination of efficiency and other technical characteristics to cost and flexibility of operation, make the measurements representative of this particular design only. However, the curves in Fig. G agree closely with calculations. All the values at the low-voltage end are apt to be misleading unless it is realized that they are based on full-load current at all voltages. Thus in the minimum-voltage position the percentage reactance and percentage regulation on this basis will nominally be infinity. This makes the other characteristics also appear abnormal at the lower voltages. Measurements on an ordinary static transformer will give similar results if expressed in the same manner.

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* G. L. PORTER: "Static Balancers," *Electrical Review*, 1933, vol. 113, p. 763, and *Electrical Times*, 1934, vol. 85, p. 172.

The leakage reactance can be reduced to any desired value by altering the frame and coil proportions and the ratio of copper to iron weights. Mr. Fairley gives an interesting qualitative illustration in Fig. J of the voltage distortion described on page 13. Such distortion quantitatively would not, of course, be permissible in practice.

The direction of winding of the moving coil C is immaterial, as suggested by Mr. H. V. Field. With regard to Fig. 2, the action of the regulator is as stated by Mr. Field, but the paper is correct in stating that an effective voltage variation of $V_3$ on the output side is obtained. The current in the short-circuited coil is substantially zero at no load for all positions, as explained on page 6. On load the value is given by formula (16), which shows that the current is directly proportional to the load and the same for all positions.

In reply to Mr. Peace, recent developments in design and the use of the astatic voltage relay have considerably increased the size of regulator which can be automatically controlled directly from the relay without intermediate contactors. For example, contactors would not now be necessary on a regulator of the size to which he refers.

Mr. Peace's description of a heavy short-circuit on this regulator is of considerable value. The manufacturer is, of course, entirely dependent upon the operating engineer for information of this kind.

The moving-coil regulator is in widespread use for X-ray work, particularly in the control of deep-therapy and treatment equipments. For radiography, where exposures of a small fraction of a second are necessary, it is usual to pre-set the regulator for the nominal setting, as described by Mr. Peace, and allow it to take care merely of incoming voltage variations. A high speed of operation may be necessary in these cases, of which an example is described in Section 6(*e*).

Dr. Walton gives a valuable explanation of the conditions affecting magnetizing current—this should be read in conjunction with the remarks given earlier in this reply. The term "leakage impedance" has been used in the paper to mean a combination of leakage reactance with resistance, as suggested by Dr. Walton. In general, as stated in Appendix 1, the resistance can be neglected.

In reply to Mr. Ball, the moving-coil regulator used as a potentiometer device is equivalent in effect to a tapped reactor. The distortion of wave-form on the secondary side is negligible unless the load is non-linear, as explained in Section 7(*a*) and illustrated in this discussion by Mr. Fairley.

Mr. D. C. Field refers to improvements in the efficiency of the later types of moving-coil regulator. These are still continuing as manufacturing and design experience is accumulated. The internal impedance of the concentric-winding type of regulator is in general so low that accentuation of transient drops due to heavy loads is negligible. The internal impedance can be deliberately increased so that the regulator shall also act as a reactor in limiting short-circuit currents and thereby give protection not only to the regulator itself but also to low-rupturing-capacity switchgear, as suggested by Mr. Field.

The torque of the induction-disc type of motor increases linearly with decrease in speed and is at maximum when stalled, so that it is inherently stable in operation. The minimum starting voltage is from one-half to two-thirds of normal, but this value can be reduced. All but the very largest regulators are now controlled directly by the astatic relay, even when several disc-type motors are required.

# A QUANTITATIVE STUDY OF ASYMMETRIC-SIDEBAND BROADCASTING

By P. P. ECKERSLEY, Member.

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## SUMMARY

The major factor which hampers the progress of broadcasting is the lack of a sufficient number of channels in which to contain the increasing number of transmissions. In the present state of the art few, except local, stations can be received without interference from other broadcasting stations on frequency-contiguous channels unless the upper frequencies of modulation of the wanted station are severely attenuated and unless, therefore, reproduction is lacking in intelligibility and naturalness. This inter-station interference can be eradicated or minimized, and quality improved as regards "top" reproduction, if all or part of one sideband of the transmitted spectrum is cut away. The cutting-away operation on the side-band, however, produces harmonic distortion in the received signals. It is the object of the asymmetric-sideband system of transmission to cut away part of one sideband without introducing audible harmonic distortion.

The paper gives a quantitative analysis which shows how asymmetry, either of phase or of magnitude, between two sideband components, created by any given modulation component, is related to harmonic distortion; and it is shown that this distortion is proportional, over a large range of values, to depth of modulation. The nature of the sound spectrum is such that the intensities in the upper register are much less than those in the lower middle register. Since the degree of asymmetry between counterpart sideband components is a measure of the attenuation of the cut-away sideband, this attenuation can be increased as the modulation frequency is made greater. This is tantamount to saying that, as the sideband frequency becomes more and more removed from the carrier frequency, it may be more and more attenuated while the distortion can be made to remain constant and small. Thus in asymmetric technique the outer parts of one sideband may be removed without the introduction of audible distortion.

Curves are derived, based upon the evaluation of expressions given in the quantitative analysis, which show the required attenuation-constant of filters which cut away part of one sideband but produce no more than a constant distortion. It is proved that phase asymmetry between counterpart sidebands may give rise to more serious distortions than are created by magnitude asymmetry acting alone, and a network is described which keeps phase asymmetry to a minimum and yet gives a close approximation to the required attenuation.

Accounts are given of practical tests wherein the quality of reproduction obtainable from an asymmetric transmitter is compared with that given by orthodox modulation. While it is true that a highly trained ear may, on rare occasions, be able to detect some slight differences between the two types of reproduction, when they are directly compared, the general public would be quite incapable of so doing, even if they possessed receiving apparatus so free from distortion itself as to give them an opportunity to do so. It is pointed out that it is better to face the occasional minute deteriorations in quality given by the asymmetric system than to suffer from the continuous hissing and splitting sounds brought about by existing inter-station interference.

In the latter part of the paper an analysis is given which shows that the asymmetric system has three possible applications to space broadcasting as it is practised to-day, namely (1) It could be used to allow carrier difference frequencies of the order of 11–12 kc. without introducing any sideband overlap interference. (2) It could be applied to existing conditions, where carrier difference frequencies are of the order of 9 kc., and would reduce inter-station interference to the order of one-tenth its present value. (3) If existing interferences are considered to be tolerable, then carrier difference frequencies can be reduced to 6 kc. without increasing interference above its existing values; this implies that 1½ times the number of stations working in Europe to-day on exclusive waves could be accommodated in the existing waveband.

It is suggested, finally, that a policy could be adopted such that the introduction of the asymmetric system would encourage receiver development along lines wherein, by an augmentation of only the carrier component at the detector, all distortion due to the use of an asymmetric spectrum would be eliminated. Thus an evolution might be envisaged which led to the introduction of the ideal carrier and single-sideband type of transmission when all receivers were adapted to reduce the resulting distortion to a tolerable amount.

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## LIST OF SYMBOLS

$a = \dfrac{M_N}{2}(1 + \xi)$.

$b = \dfrac{M_N}{2}(1 - \xi)$.

$d = 4\delta/M_N$, $d$ having suffixes to distinguish the nature of $\delta$.

$f_m$ = mean pass frequency of band-pass filter.

$f_s$ = frequency of sideband component.

$f_0$ = carrier frequency.

$f_1$ = lower frequency of cut-off in band-pass filter.

$f_2$ = upper frequency of cut-off in band-pass filter.

$\Delta f$ = frequency of modulating component.

$\Delta f_\infty$ = a modulating frequency which, added to or subtracted from the carrier frequency, expresses a frequency at which $\alpha$ may be infinite.

$M_A$ = modulation factor of carrier after passing through asymmetric filter.

$M_{A\xi}$ = modulation factor of carrier after passing through asymmetric filter when only magnitude asymmetry exists.

$M_{A\phi}$ = modulation factor of carrier after passing through asymmetric filter when only phase asymmetry exists.

### LIST OF SYMBOLS—*continued*

$M_N$ = modulation factor of carrier before passing through asymmetric filter.
$S$ = magnitude of one sideband-component vector.
$t$ = time.
$\alpha_\epsilon$ = attenuation constant of equalizer.
$\alpha_R$ = desirable attenuation constant of asymmetric carrier-frequency filter if no demodulation correction takes place.
$\alpha_T$ = desirable attenuation constant of asymmetric carrier-frequency filter with demodulation correction.
$\delta$ = distortion factor, i.e. ratio of second-harmonic amplitude to fundamental amplitude.
$\delta_\xi$ = distortion factor produced by magnitude asymmetry alone.
$\delta_\phi$ = distortion factor produced by phase asymmetry alone.
$\delta_0$ = constant value of distortion chosen as tolerable or negligible in asymmetric systems.
$\theta_1$ and $\theta_2$ = phase angles made by two sideband components, created by the same modulating component, with carrier component.
$\xi$ = ratio ($< 1$) of amplitudes of two sideband components created by the same modulating component.
$2\phi$ = algebraic sum of $\theta_1$ and $\theta_2$.
$\psi$ = angle between vector resultant of two unequal-magnitude sideband vectors and the vertical.
$\Delta\omega = 2\pi\Delta f$.

## (1) INTRODUCTION

A paper outlining the principles of the asymmetric-sideband system has already been published.[*] That paper, which was designed to be no more than a preliminary survey of principle, is supplemented herein by a quantitative analysis which forms a guide to practical design.

The object of the asymmetric-sideband system of transmission is to reduce the frequency band width normally occupied when a carrier is modulated (and when both sidebands are fully transmitted) without the introduction of audible distortion into the received signals. If this aim could be practically achieved, then the practice of broadcasting could be benefited in any or some of the following ways:—

(*a*) The existing carrier difference frequencies could be slightly increased and all sideband overlap interference would thereby be eliminated, a benefit *per se*, and at the same time the quality of reproduction of radio programmes with respect to the reproduction of the higher audio frequencies could be greatly improved.

(*b*) The interferences associated, in the modern practice of broadcasting, with the working of stations at carrier difference frequencies less than twice the highest modulation frequency transmitted, would be reduced or eliminated.

(*c*) If the interferences referred to in (*b*) are considered to be, in all the circumstances, tolerable, then, if the asymmetric system was used, existing carrier difference frequencies could be reduced, and the number of stations which could work simultaneously in a given continental area could be thereby increased.

One of the most important features of the system is that no alteration to existing receivers would be immediately necessary when the transmitters were modified to radiate the asymmetric spectrum; so far as the ordinary listener is concerned the service from local stations would appear to continue as before the changes were made, but some amelioration of inter-station interference would take place. Some modifications in reception technique would eventually be desirable, however, if the listener were to benefit to the greatest possible extent from the improved system of transmission.

## (2) OUTLINE OF PRINCIPLE

It is proposed first to outline the basic principles of the asymmetric system qualitatively and thereafter to give a quantitative analysis forming a guide to practical design.

When a carrier is modulated by a lower frequency three component frequencies are created, namely the carrier, an upper-frequency sideband component, and a lower-frequency sideband component. The asymmetric system is based upon the use of a filter which gives, at the higher modulation frequencies, a differential attenuation between the upper- and lower-frequency sideband components created by a given modulation component. This filter or a series of filters cutting away part of one sideband is connected at any point in the chain of circuits between the output of the circuit where the normally modulated carrier is generated and the input to the receiver detector. When the asymmetric system is applied, as for its greatest usefulness it should be, to the transmitter, a sideband-cutting filter is connected at any point in the chain of circuits between the terminals where the normally modulated carrier is generated and the input to the transmitting aerial.

The filter used for cutting away part of one sideband is designed to transmit the two sideband components created by low frequencies of modulation (and therefore having frequencies close to the carrier frequency) at substantially equal amplitude but to increase the attenuation of one sideband component relatively to its fully transmitted counterpart as the frequency of modulation (and hence the frequency difference of the sideband components from the carrier frequency) becomes greater. Fig. 1 illustrates the principle of the system. The Figure shows the frequency/attenuation and frequency/phase characteristics of a band filter. $f_0$ is the carrier frequency, located asymmetrically with respect to the mean pass frequency of the band filter $f_m$. $(f_0 \pm \Delta f_1)$ and $(f_0 \pm \Delta f_2)$ are sideband component frequencies corresponding to modulation frequencies $\Delta f_1$ and $\Delta f_2$ respectively. When the modulation frequency is relatively low and has a value $\Delta f_1$ both sideband components are substantially equally transmitted, but when the modulation frequency has a value $\Delta f_2$, greater than $\Delta f_1$, one sideband component, of frequency $(f_0 - \Delta f_2)$, is attenuated, while the other, of frequency $(f_0 + \Delta f_2)$, is fully transmitted.

Asymmetry between two counterpart sideband components introduces harmonic distortion in the received signals. This asymmetry may either be an asymmetry of magnitude or the sideband components may have a phase asymmetry to the carrier. Both forms of asymmetry may be present simultaneously, both produce harmonic distortion. It will be shown in the quantitative analysis that the distortion introduced is, in practical conditions of working, generally proportional to modulation depth and is a function of the degree of asymmetry.

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[*] *Journal I.E.E.*, 1935, vol. 77, p. 517.

It is known that the intensity of the components in the spectrum of sounds forming typical speech and music radio programmes falls rapidly as the frequency of these components increases above a certain frequency (of the order of 1 000–1 500 cycles per sec.). Hence if some value of distortion can be regarded, thanks to the ear's tolerance, as negligible, this distortion may be kept constant (and negligible) in an asymmetric-sideband system provided the degree of asymmetry between counterpart sideband components is made a function of the probable depth of modulation occurring in practice. The fact that modulation depth decreases as modulation frequency increases makes it possible to cut away the components of one sideband more and more as they are of greater frequency-difference from the carrier without introducing more than a constant and negligible distortion. Thus in the asymmetric system the bulk of the modulated energy is radiated in a symmetric form; asymmetry is only introduced in frequency bands where the sideband energy is relatively small. Since in the proposed adaptation of orthodox technique a double-sideband system carries the bulk of the energy, and because the distortion due to the cutting-away of the remaining spectrum is so slight, it is obvious that the average receiver will not remark any difference in harmonic distortion between the symmetric and asymmetric transmissions.

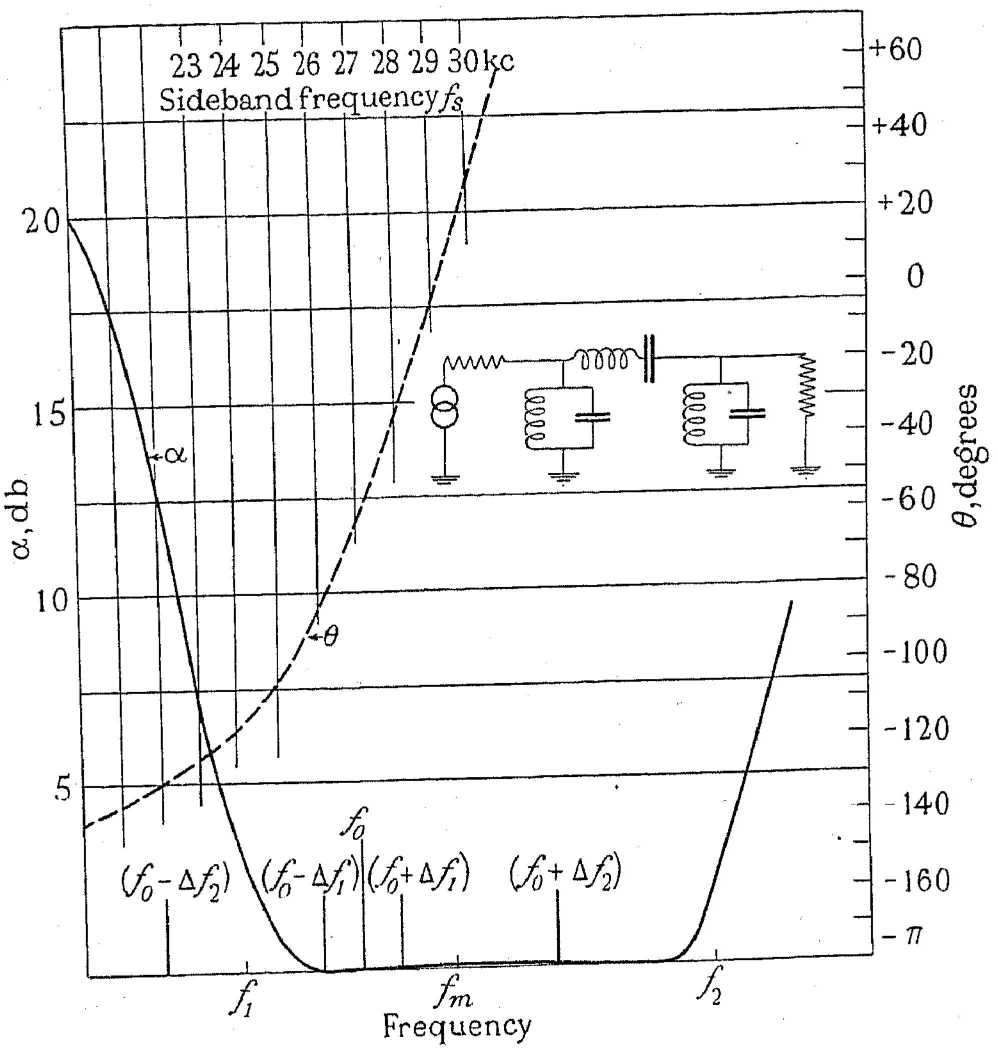

**Fig. 1**

Points obtained experimentally. $f_0 = 27$ kc./sec., $\Delta f_1 = 1$ kc./sec., $\Delta f_2 = 5$ kc./sec.

The differential treatment of the sidebands, however, introduces not only harmonic distortion but, unless precautions are taken to correct it, frequency-characteristic distortion as well. By the frequency characteristic is meant the curve showing the intensity of the modulated output plotted against frequency, for a constant input modulation. Let $M_N$ be the modulation factor of the carrier before passing through the asymmetric filter and let $M_A$ be the modulation factor of the carrier after passing through the asymmetric filter. Suppose, further, that the modulation frequency be varied while $M_N$ is kept constant. At low frequencies of modulation both sideband components are equally transmitted by the filter, and $M_A = M_N$. At a frequency of modulation so high that one sideband component is relatively of zero amplitude compared with the other, the effective modulation of the carrier at the filter output terminals will be half its value at the input to the filter, or, for high modulation frequencies, $M_A = \frac{1}{2}M_N$. Thus the overall frequency-characteristic of an "uncorrected" asymmetric-sideband transmitter would be represented by a curve showing a maximum at low modulation frequencies and falling through intermediate values to half that maximum at high modulation frequencies. Thus the action of the sideband-cutting filter is partly to demodulate the normally modulated carrier. The distortion of the frequency characteristic may be corrected by connecting somewhere, between the input modulation terminals and the input to the modulating valve, a frequency-discriminating network designed to attenuate the modulations by half for the lower frequencies but to give them full transmission at high frequencies.

It is necessary that the depth of modulation of the carrier in an asymmetric transmitter shall be sensibly the same as that obtainable in normal practice. Thus the asymmetric transmitter must embody, as well as the equalizer which attenuates the lower frequencies of the modulating spectrum, extra amplification, above that necessary for conventional technique, to raise the level of the modulations by an amount equal to the maximum equalizer attenuation.

The result of amplification and attenuation in the correction system is to leave the intensity of the lower-frequency modulating components the same as if no corrector circuits had been interposed. The higher-frequency components of modulation are more and more amplified as their frequency is higher, the maximum occurring when the equalizer is fully transmitting and when the amplification doubles the normal intensity. The amplitude of any higher frequency can never exceed the normal maximum because the design of the equalizer circuits is proportioned by the audio spectrum intensities and so no overload takes place albeit the apparent modulation in each system, asymmetric and normal, is the same.* It is thus seen that, provided the circuits are correctly designed, the asymmetric system need introduce neither serious harmonic distortion nor any frequency-characteristic distortion even though a part of the normally radiated spectrum is cut away.

The root question, the answer to which will alone determine whether or not the scheme has practical value, is whether the amount by which the normal spectrum may be narrowed is sufficient to justify the introduction of the resulting harmonic distortion. Evidently the quantities involved might be such that, in order to keep the distortion negligible, it would not be possible to cut away more than a small outer part of one sideband. On the other hand, a study might reveal that one sideband could be almost wholly eliminated without the harmonic distortion becoming intolerable. The question at issue can obviously only be answered in terms of a quantitative analysis, and it is therefore necessary to determine the relationship between the relevant parameters. More specifically, a quantitative analysis must give:—

(*a*) The relationship between harmonic distortion, modulation factor, and the degree of asymmetry between counterpart sideband components.

(*b*) The relationships between frequency-characteristic distortion and the degree of asymmetry between counterpart sideband components.

(*c*) The relationship between modulation frequency and modulation factor in the spectrum of sounds forming typical broadcast intelligence, i.e. the frequency/intensity relationship in the spectrum of normal sounds transmitted in radio broadcasting.

## (3) QUANTITATIVE ANALYSIS

### General

Fig. 1, already referred to, shows a typical attenuation and phase characteristic of a single-section constant-$k$ type of band filter working between resistance terminations which match, or tend to match, the filter image impedance. $f_1$ and $f_2$ are the cut-off frequencies, and $f_m$ = mean pass frequency = $\sqrt{(f_1 f_2)}$. In the asymmetric system the carrier frequency is located between $f_1$ and $f_m$ but closer to $f_1$, or between $f_2$ and $f_m$ but closer to $f_2$. If, in Fig. 1, the frequency of modulation is $\Delta f_2$, then the amplitude of one sideband component of frequency $(f_0 - \Delta f_2)$ is less than the amplitude of the counterpart sideband component of frequency $(f_0 + \Delta f_2)$. But if the frequency of modulation is comparatively low and equal to $\Delta f_1$ then both the sideband components are substantially equally transmitted.

If the ratio of amplitudes of counterpart sideband components created by a given frequency of modulation is $\xi$, a ratio less than unity, then the magnitude of one sideband component (in the case considered that of lower frequency) is $\xi$ times that of the other. We may define $\xi$ as the magnitude asymmetry factor, the value of which may vary between zero and unity.

A consideration of the phase characteristic of the filter will show that if the carrier is located near one cut-off frequency then the phase angle of one sideband component to the carrier is different from the phase angle of the other sideband component to the carrier. If $\theta_1$ and $\theta_2$ be these angles, then the algebraic sum $(\theta_1 + \theta_2)$ may be finite. We shall, for reasons which will be clear later, write $(\theta_1 + \theta_2)/2 = \phi$, and call $\phi$ the phase-asymmetry factor. If the carrier has a frequency $f_m$, $\phi$ is always zero and phase asymmetry is wholly absent because only at this frequency is one half of the phase characteristic of the filter a mirror image of the other half.

It will be helpful in developing the analysis to consider special cases, as follows:—

(i) $\xi = 1$, $\phi = 0$ (symmetric and normal transmission);
(ii) $\xi < 1$, $\phi = 0$ (asymmetric transmission with amplitude asymmetry only);
(iii) $\xi = 1$, $\phi$ finite (phase asymmetry only);
(iv) $\xi < 1$, $\phi$ finite (asymmetric transmission with both amplitude and phase asymmetry, i.e. the general case).

### Harmonic distortion when $\xi = 1$, $\phi = 0$ (symmetric transmission, Fig. 2)

It is convenient, both to illustrate the method of the more complete analysis and to define symbols, to recapitulate a well-known analysis of modulation.

It is the object throughout this Section to derive expressions which define the shape of the wave envelope resulting from the modulation of a high or carrier frequency by a lower frequency. The shape of the wave envelope is given (sufficiently accurately, even when distortions are present) by a curve passing through points the ordinates of which are the positive (or negative) peak amplitudes of the modulated carrier while the abscissae represent the times at which these peak amplitudes occur. It is assumed throughout the work that the modulation is sinusoidal, that the unmodulated-carrier vector amplitude is unity, and that rectification is "linear."

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* See the Section on "Equalizer Design" (page 52).

Fig. 2 shows the relevant vector diagram. OF, the unit-magnitude carrier vector, is considered to remain stationary and vertical. The sideband vectors FA and FB, each of magnitude $S$, are supposed to rotate in opposite directions at the same angular velocity, given by $\Delta\omega t = 2\pi\Delta f t$, where $\Delta f$ is the modulation frequency and $t$ is the time. An auxiliary circle is constructed with centre F and radius FR′ equal to the arithmetic sum $2S$ of the sideband-vector amplitudes. It is evident from a consideration of the triangle FR′R that $FR = 2S \cos \Delta\omega t$ and that the vector sum of the sideband vectors and the unit-magnitude carrier vector is $OR = 1 + 2S \cos \Delta\omega t$. If the magnitude of OR is plotted against $\Delta\omega t$ the resulting curve is a cosine curve. The peak and minimum values of the resultant are $(1 + 2S)$ and $(1 - 2S)$ respectively, corresponding to values of $\Delta\omega t$ of 0 and 180° respectively.

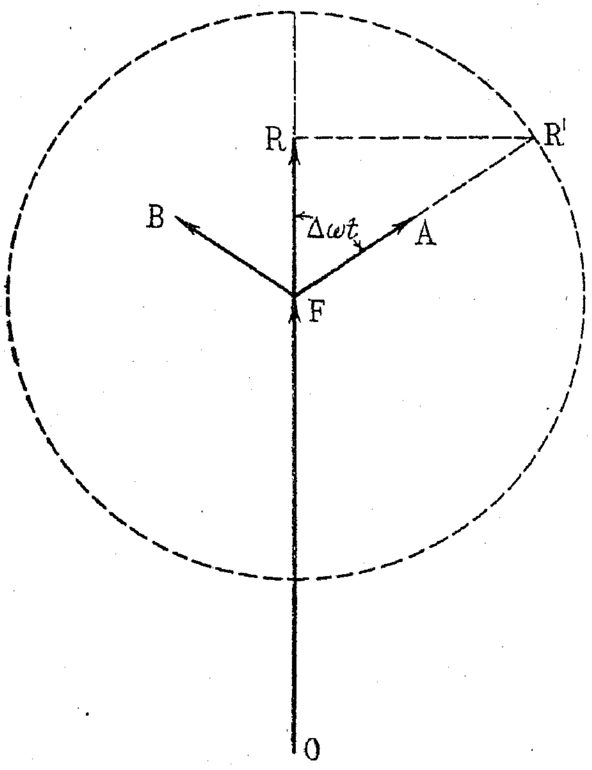

**Fig. 2.**—Symmetric transmission.

$\xi = 1$, $\phi = 0$, $M_N = 0{\cdot}6$.
—·—·— Locus of R.
- - - - - Construction lines.
———— Vectors.

The process of rectification eliminates the carrier; and the resulting audio component, if there are no losses, and if the rectifier is linear, is given by the function $1 + 2S \cos \Delta\omega t - 1 = 2S \cos \Delta\omega t$. Thus the audio-component intensity is represented by the wave-envelope curve if this is drawn about a mean line whose height, above the horizontal zero axis passing through 0 (Fig. 2), is equal to the carrier amplitude.

The modulation constant is defined as the ratio of the incremental peak volts, when modulation takes place, to the value of the carrier peak volts, or

$$M_N = \frac{2S}{1} = 2S \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Thus the values of the audio components after rectification are proportional to $M_N$ and there is no distortion.

### Harmonic distortion when $\xi < 1$, $\phi = 0$ (asymmetric transmission with only magnitude asymmetry, Fig. 3)

Fig. 3 is identical with Fig. 2 except that the vector FB is $\xi$ times FA, $\xi$ being the magnitude-asymmetry factor and having any value between unity and zero. An auxiliary circle is drawn of radius FR′ which is the arithmetic sum of FB and FA. A point G is chosen on a horizontal line passing through F so that FG is the arithmetic difference between FA and FB. FR (not drawn) is the resultant of FA and FB. Knowing that FB = $\xi$FA, that FA = $S$, and that $S = M_N/2$ gives the equivalences

$$FR' = a \text{ and } FG = b$$

where
$$a = \frac{M_N}{2}(1 + \xi) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and
$$b = \frac{M_N}{2}(1 - \xi) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

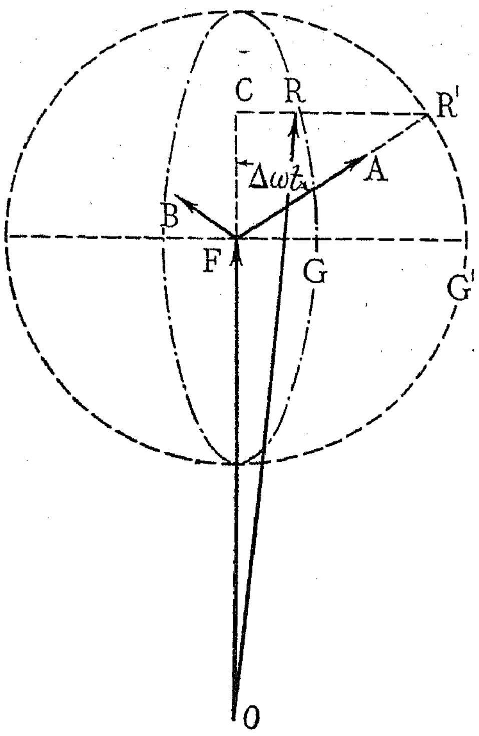

**Fig. 3.**—Asymmetric transmission, magnitude asymmetry only.

$\xi = 0{\cdot}5$, $\phi = 0$, $M_N = 0{\cdot}6$, $M_{A\xi} = 0{\cdot}45$.
—·—·— Locus of R.
- - - - - Construction lines.
———— Vectors.

The resultant of the three vectors OF (the carrier vector) and FB and FA (the sideband component vectors) is OR. The value of OR may be found at any instant of time by finding expressions for the $x$ and $y$ co-ordinates, namely CR and (OF + FC), of the point R.

It can be shown that

$$x = b \sin \Delta\omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and
$$y = 1 + a \cos \Delta\omega t \quad . \quad . \quad . \quad . \quad (5)$$

whence

$$OR = (a^2 \cos^2 \Delta\omega t + b^2 \sin^2 \Delta\omega t + 2a \cos \Delta\omega t + 1)^{\frac{1}{2}} \quad (6)$$

It would be possible to derive the fundamental and harmonic values from expression (6). It is more logical, however, to derive a comprehensive expression dealing with the general case where $\xi$ is less than unity and $\phi$ is finite, when the value of the distortion component can be found for particular cases by writing $\phi = 0$ and $\xi < 1$, or $\xi = 1$ and $\phi$ finite, in such a general expression. In order, however, to check the findings of the analysis, Appendix I gives a simple method for finding an expression for the distortion when $\phi = 0$ and $\xi < 1$. In the meantime a further analysis of the case here dealt with may help to clarify what follows.

The locus of R is an ellipse having axes given by the vector magnitude of FR. Evidently, from (4) and (5),

$$\mathrm{FR} = (b^2 \sin^2 \Delta\omega t + a^2 \cos^2 \Delta\omega t)^{\frac{1}{2}}$$

or, substituting the values of $a$ and $b$ in terms of $M_N$ and $\xi$,

$$\mathrm{FR} = \frac{M_N}{2}(1 + 2\xi \cos 2\Delta\omega t + \xi^2)^{\frac{1}{2}} \quad . \quad . \quad (7)$$

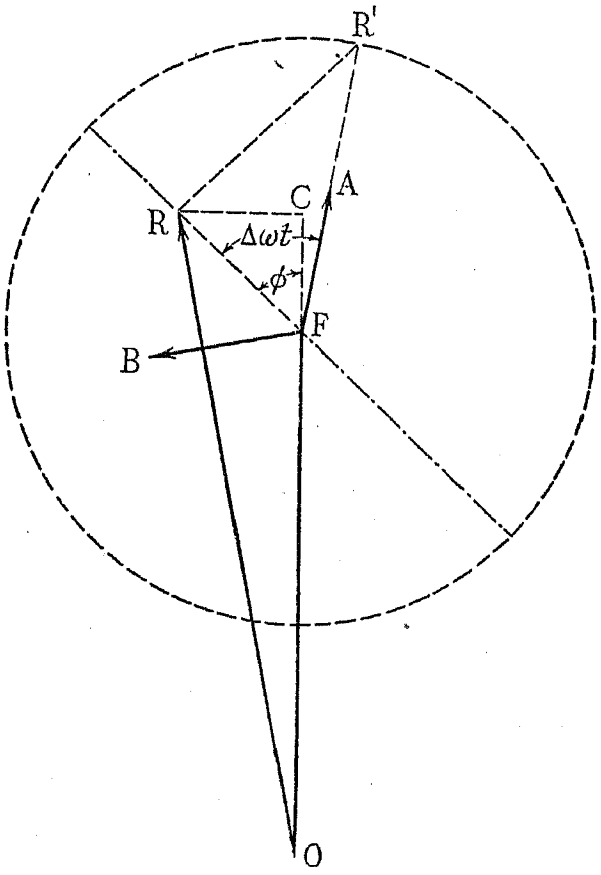

**Fig. 4.**—Asymmetric transmission, phase asymmetry only.

$\xi = 1$, $\phi = 45°$, $M_N = 0{\cdot}6$, $M_{A\phi} = 0{\cdot}54$.

—·—·— Locus of R.
- - - - - Construction lines.
——— Vectors.

The angle made by FR with the vertical is

$$\psi = \Delta\omega t - \arc\tan \frac{\xi \sin 2\Delta\omega t}{1 + \xi \cos 2\Delta\omega t} \quad . \quad . \quad (8)$$

If $\Delta\omega t = 0$ in equations (7) and (8),

$$(\mathrm{FR})_{\Delta\omega t=0} = \frac{M_N}{2}(1 + \xi) = a, \text{ and } (\mathrm{FR})_{\Delta\omega t=0} \text{ is vertical.}$$

Or, if $\Delta\omega t = 90°$,

$$(\mathrm{FR})_{\Delta\omega t=90°} = \frac{M_N}{2}(1 - \xi) = b, \text{ and } (\mathrm{FR})_{\Delta\omega t=90°} \text{ is horizontal}$$

which is obvious from Fig. 3.

In the limits $\xi = 1$, $\xi = 0$, the locus of R becomes either a straight line (symmetric modulation) or a circle (single-sideband transmission). A diagram showing the locus of R and the wave-form resulting for the values $\xi = 0{\cdot}2$, $M_N = 1$, is given in Fig. 6.

The increment of peak volts above the carrier due to modulation, divided by the carrier amplitude, is called the modulation factor. Asymmetry between counterpart sidebands introduces a demodulating effect, and the modulation factor no longer has a value $M_N$ but a new and smaller value $M_A$.

From the expressions derived, and by a consideration of Figs. 3 and 6, it can be seen that we may write

$$M_A = a = \frac{M_N}{2}(1 + \xi) \quad . \quad . \quad . \quad . \quad (9)$$

and that $M_A = M_N$ when $\xi = 1$ (symmetric modulation) and that $M_A = \frac{1}{2}M_N$ when $\xi = 0$ (single-sideband transmission).

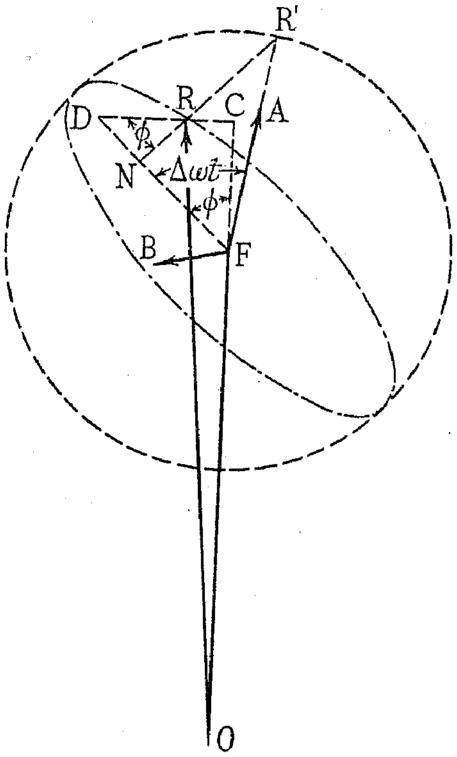

**Fig. 5.**—Asymmetric transmission, general case.

$\xi = 0{\cdot}5$, $\phi = 45°$, $M_N = 0{\cdot}6$, $M_A = 0{\cdot}41$.

—·—·— Locus of R.
- - - - - Construction lines.
——— Vectors.

It is seen that the resultant vector OR swings from one side to the other of the vertical and that this swinging is a measure of the distortion. As $\xi$ decreases so the divergence of the magnitude of the resultant from its value when $\xi = 1$ becomes greater, reaching a maximum when $\xi = 0$, i.e. when one sideband is completely removed. The divergence is a maximum when $\Delta\omega t = 90°$ and 270°, i.e. when, in symmetric modulation, the carrier has its unmodulated amplitude. Clearly the distortion is almost wholly in the nature of a 2nd harmonic. It will be assumed hereafter that the distortion factor is measured wholly by the amplitude of the 2nd harmonic divided by the fundamental.

### Harmonic distortion when $\xi = 1$ and $\phi$ is finite (asymmetric transmission with only phase asymmetry, Fig. 4)

It is extremely unlikely, in the practice of asymmetric transmission or reception, that phase asymmetry will be considerable if the magnitude ratio of the sidebands is unity, but the case is treated separately here in order to lead up in a logical way to the general case where $\xi$ and $\phi$ are both variables. As has been shown elsewhere,* when the sideband-component vectors have equal amplitude but unequal phase-angles with respect to the carrier vector the locus of their resultant (R, Fig. 4) lies along a straight line inclined to the vertical at an angle $\phi$, which is equal to $(\theta_1 + \theta_2)/2$ where $\theta_1$ and $\theta_2$ are the angles made by the sideband-component vector with the carrier. The angle $2\phi$ is in fact the algebraic sum of the sideband phase-angles and is finite when $\theta_1 \not\leq \theta_2$. $\phi$ is thus the phase-asymmetry distortion-factor.

A diagram illustrating the case is given in Fig. 4, and it can be shown from this that the co-ordinates of R are given by

$$x = M_N \cos \Delta\omega t \sin \phi \quad . \quad . \quad . \quad . \quad (10)$$

$$y = 1 + M_N \cos \Delta\omega t \cos \phi \quad . \quad . \quad . \quad (11)$$

whence

$$\text{OR} = (M_N^2 \cos^2 \Delta\omega t + 2M_N \cos \Delta\omega t \cos \phi + 1)^{\frac{1}{2}} \quad (12)$$

Evidently, by writing $\Delta\omega t = 0$ or $\Delta\omega t = 180°$ respectively,

$$(\text{OR})_{max.} = (M_N^2 + 2M_N \cos \phi + 1)^{\frac{1}{2}} \quad . \quad (13\text{A})$$

$$(\text{OR})_{min.} = (M_N^2 - 2M_N \cos \phi + 1)^{\frac{1}{2}} \quad . \quad (13\text{B})$$

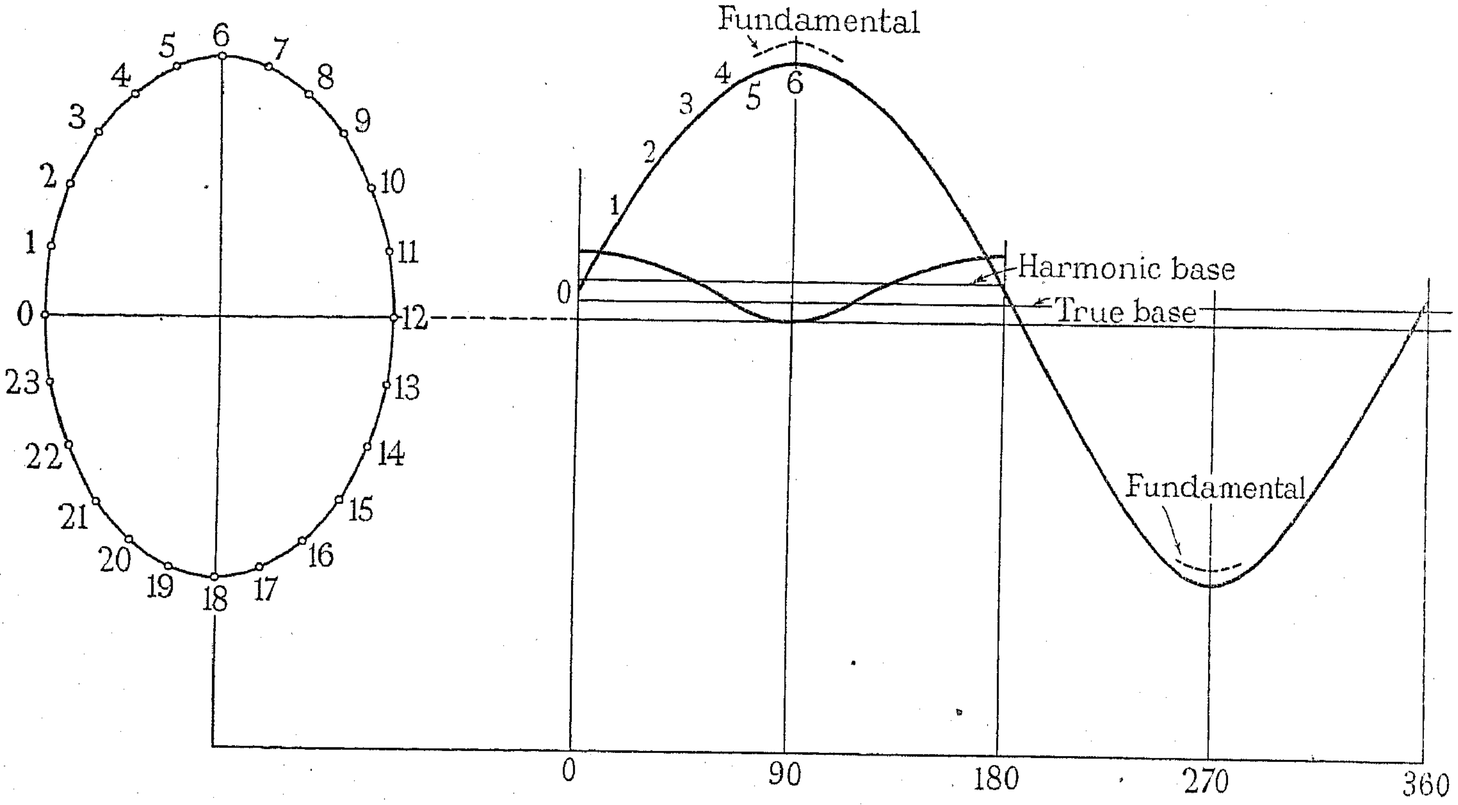

**Fig. 6**

$M_N = 1$, $\xi = 0{\cdot}2$, $2\phi = 0$, $M_A = 0{\cdot}6$, $\delta_\xi = 7{\cdot}0$ %.

If $\phi = 90°$ $(\theta_1 + \theta_2 = 180°)$,

$$(\text{OR})_{max.} = (\text{OR})_{min.} = (M_N^2 + 1)^{\frac{1}{2}}$$

and the resulting wave-envelope contains no fundamental component and is wholly of the form of a harmonic of the modulating component. This fact was revealed by Johnstone and Wright.*

A diagram showing the resulting wave-form distortion is given in Fig. 7 for certain values of $M_N$ and $\phi$. The maximum distortion is seen to occur when $\Delta\omega t = 0$ and 180°; this proves that, while phase asymmetry, like magnitude asymmetry between sideband components, produces a swinging of the resultant OR about the vertical, the resulting harmonic, which is also mainly of double the fundamental frequency, has a 90° phase-difference from that produced by magnitude asymmetry.

An approximate expression for the distortion is given in Appendix I.

### Harmonic distortion when $\xi < 1$ and $\phi$ is finite (general case of asymmetric transmission)

From previous reasoning it is clear that when $\xi < 1$ and $\phi$ is finite the locus of R lies on an ellipse the major axis of which is inclined at an angle $\phi$ to the vertical. Three examples are illustrated in Figs. 8, 9, and 10.

It can be proved from Fig. 5 that

$$x = a \cos \Delta\omega t \sin \phi + b \sin \Delta\omega t \cos \phi \quad . \quad . \quad (14)$$

$$y = 1 + a \cos \Delta\omega t \cos \phi - b \sin \Delta\omega t \sin \phi \quad (15)$$

Appendix II shows that an approximate value for the distortion and demodulation caused by both magnitude and phase asymmetry of counterpart sidebands may be obtained by writing

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* *Wireless Engineer*, 1936, vol. 13, pp. 517, 534.

* *Wireless Engineer*, 1936, vol. 13, p. 534.

$$(x^2+y^2)^{\frac{1}{2}} \simeq y\left(1+\frac{x^2}{2y^2}\right) = y+\frac{x^2}{2y} \simeq y+\frac{x^2}{2}$$

There are thus two approximations: one is based upon the fact that $\sqrt{(1+Z)} \simeq 1+\frac{1}{2}Z$, where $Z$ is a quantity not much greater than 1; the other is to assume that $y$ in the denominator of the fraction $x^2/(2y)$ is always equal to the unmodulated carrier amplitude (assumed to be unity). It will be seen that, in the majority of cases, $x$ does not in fact exceed $y$. Consideration also shows that the assumption that $y$ can be written as unity in the ratio $x^2/(2y)$ is justified in the majority of cases because, where FC does in fact greatly exceed OF (Fig. 5) and where, therefore, $y$ is considerably greater or less than unity, $x$ is very small and therefore

$$y+\frac{x^2}{2y} \simeq y+\frac{x^2}{2}$$

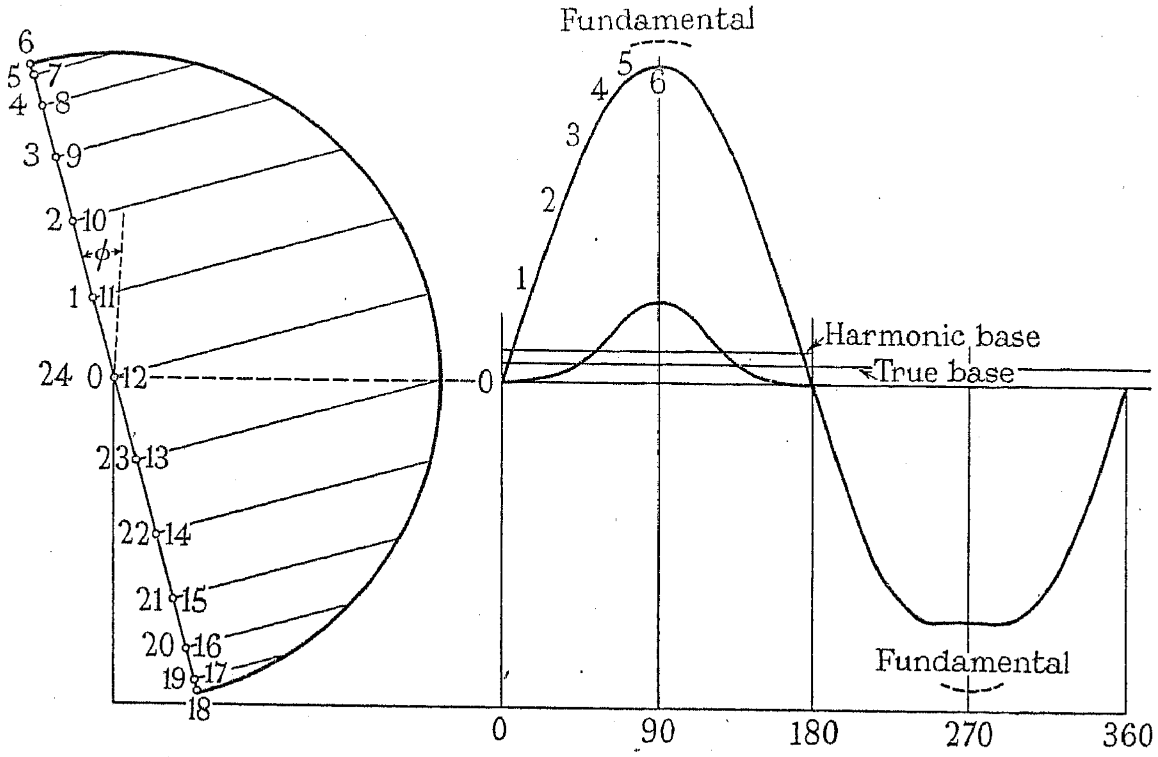

**Fig. 7**

$M_N = 1$, $\xi = 1$, $2\phi = 30°$, $M_{A\phi} = 0{\cdot}95$, $\delta_\phi = 7{\cdot}4$ %.

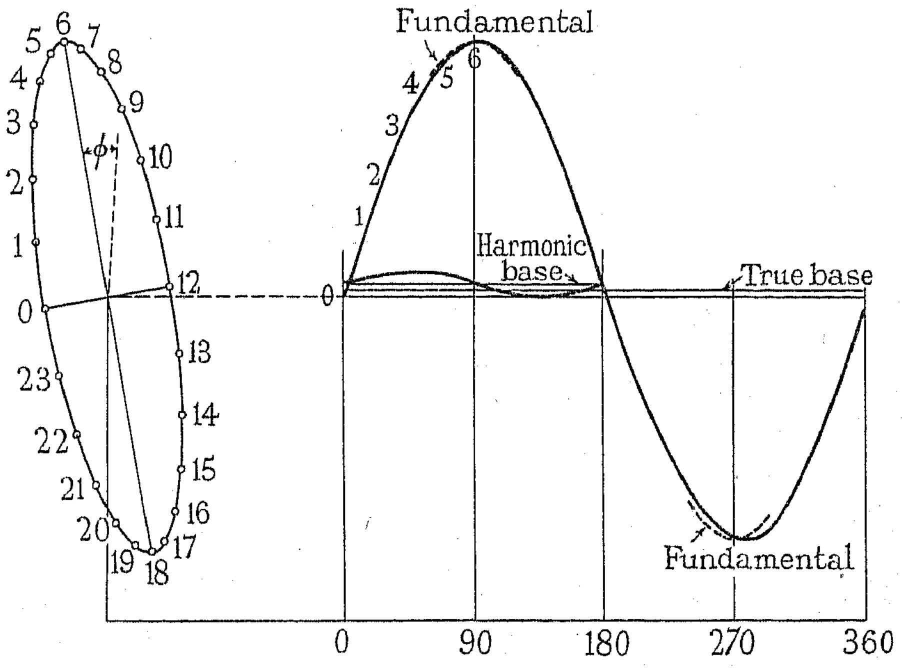

**Fig. 8**

$M_N = 1$, $\xi = 0{\cdot}6$, $2\phi = 20°$, $M_A = 0{\cdot}77$, $\delta = 2{\cdot}8$ %.

There are, however, possible conditions where the approximation cannot be justified, and these occur when $\xi$ is unity or nearly unity, $\phi$ is of moderate value, and $M_N$ is large. But if $\xi$ and $M_N$ have the value unity and $\phi$ is large, the approximation gives a true result just as it does if, whatever the values of $\xi$ and $M_N$, $\phi$ is zero, i.e. the approximation is justified in the case where only magnitude asymmetry exists.

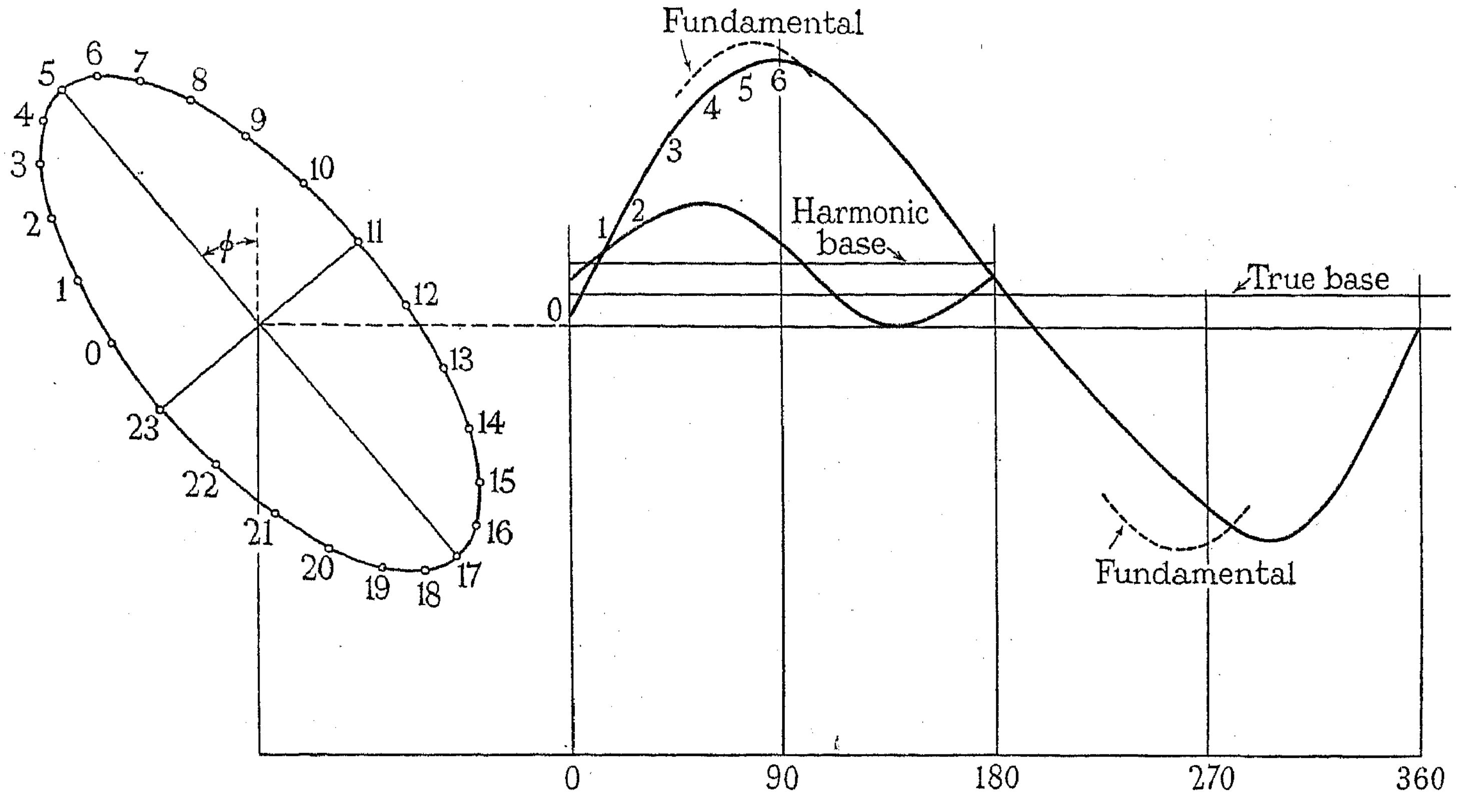

**Fig. 9**

$M_N = 1$, $\xi = 0{\cdot}4$, $2\phi = 80°$, $M_A = 0{\cdot}585$, $\delta = 12\,\%$.

The accuracy of the formula derived from the approximation can be judged from a study of Table 1, which is referred to again hereafter. It should, lastly, be noted that in the practice of asymmetric technique the approximation is practically always justified.

and that the modulation factor is given as

$$M_A = \frac{M_N}{2}(1 + \xi^2 + 2\xi \cos 2\phi)^{\frac{1}{2}} \quad . \quad . \quad (17)$$

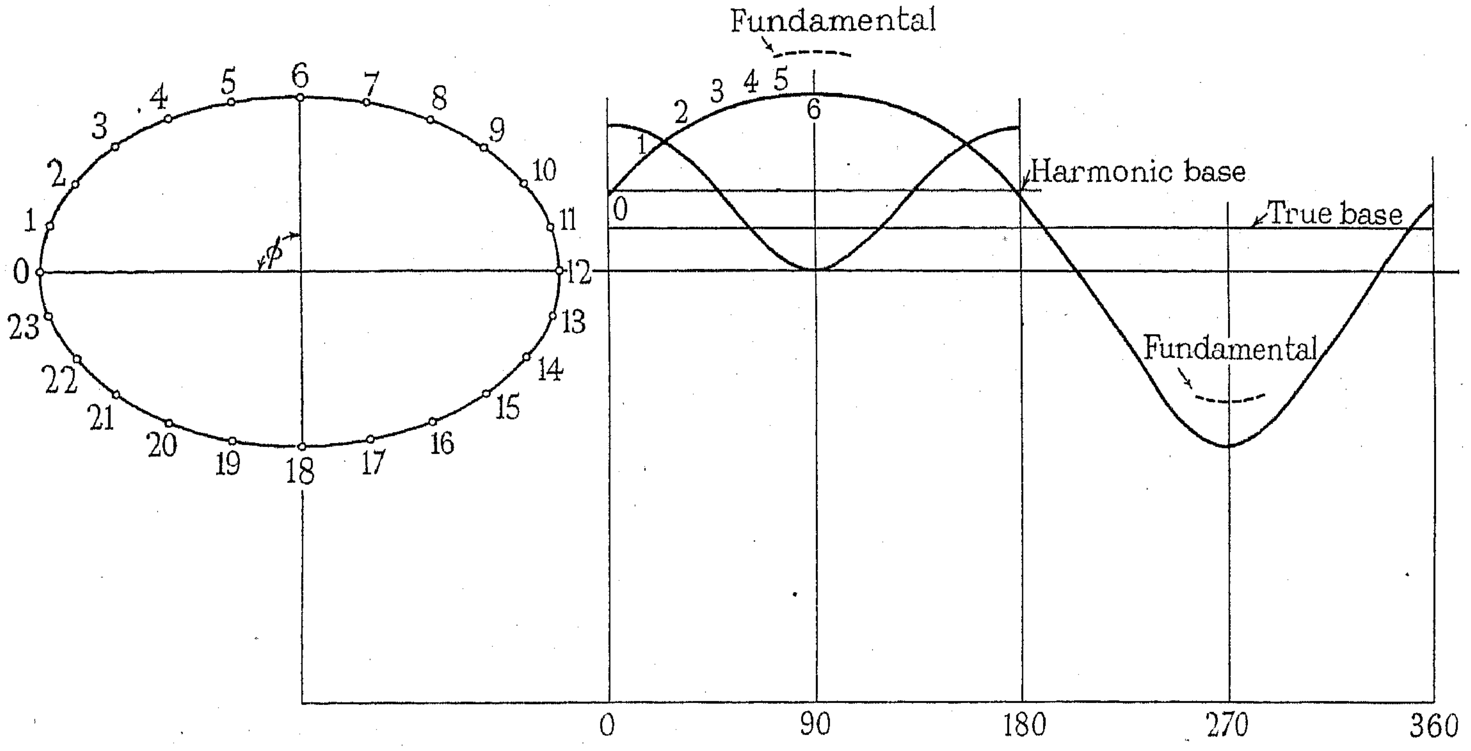

**Fig. 10**

$M_N = 1$, $\xi = 0{\cdot}2$, $2\phi = 180°$, $M_A = 0{\cdot}4$, $\delta = 24{\cdot}6\,\%$.

Thus Appendix II shows that the general case of asymmetric transmission can be analysed by writing that, if $\xi < 1$ and $\phi$ is finite, the distortion is given by

$$\delta = \frac{M_N}{8}\,\frac{1 + \xi^2 - 2\xi \cos 2\phi}{(1 + \xi^2 + 2\xi \cos 2\phi)^{\frac{1}{2}}} \quad . \quad . \quad (16)$$

The expressions for the special cases can be written as follows:—

When $\xi < 1$, $\phi = 0$ (magnitude asymmetry only),

$$\delta_\xi = \frac{M_N}{8}\,\frac{(1 - \xi)^2}{(1 + \xi)} \quad . \quad . \quad . \quad . \quad (18)$$

(see Appendix I)

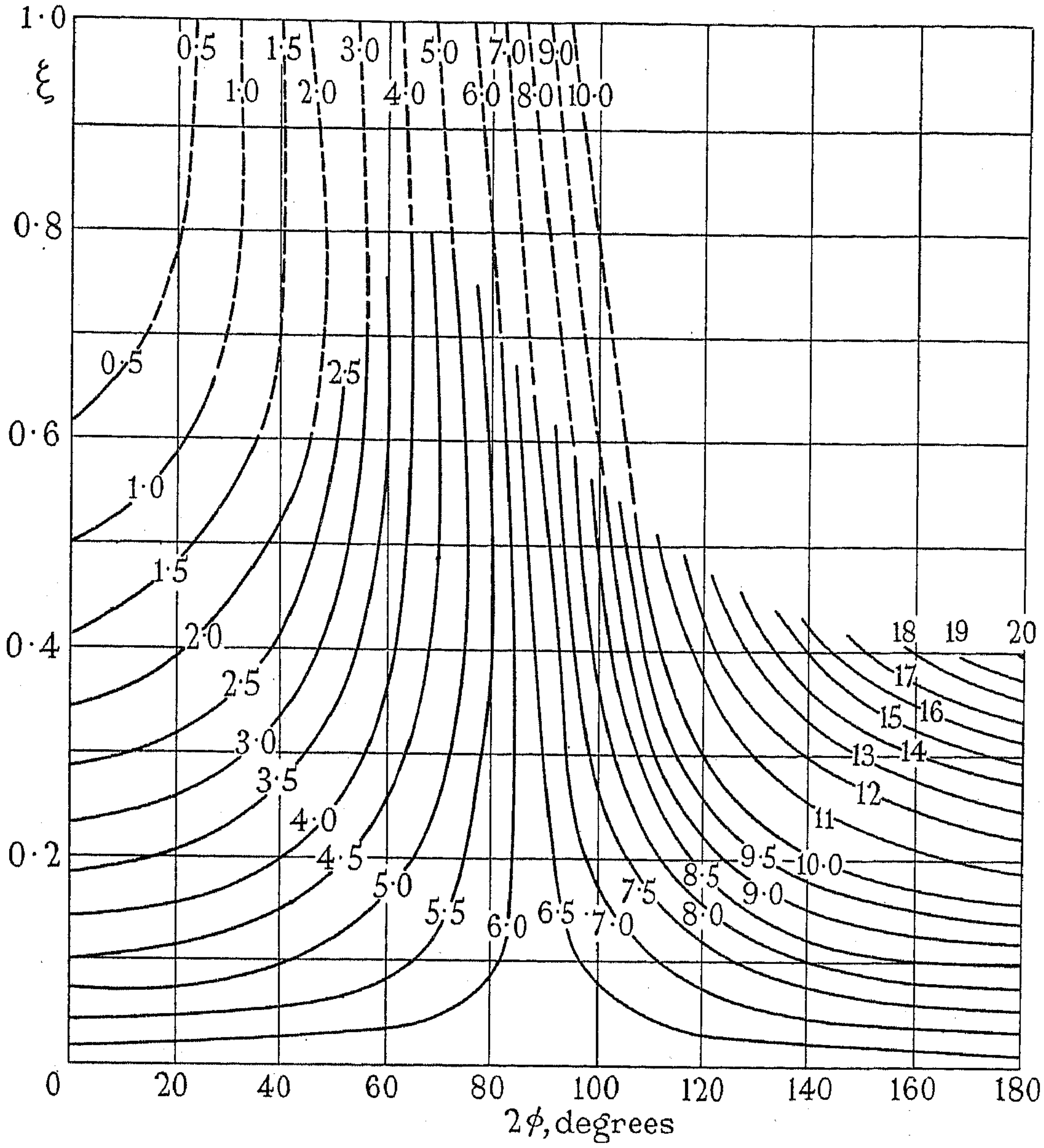

**Fig. 11A**

$M_N = 0{\cdot}5$. Figures on curves denote values of $\delta$, expressed as percentages.

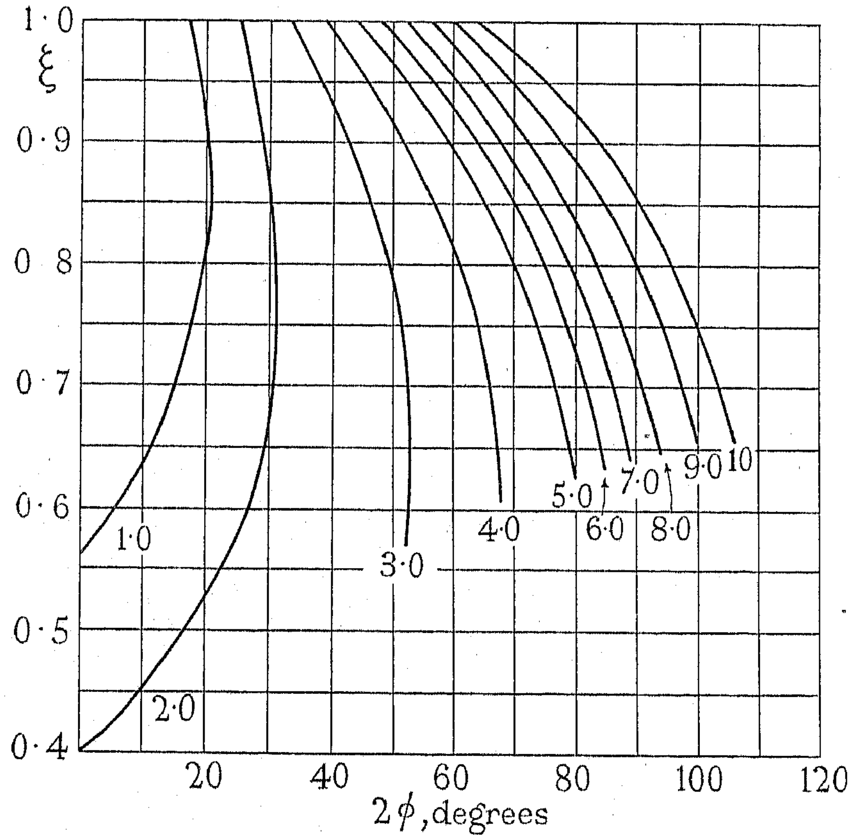

**Fig. 11B**

$M_N = 0{\cdot}75$. Figures on curves denote values of $\delta$, expressed as percentages.

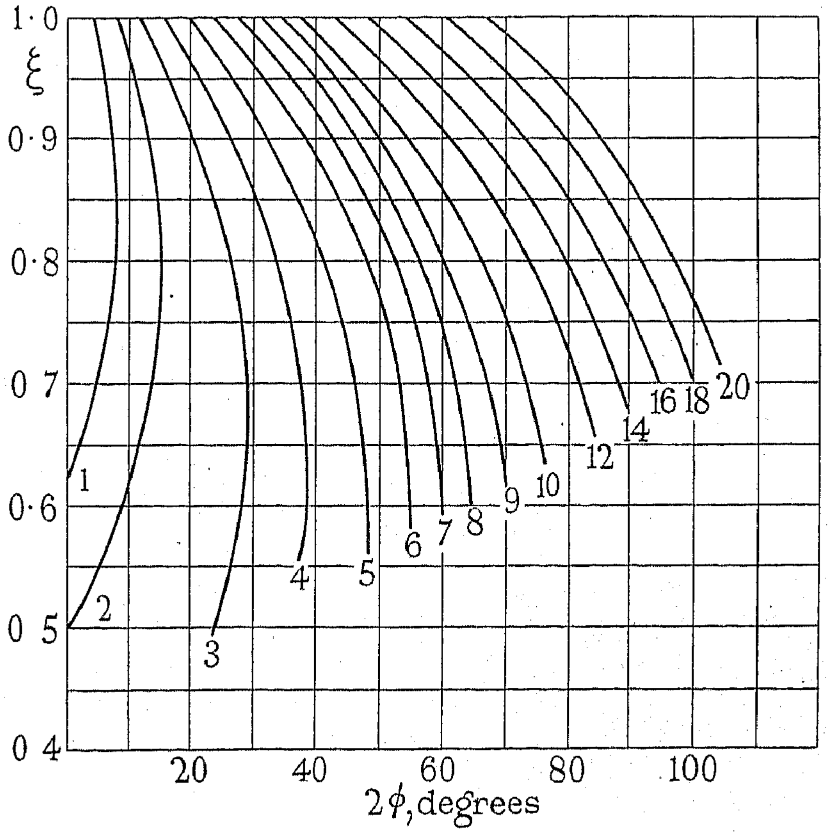

**Fig. 11C**

$M_N = 1$. Figures on curves denote values of $\delta$, expressed as percentages.

$$M_{A\xi} = \frac{M_N}{2}(1 + \xi) \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

[see equation (9)]

When $\xi = 1$, $\phi$ is finite (phase asymmetry only),

$$\delta_\phi = \frac{M_N\sqrt{2}}{8}\frac{(1 - \cos 2\phi)}{(1 + \cos 2\phi)^{\frac{1}{2}}} \quad . \quad . \quad . \quad (20)$$

$$M_{A\phi} = M_N\sqrt{2}(1 + \cos 2\phi)^{\frac{1}{2}} \quad . \quad . \quad . \quad (21)$$

It is relevant to note in (21) that, if $\phi = 90°$ $(\theta_1 + \theta_2 = 180°)$, $M_{A\phi} = 0$; showing, what has been previously mentioned, that, in this condition, the carrier is completely demodulated. Since this fact can be proved from (12) and (13), which are not approximations, evidently (20) and (21) are accurate whatever the values of $\xi$ and $M_N$ provided that $\phi$ is large.

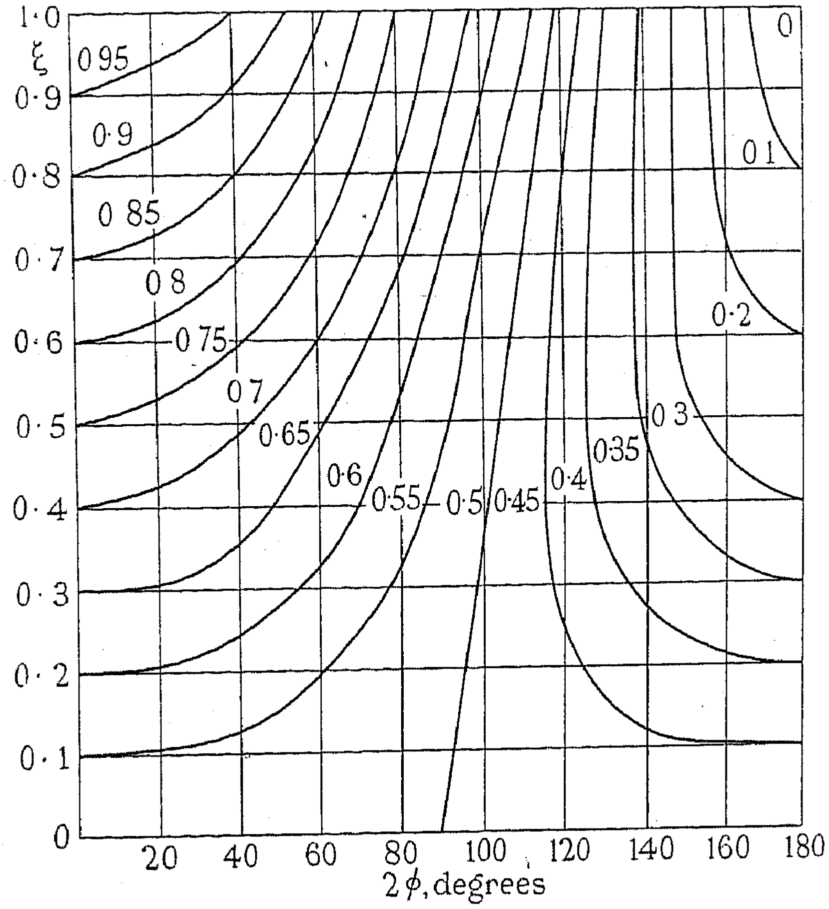

**Fig. 12**

$M_N = 1$. Figures on curves denote values of $M_A$.

In order, however, to supplement the information given by equations (16) to (21), derived from the approximations, it has been thought advisable to determine the degree of distortion when $\xi = 1$ and $\phi$ is finite and of medium value. It is a pity that the analysis given in the paper by Johnstone and Wright cannot be used, but, as has been pointed out,* it does not appear to check with straightforward graphical analysis. The author felt that in these circumstances it would be best to resort to a graphical method to derive the distortion.† The results are shown in Figs. 13A and 13B, and prove (Fig. 13A) that, when $\xi = 1$ and $\phi$ is finite, for values of modulation below 50 % the distortion is proportional to modulation, and (Fig. 13B) that the approximate equation (20) can be used for medium values of $\phi$ without introducing an error much above 10 % provided the modulation is less than 50 %.

A great many cases were analysed by graphical methods in the way illustrated in Figs. 6–10 inclusive, and Table 1 gives a comparison of the results (for $M_N = 1$) with those derived from equations (16) and (17). It is seen that the formula is accurate within the limits given.

## Analysis of the Sound Spectrum

The conception of the asymmetric system is based upon the assumption that distortion may be kept constant and negligible, because, as asymmetry is increased between sideband components by the sideband-cutting filter, the modulation factor, on account of the nature of the sounds transmitted, becomes smaller. The foregoing expressions relating distortion and demodulation with phase and magnitude asymmetry would be useless in determining the requirements of the sideband-cutting filter unless supplemented by a knowledge of the relationship between $M_N$ and the modulation frequency $\Delta f$.

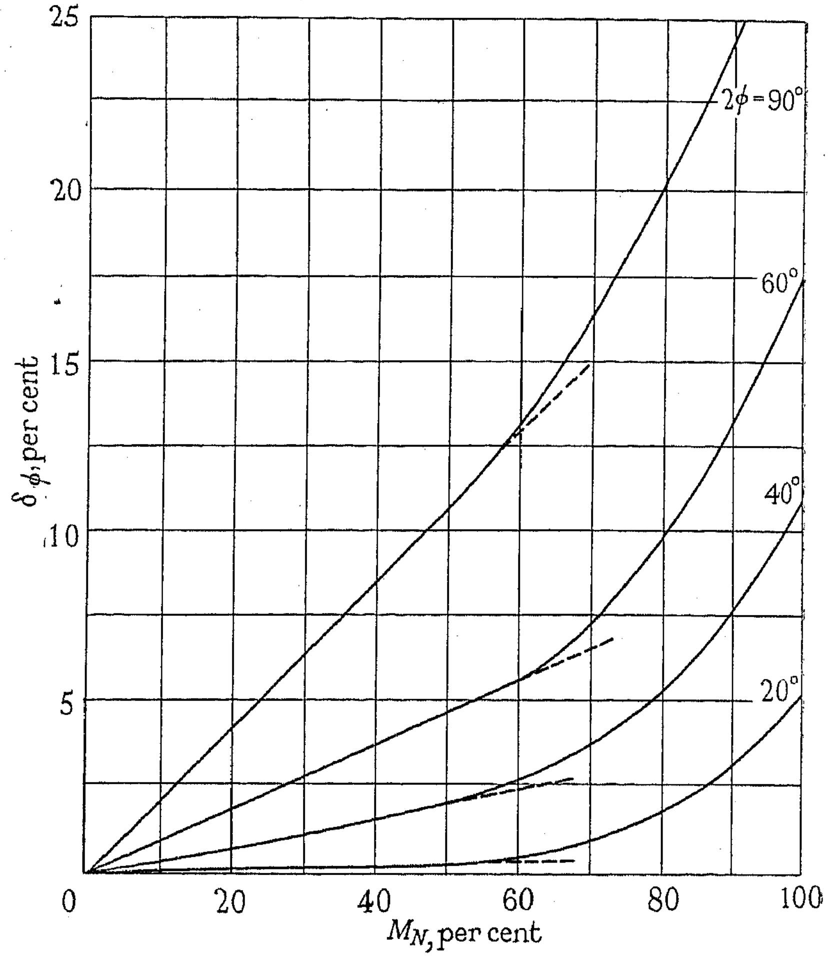

**Fig. 13A**

A great deal of work has been done in attempting to determine the nature of the sound spectrum. It is not proposed here either to discuss the methods adopted by other workers or necessarily to use the results so obtained. What we require to know in designing the asymmetric system is more a probability of the distribution of the magnitudes of the components within the spectrum of typical speech and music sounds than to be influenced by exceptional conditions.

The author some 7 years ago devised a method for analysing this probability. The circuits used comprised a resistance-coupled tuned circuit tuned to different audio frequencies, and the momentary peak maxima developed across this circuit, when energized from a microphone output, were measured by a "slide back" voltmeter. The measurements were repeated in 1936 and gave substantial agreement with the earlier results. Hence the curve of Fig. 14, derived from an average of hundreds of observations, is taken as a guide for designing asymmetric-sideband filters.

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* *Wireless Engineer*, 1936, vol. 13, p. 517.

† An approximate formula, given in Appendix I, applies when $\phi$ is of moderate values and $M_N$ is large (see Fig. 13B).

It is not said that the curve is truly representative of the possible maxima which may on rare occasions occur, but it is argued that if it is used to guide the design of the system under discussion it is unlikely that such a system will give, over a long period of listening, any distortions which could be considered intolerable. Indeed, the choice of the values in Fig. 14 can only be criticized in terms of the performance of the asymmetric transmission system, rather than in terms of its comparison with other results obtained with a different object in view.

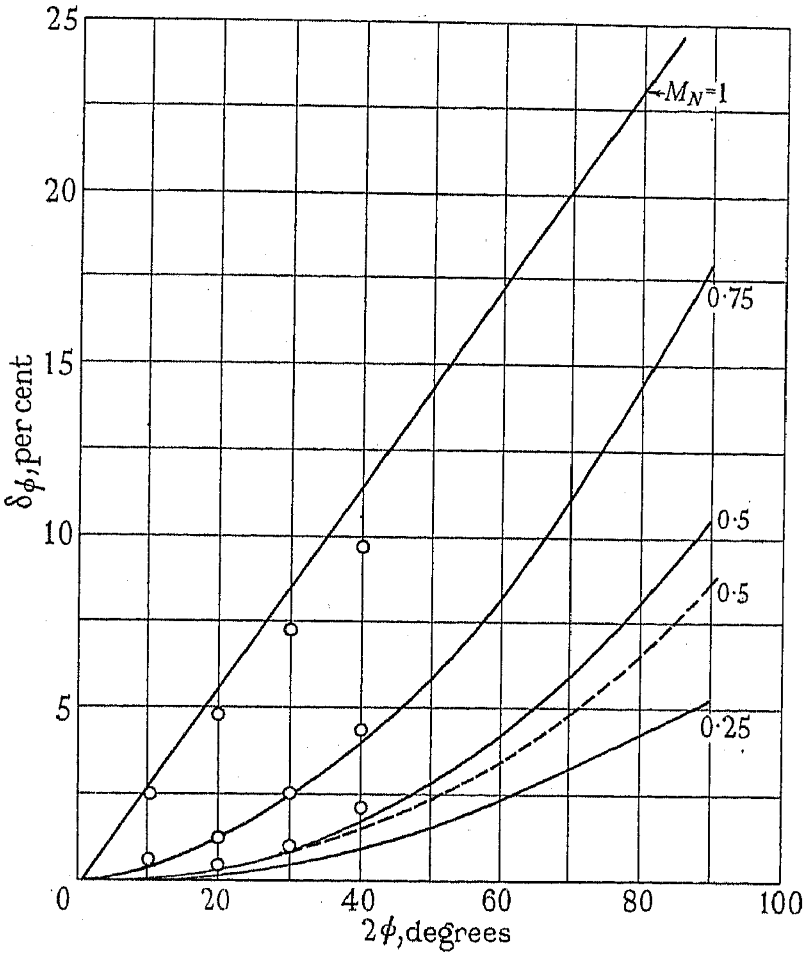

**Fig. 13B**

o o o Points derived from expression in Appendix II.
——— Graphical analysis.
- - - - - - From approximate formula (16).

## Summary of the Analysis

The analysis shows that both phase and magnitude asymmetry between counterpart sideband components created by any given modulation frequency produce a distortion of the modulated-wave envelope and that this distortion takes chiefly the form of a 2nd harmonic of the fundamental.

The conception of the asymmetric systems implies that magnitude asymmetry between sideband components must be present. The analysis, however, reveals a point of major importance, namely that if both phase and magnitude asymmetry exist together in considerable degree, then the distortion will be very much greater than if magnitude asymmetry existed alone. It is practically impossible in the asymmetric system for phase asymmetry to exist without some magnitude asymmetry, but evidently phase asymmetry is dangerous when magnitude asymmetry is not very pronounced. Phase asymmetry may introduce a very serious demodulation of the carrier, a demodulation which could be, even in practical conditions of working, far greater than that introduced by magnitude asymmetry. It will be shown later, however, that it is possible to avoid the effects of phase asymmetry and yet produce a very large attenuation of a part of one sideband.

### Curves relating $\xi$ and $\phi$ for constant values of $\delta$.

The summary to this analysis is given in a form convenient for direct consultation by engineers engaged in designing asymmetric-sideband systems. This summary is to be found in Figs. 11A, 11B, and 11C, where curves of constant distortion are plotted against $\xi$ and $2\phi$.

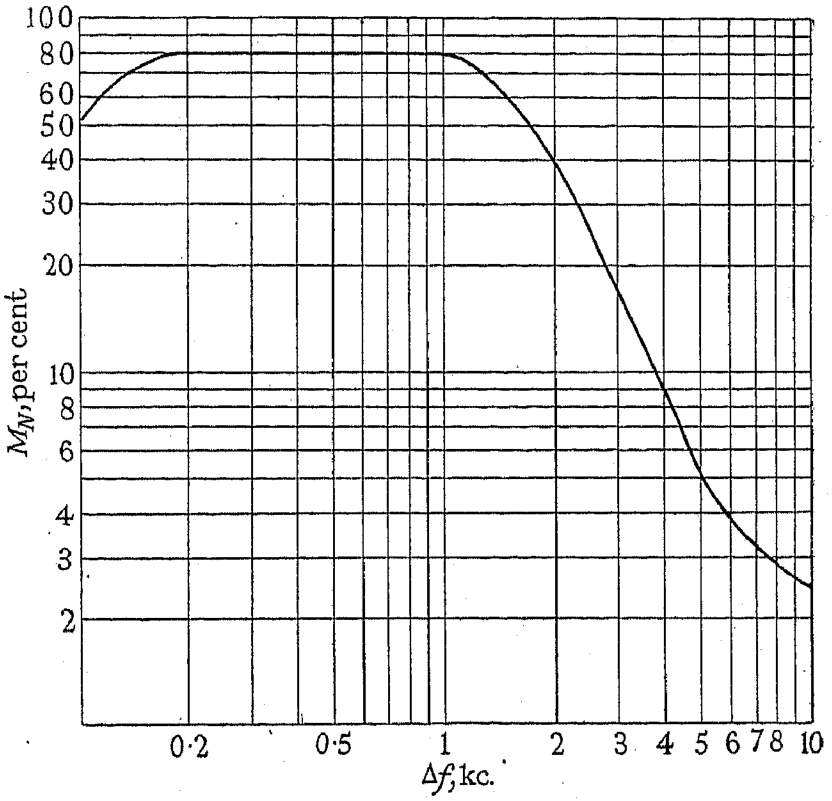

**Fig. 14.**—Analysis of sound spectrum (author's results).

Fig. 11A gives a set of curves derived from equation (16) for $M_N = 0\cdot5$ (50 % modulation). Values are only given which are likely to have practical relevance to asymmetric technique. The full-line curves represent the cases where the approximate formula applies for all values of $M_N$, and the broken-line curves apply for all values of $M_N$ below and including 0·5. The distortion can thus be assessed for any value of $M_N$, if the full-line curves are used, by multiplying the distortion found from Fig. 11A by the new modulation factor, at which it is required to find the distortion, divided by 0·5. The dotted lines are only used if $M_N$ exceeds 0·5. Figs. 11B and 11C show the curves which apply where

$\delta$ is not always proportional to $M_N$. The curves are drawn for modulation factors of 0·75 and 1·0 respectively. These curves are derived from the information given in Figs. 13A and 13B, which were in turn derived from a graphical analysis. The charts, taken together, give all the required information and meet all practical cases.

The points on the curves of Fig. 11A where $\xi < 1$ and $\phi$ is finite were found by plotting curves of $\delta$ against $\phi$ for different values of $\xi$. The values of $\xi$ and $\phi$ for constant values of $\delta$ were then read off directly and transferred to Fig. 11A. This simple method was adopted because the conversion of (16) to express $\xi$ in terms of the constants $M_N$ and $\delta$ and the variable $\phi$ (or vice versa to express $\phi$ in terms of the constants $M_N$ and $\delta$ and the variable $\xi$) involves rather heavy algebra and it was thought that the use of curves would be quicker, simpler, and probably more reliable.

### Table 1

For $M_N = 1$

| Values of $\xi$ and $2\phi$ | | Results of graphical analysis | | Results from chart of Figs. 11 and 12 | | Ratio of results of graphical analysis to those from charts | |
|---|---|---|---|---|---|---|---|
| $\xi$ | $2\phi$ | $\delta$ | $M_A$ | $\delta$ | $M_A$ | $\delta$ | $M_A$ |
| 1 | 30 | 7·4 | 0·95 | 2* | 0·97 | 3·7 | 0·98 |
| 0·8 | 20 | 2·8 | 0·88 | 0·8* | 0·875 | 3·5 | 1·005 |
| 0·8 | 40 | 6·8 | 0·89 | 3·0* | 0·85 | 2·26 | 1·05 |
| 0·6 | 20 | 2·8 | 0·77 | 1·8 | 0·78 | 1·55 | 0·99 |
| 0·6 | 40 | 4·0 | 0·77 | 5·5 | 0·75 | 0·73 | 1·025 |
| 0·6 | 100 | 14·2 | 0·6 | 18·5 | 0·52 | 0·77 | 1·15 |
| 0·4 | 40 | 6·3 | 0·66 | 5·0 | 0·66 | 1·26 | 1·00 |
| 0·4 | 80 | 12·0 | 0·585 | 11·1 | 0·57 | 1·08 | 1·02 |
| 0·4 | 100 | 16·0 | 0·52 | 16·0 | 0·51 | 1·00 | 1·02 |
| 0·2 | 0 | 7·0 | 0·6 | 6·8 | 0·6 | 1·03 | 1·00 |
| 0·2 | 40 | 7·3 | 0·565 | 8·0 | 0·57 | 0·91 | 0·99 |
| 0·2 | 90 | 12·1 | 0·495 | 13·2 | 0·51 | 0·92 | 0·97 |
| 0·2 | 180 | 24·6 | 0·4 | 22·3 | 0·4 | 1·1 | 1·00 |

* These values are for cases where the approximation is no longer justified and where Figs. 11B and 11C must be used. Note the good agreement for $M_A$, which does not depend upon an approximation.

Certain fixed points can, however, be obtained by giving $\phi$ certain specific values and then expressing $\xi$ in terms of $M_N$ and $\delta$. Thus when $\phi = 0$ it can be shown from (18) that $\xi$ is given by

$$(\xi)_{\phi=0} = (1 + d_1) - \left[d_1(d_1 + 4)\right]^{\frac{1}{2}} \quad . \quad . \quad (22)$$

where $d_1 = 4\delta_\xi / M_N$

and from (16) that, when $2\phi = 180°$,

$$(\xi)_{2\phi=180°} = \left[d_2(d_2 + 4)\right]^{\frac{1}{2}} - (1 + d_2) \quad . \quad (23)$$

where $d_2 = 4\delta_\phi / M_N$.

Points derived from (22) and (23) can be plotted directly on the charts without reference to curves. We may also convert (20) for Fig. 11A to give cos $\phi$ in terms of $M_N$ and $\delta$. If $d_3 = 4\delta_\phi / M_N$, then

$$2\phi = \text{arc cos}\left[1 + d_3^2 - d_3(d_3^2 + 4)^{\frac{1}{2}}\right] \quad . \quad (24$$

**Curves relating $\xi$ and $\phi$ for constant values of $M_A$.**

The degree of demodulation of the carrier, given certain values of magnitude and phase asymmetry, can be readily found from the chart of Fig. 12. In order to plot the required curves we may resolve (17) and show that

$$\xi = -\cos 2\phi \pm \left[\cos^2 2\phi + (4M_A^2 - 1)\right]^{\frac{1}{2}}$$

and so give $M_A$ constant chosen values and find $\xi$ for a given value of $2\phi$. Evidently $\xi$ must lie between 0 and 1. It can have alternative values of

$$\xi = -\cos 2\phi + \left[\cos^2 2\phi + (4M_A^2 - 1)\right]^{\frac{1}{2}}$$

and $$\xi = -\cos 2\phi - \left[\cos^2 2\phi + (4M_A^2 - 1)\right]^{\frac{1}{2}}$$

provided always the resulting values lie between 0 and 1. If $M_A = 0·5$, then

$$(\xi)_{M_A=0·5} = -2\cos 2\phi$$

so that, to be real, $2\phi$ must exceed 90°. Thus when

$$2\phi = 90°,\ (\xi)_{M_A=0·5} = 0;$$

and when $$2\phi = 120°,\ (\xi)_{M_A=0·5} = 1.$$

## (4) DESIGN OF ASYMMETRIC-SIDEBAND TRANSMISSION SYSTEM

### Asymmetric Carrier-Frequency Filter

The foregoing quantitative analysis furnishes all the information necessary to enable an expression to be determined for the required attenuation/frequency characteristic of a sideband-cutting filter which would give a constant and, it is believed, negligible distortion.

The increasing asymmetry factor as the modulation frequency increases is obviously a measure of increasing attenuation and distortion. But as the modulation frequency increases the modulation factor decreases, and so distortion is kept constant and negligible.

If $\xi$ is known for any given modulation frequency (and hence sideband-component frequency), then, assuming that one sideband component is fully transmitted, $\alpha$ (the attenuation constant), which is a function of the reciprocal of $\xi$, will also be known. Since expressions exist to relate $\xi$ with $\delta$ and $M_N$, therefore it is possible to find a value of $\alpha$ which, with a given value of $M_N$, keeps $\delta$ constant for any given sideband frequency.

It has been shown that, if phase asymmetry is neglected, then

$$\delta_\xi = \frac{M_N}{8}\,\frac{(1-\xi)^2}{1+\xi}$$

This equation must be converted so that $\xi$ is given as a function of $M_N$ and $\delta_{\xi 0}$; the latter being a constant, small, and negligible value of $\delta_\xi$. This will give a value of $\xi$, at any particular modulation frequency, given by the value of $M_N$ at that modulation frequency and the constant $\delta_{\xi 0}$. The values of $\xi$ required at any given modulation frequency can then be converted to the values of $\alpha$ at the corresponding sideband frequency. Such values of $\alpha$ plotted against sideband frequency will then give the attenuation curve of a filter producing a constant distortion $\delta_{\xi 0}$.

The required curve having been found, the procedure will then be to choose a network giving as nearly as possible the required attenuation/frequency characteristic. The phase asymmetry introduced by such a network can be assessed either by calculation or by experiment. This will give values of both $\xi$ and $\phi$, and, from the charts of Figs. 11 and 12, the total distortion and demodulation can be found. If phase asymmetry appears to add more than a tolerable amount of distortion to that inevitably existing owing to magnitude asymmetry, alternative networks must be studied until a satisfactory compromise is reached; design requirements are revealed through trial and error.

Equation (22) expresses $\xi$ in terms of $M_N$ and $\delta$. This, however, applies to a system in which the overall characteristic of the transmitter would not be level, since as $\xi$ decreases the effective modulation of the carrier also decreases. This demodulation must be corrected in some part of the system. Correction must take the form either of multiplying the input modulations or of multiplying the output from the detector by a factor $M_N/M_A$. Equation (22) expresses the case where no correction is applied.

It is, in theory, immaterial whether correction takes place in the receiver (as an audio compensation in the post-detector circuits) or in the transmitter. In practice, however, correction of the asymmetric filter demodulation must take place in the transmitter, otherwise existing receivers would remark the changes when any transmitter was adapted to the asymmetric system. It is, however, pertinent to observe that correction for demodulation makes it necessary to include more of the cut-away sideband than would be necessary if demodulation correction were not applied. In order to illustrate this point two cases are considered throughout the paper, namely one in which correction is applied and the other where it is assumed to be absent.

If demodulation correction is used, then the equation for $\delta_{\xi 0}$ is given by multiplying the value of $\delta_\xi$ obtained from (22) by $M_N/M_{A\xi}$.

Thus $$\delta_{\xi 0} = \frac{M_N}{8} \times \frac{M_N}{M_{A\xi}}\,\frac{(1-\xi)^2}{1+\xi}$$

Or, from (19),

$$\delta_{\xi 0} = \frac{M_N}{4}\left(\frac{1-\xi}{1+\xi}\right)^2$$

If again we write $d = 4\delta_{\xi 0}/M_N$, then it can be proved that

$$\xi_T = \frac{(\sqrt{d}-1)^2}{d-1} \quad . \quad . \quad . \quad . \quad (25)$$

showing the value of $\xi (= \xi_T)$ when demodulation correction is applied.

Equation (22) expresses the value of $\delta$ when the frequency characteristic is not level. Converting (22) to give $\xi_R$,

$$\xi_R = (1+d) - \left[d(d+4)\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad (26)$$

We may write that the attenuation constant of the sideband components of reduced amplitude is given in decibels by

$$\alpha = 20 \log_{10} \frac{1}{\xi}$$

Or, from (25) and (26),

$$\alpha_T = 20 \log_{10} \frac{d-1}{(\sqrt{d}-1)^2} \quad . \quad . \quad . \quad (27)$$

$$\alpha_R = 20 \log_{10} \frac{1}{(1+d) - \left[d(d+4)\right]^{\frac{1}{2}}} \quad . \quad (28)$$

Equation (27) shows that as $d$ gets nearer and nearer to unity, $\alpha_T$ approaches infinity. Equation (28) shows that when $d = 0 \cdot 5$, $\alpha_R$ is infinity.

Thus it is seen that, with demodulation correction, distortion will not be increased above a certain value even though one side-band component is completely cut away, always provided that the modulation factor $M_N$ does not exceed a value $4\delta_{\xi 0}$. Similarly, when the frequency characteristic is not level, the sideband may be wholly removed provided the modulation factor does not exceed $8\delta_{\xi 0}$. Writing the particular values of these modulation factors as $M_{N\infty T}$ and $M_{N\infty R}$ respectively, evidently $M_{N\infty R} = 2M_{N\infty T}$.

But $M_N$ is a function of the modulating frequency $\Delta f$ and decreases as $\Delta f$ increases (see Fig. 14), so that $M_{N\infty T}$ may be related to a modulating frequency $\Delta f_{\infty T}$ and $M_{N\infty R}$ to a modulating frequency $\Delta f_{\infty R}$. Since $M_{N\infty R} = 2M_{N\infty T}$, therefore $\Delta f_{\infty R} < \Delta f_{\infty T}$. But the filter attenuation characteristic is a curve plotting $\alpha_T$ or $\alpha_R$ against carrier frequencies $(f_0 \pm \Delta f)$, and the frequencies above or below which the attenuation constants may be infinite may be expressed as $(f_0 \pm \Delta f_{\infty T})$ or $(f_0 \pm \Delta f_{\infty R})$, showing that since $\Delta f_{\infty R} < \Delta f_{\infty T}$ the frequency at and above or below which the attenuation may be infinite is closer to the carrier frequency $f_0$ if no demodulation correction takes place than when it is applied; or, by implication, that more of one sideband may be cut away if the frequency characteristic of the transmitter is allowed to remain unlevel than if it is arranged to be corrected.

This fact is obvious when it is realized that, to correct the frequency characteristic, $M_N$ must be multiplied by $2/(1+\xi)$, and that this increasing modulation of the carrier as $\xi$ decreases must introduce a greater distortion which can only be compensated for by allowing for less asymmetry at a given modulation frequency than if $M_N$ did not have to be increased above its normal value.

The curves in Figs. 15A and 15B show the values of attenuation plotted against sideband frequency (expressed as $\Delta f = f_0 - f_s$) for the two cases.

It will be appreciated that, if $\delta_{\xi 0}$ (and hence $d$) has a value greater than zero, then even at sideband-component frequencies infinitely close to the carrier frequency $\alpha$ also will be greater than zero. But since one set of sideband components is transmitted with zero attenuation the curve plotting the attenuation constant against frequency for the requirements implicit in (27) and (28) will show a discontinuity at the carrier frequency. This is because it has been assumed that some distortion is tolerable at all sideband frequencies. Actually, equations (27) and (28) should be considered to apply only for those sideband-component frequencies created by modulation components having frequencies greater than that (say 1 200 cycles per sec.) where modulation decreases with increasing frequency of modulation. In drawing the criterion attenuation/frequency characteristics for asymmetric-sideband carrier-frequency filters it will be necessary, therefore, only to use (27) and (28) for sideband-frequency values above or below $(f_0 \pm 1\,200)$ cycles per sec., say, and to join the point showing the value of $\alpha_T$ or $\alpha_R$ for a frequency of $(f_0 \pm 1\,200)$ cycles per sec. to a point representing $f_0$ and zero attenuation by a curve estimated to represent the probable curve obtained in practice. Curves derived from (27) and (28) and smoothed to fit practical working conditions are given in Figs. 15A and 15B. The broken lines are values given by (27) and (28) at the lower frequencies. It has been assumed that it is the lower-frequency sideband which is cut away; it might equally have been the higher.

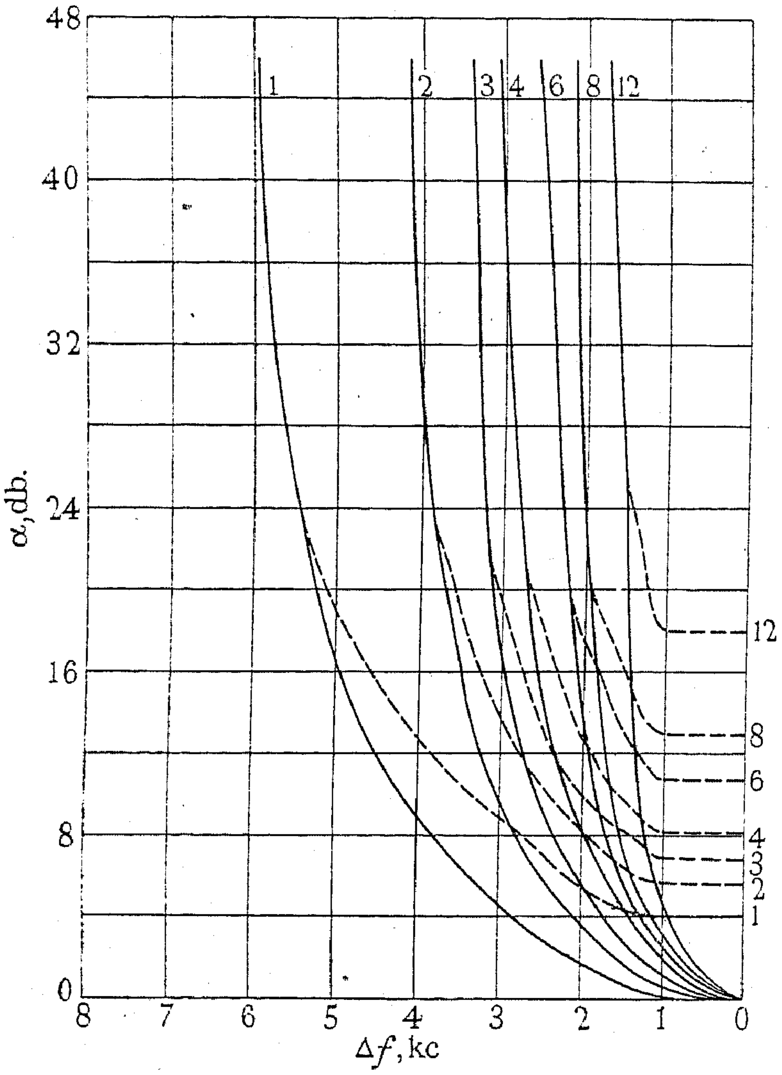

**Fig. 15A.**—Modulation-characteristic corrected.

Figures on curves denote values of $\delta_{\xi_0}$, expressed as percentages.

——— Desired attenuation. - - - - - - From $\dfrac{1-d}{(\sqrt{d}-1)^2}$

The required attenuation curve of the band filter wherein the lower-frequency sideband is cut away is thus one in which attenuation should increase gradually from the cut-off frequency as the frequency decreases, but wherein, as this frequency decreases still further, the attenuation may increase very rapidly and below a certain side-band frequency may become infinite.

The problem of designing such a filter would be most nearly solved, so far as the production of the correct frequency/attenuation characteristic is concerned, by the use of a single 3- or 4-element section matched with a section having $f_\infty$ a little lower than $(f_0 - \Delta f_\infty)$. This solution fails, however, because the phase characteristic of such a combination would give intolerable phase-asymmetry distortion. A single section would tend to give the correct attenuation/frequency characteristic over a range of frequencies close to the carrier frequency but would not greatly attenuate the outer parts of one sideband, thus not being as effective as desirable in bringing out the possible merits of the system.

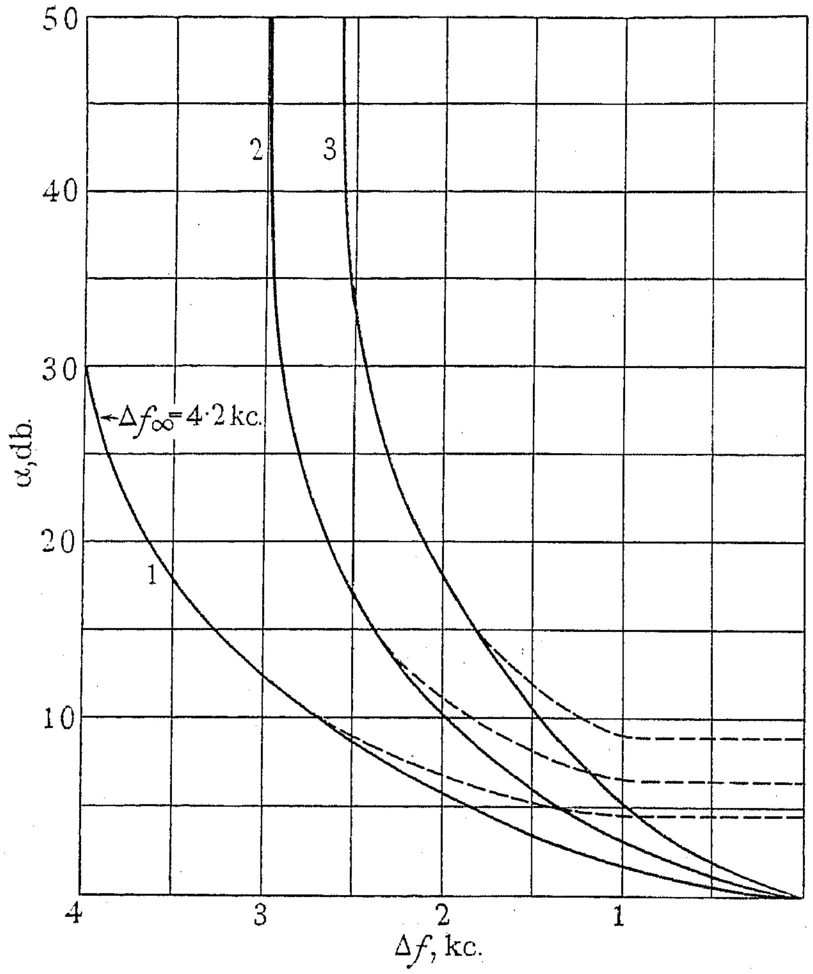

**Fig. 15B.**—Modulation-characteristic uncorrected.

Figures on curves denote values of $\delta_{\xi_0}$, expressed as percentages.

——— Desired attenuation. - - - - - - From $\dfrac{1}{1+d} - [d(d+4)]^{\frac{1}{2}}$

It might be possible to produce the required characteristic by associating filter sections having "staggered" cut-off frequencies. According to this idea, and if the lower-frequency sideband were being attenuated, one filter section might have a lower cut-off frequency at $f_0$ kc./sec., the next at $(f_0 - 1)$ kc./sec., the next at $(f_0 - 2)$ kc./sec., and so on. It would be necessary, since their cut-off frequencies would be different, to decouple each section from the other, and this would be very simply done either by using decoupling-resistance networks matching or tending to match the filter-section image impedances, or by valves. Some amplification would in any case be necessary to compensate for the losses in the resistance networks, and therefore some sections could be coupled by valves directly and others by resistance networks. The phase characteristics of such combinations would tend to be more uniform than if an $f_\infty$ section were used, because the carrier would be located nearer and nearer (in the case considered where the lower-frequency sideband was attenuated), in each successive filter, to $f_m$. A quantitative study of such a system shows, however, that while the expected tendencies are revealed the actual phase distortion becomes too large to be acceptable.

The preferred solution to the problems lies in using a constant-*k* band-pass filter section in combination with a constant-*k* band-elimination filter section. A network of this form is shown in Fig. 16. The resistance network, which is, more suitably, replaced by valves, is introduced in order that the two sections, having different image impedance/frequency characteristics, may be dissociated electrically from one another. The importance of the scheme lies in its facility to eliminate phase distortion and at the same time to give the correct form of frequency/attenuation characteristic.

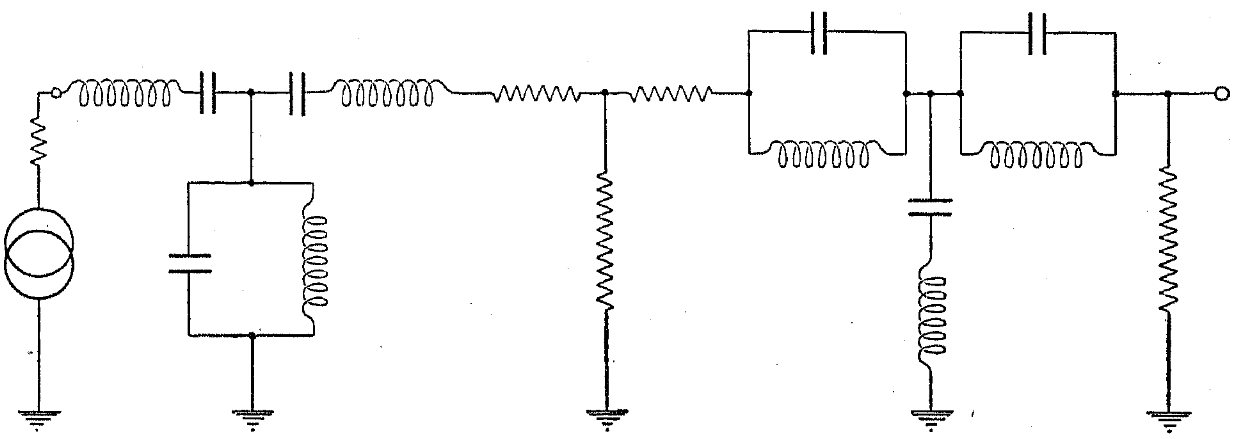

Fig. 16

Let $\theta$ be the phase angle between currents entering and leaving a filter. Let $\theta_0$ be the value of $\theta$ at a carrier frequency $f_0$, and let $\theta_1$ and $\theta_2$ respectively be the values of $\theta$ for frequencies corresponding to any two sideband frequencies $(f_0 + \Delta fx)$ and $(f_0 - \Delta fx)$.

The requirement for phase symmetry between counterpart sideband components is that the total effect of the network shall be to make $(\theta_0 - \theta_1) = (\theta_2 - \theta_0)$, it being assumed that $\theta_2 > \theta_0 > \theta_1$. This condition is fulfilled if, in two networks, such as those shown in Fig. 16 and connected so that the final output is determined by their combined performance, the sum of the differentials expressing the rate of change of $\theta$ with frequency, at a number of frequencies in the working sideband range, is equal to a constant. This implies that either of the filters alone may have a flexure in its phase characteristic provided always the flexures in each filter are equal and opposite to one another.

Considering the case in which the lower sideband is attenuated, consultation of textbooks on wave filters will show that a single-section constant-*k* band-pass filter terminated in a resistance has a phase characteristic which, over the range of frequencies between its lower cut-off frequency $f_{1b}$ and frequencies less than $f_{1b}$, has a flexure such that the rate of change of $\theta$ with frequency decreases as the frequency decreases. The curve thus flattens out as the frequency decreases, becoming asymptotic to a zero value of $\theta$ at zero frequency. In a constant-*k* band eliminator, however, the rate of change of $\theta$ with frequency increases as frequency is decreased from a value greater than $f_{2\epsilon}$ (the upper cut-off frequency of the band eliminator) to $f_{2\epsilon}$. Suppose the networks of Fig. 16 are arranged so that the carrier frequency $f_0$ is coincident with the lower cut-off frequency of the band filter and that the value $f_{m\epsilon}$ for the band eliminator approximates to $(f_0 - \Delta f_\infty)$ while the upper cut-off frequency $f_{2\epsilon}$ of the band eliminator is a little greater than $f_{m\epsilon}$ but less than $(f_0 - \Delta f_\infty)$. Consideration will show that, given these conditions over a working range of frequencies $f_0$ to $(f_0 - f_{2\epsilon})$ and because the flexures of the two phase characteristics are inverse, the sum of their differential constants will tend to be a constant, i.e. the combination of the two characteristics tends to be a straight line of the form $y = kx$.

At frequencies less than $(f_0 - f_{2\epsilon})$, i.e. within the attenuation band of the band eliminator, very violent phase-changes take place and phase asymmetry is extremely pronounced. But obviously, in this band of frequencies, attenuation of one sideband component is so great, band eliminator and band pass combining their attenuations, that $\xi$ is virtually zero and no added distortion due to phase asymmetry can take place. At frequencies greater than $f_0$ the rate of change of the band-eliminator phase characteristic tends to become very small while that of the band-pass filter is constant. Thus phase distortion is virtually eliminated by the combination of these inverse networks, while, thanks to the contributions made by the band eliminator to the attenuation given by the band-pass filter, the outer parts of one sideband are very greatly attenuated.

This result can be seen to be only obtainable by combining the performance of different types of filters having phase characteristics which are arranged to have inverse flexures over bands of frequencies wherein, in asymmetric technique, one sideband component is of magnitude comparable with its counterpart.

## Equalizer Design

It has been shown that when a modulated carrier is passed through the sideband-cutting filter it suffers a demodulation at the output terminals of that filter. The demodulation ratio is given by $M_A/M_N$, or, if $M_N = 1$, as in Fig. 12, by $M_A$. Evidently the compensation required is to arrange for the input modulations to be multiplied by $1/M_A$. Let this multiplication factor be written $p = 1/M_A$. Now $M_A$ has boundary values corresponding to $\xi = 1$ and $\xi = 0$ (low modulation frequency and high modulation frequency, respectively) of 1 and 0·5. Evidently, since $M_A$ has values less than unity, $p$ has values greater than unity, implying that the compensation circuits must embody an amplifier and, because $M_A$ varies with modulation frequency, a passive frequency-discriminative network.

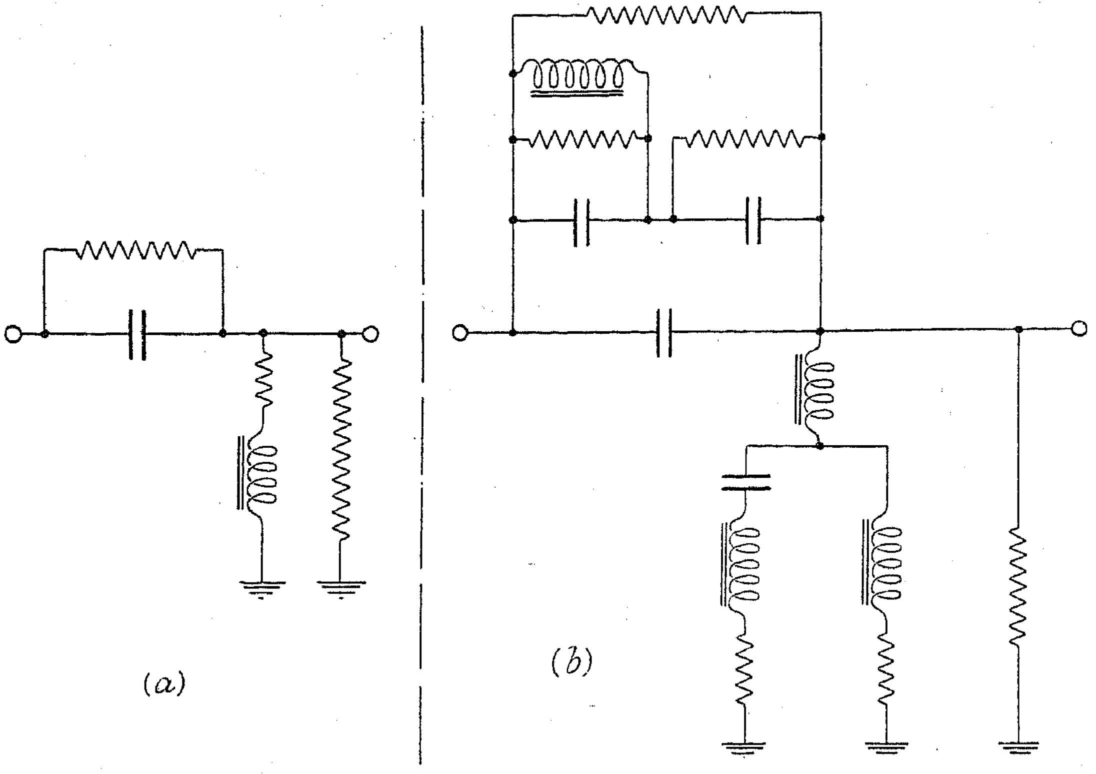

Fig. 17

We may express this by writing $p = 1/(2M_A) \times m$, where $m$ is the magnification constant of the required amplifier. If $M_A$ is never less than 0·5 and $m$ is constant and equal to 2, then $p \equiv 1/M_A$, which is the required condition. If $M_A$ is, at any modulation frequency, less than 0·5, then, to maintain $p = 1/M_A$, $m$ must exceed 2. But at the two boundary conditions $\xi = 1$, $\xi = 0$, it is seen that $m$ must be equal to 2. This proves that if $M_A$ ever falls below 0·5, a condition which can only arise if phase and magnitude asymmetry in the sideband components combine in certain proportions, the equalizer amplifier must be frequency-discriminative.

Thus the requirement of the equalizer circuit is that a passive network in association with an amplifier is required to be connected between the source of the audio-frequency modulations and the input to the modulating valve, or between the detector output and the loudspeaker, the network being characterized by an attenuation constant given by

$$\alpha_e = 20 \log_{10} (2M'_A) \quad . \quad . \quad . \quad . \quad (29)$$

and an amplifier having a magnification constant given by $m = 1/M''_A$; where $M'_A$ is any value of $M_A$ exceeding or equal to 0·5 and being written 0·5 if $M_A$ is less than 0·5, and $M''_A$ is any value of $M_A$ less than or equal to 0·5 and being written 0·5 if $M_A$ is greater than 0·5. The values of $M_A$ can be found, given the values of $\xi$ and $\phi$, for any filter from the charts of Fig. 12. If phase asymmetry can be neglected,

$$\alpha_{e\xi} = 20 \log_{10} (1 + \xi)$$

and $m = 2$ always. This is the practical case because the filter network suitable for asymmetric technique should be designed to give, so far as possible, no phase asymmetry when $\xi$ is large.

A discussion of how networks, usually known as equalizers, are built up from simple elements is given in standard textbooks. It is essential that such networks should be designed for constant impedance, since they will be fed in practice from sources having by no means negligible internal impedance.

A simple form of network suitable in cases where demodulation of the carrier by the asymmetric filter is not excessive is shown in Fig. 17(*a*), while a more complex form used in certain experiments described later, where phase asymmetry was very pronounced, is shown in Fig. 17(*b*).

## Practical Designs

It is proposed now to study different types of filter networks as regards their suitability for asymmetric-sideband transmission.

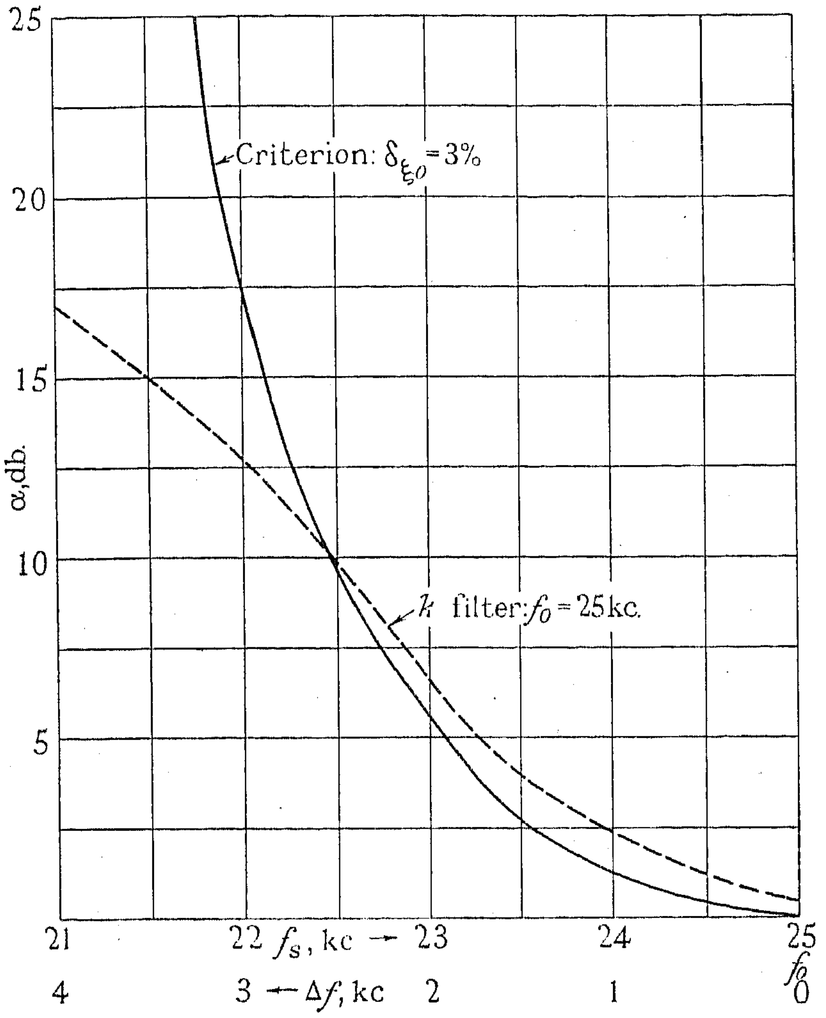

**Fig. 18.**—Constant-$k$ single-section filter. Points obtained experimentally.

**Constant-$k$ single-section filter ($f_0 = 25$ kc.).**

Fig. 1 shows the attenuation characteristic of a constant-$k$ band-pass filter. Fig. 18 repeats Fig. 1 except that $f_0$ is chosen to have a frequency of 25 kc. instead of 27 kc. Fig. 18 compares the criterion attenuation curve for 3 % distortion (a figure chosen to represent maximum tolerable distortion) with the attenuation curve of the constant-$k$ filter with the carrier located at 25 kc. Fig. 19 is derived from Fig. 1 and Fig. 18, and shows in full lines the values of $\xi$ and $\phi$ for the conditions chosen for the particular filter under study.

We may now make a Table and derive the distortion and demodulation. Thus Table 2 shows in col. 1 the values of $\Delta f$ and in col. 2 the corresponding values of $M_N$. This is obtained from Fig. 14. Cols. 3 and 4 respectively give the values of $\xi$ (from Fig. 1 or Fig. 18) and $\phi$ (from Fig. 1), corresponding to the given values of $\Delta f$ in col. 1.

Given $\xi$ and $\phi$, we can write in col. 5 the corresponding value of $M_A$ using the chart of Fig. 12. Col. 6 gives $M_{Nx}$, the actual maximum value of modulation taking place, and is obtained by dividing $M_{Np}$ (the maximum value of $M_N$ input to the transmitter modulator terminals) by $M_A$. (This increase of $M_N$ is brought about by the action of the equalizer compensating for the demodulation.) Cols. 3, 4, and 7 now give all the information necessary to obtain $\delta$ from the charts of Figs. 11A, 11B, or 11C. If $\xi, \phi$, and $M_{Nx}$ are of suitable values, Fig. 11A can be used directly; if not, Fig. 11B or Fig. 11C must be used. Table 2 is arranged to show which charts are used and how the distortion is derived. Fig. 19 is now

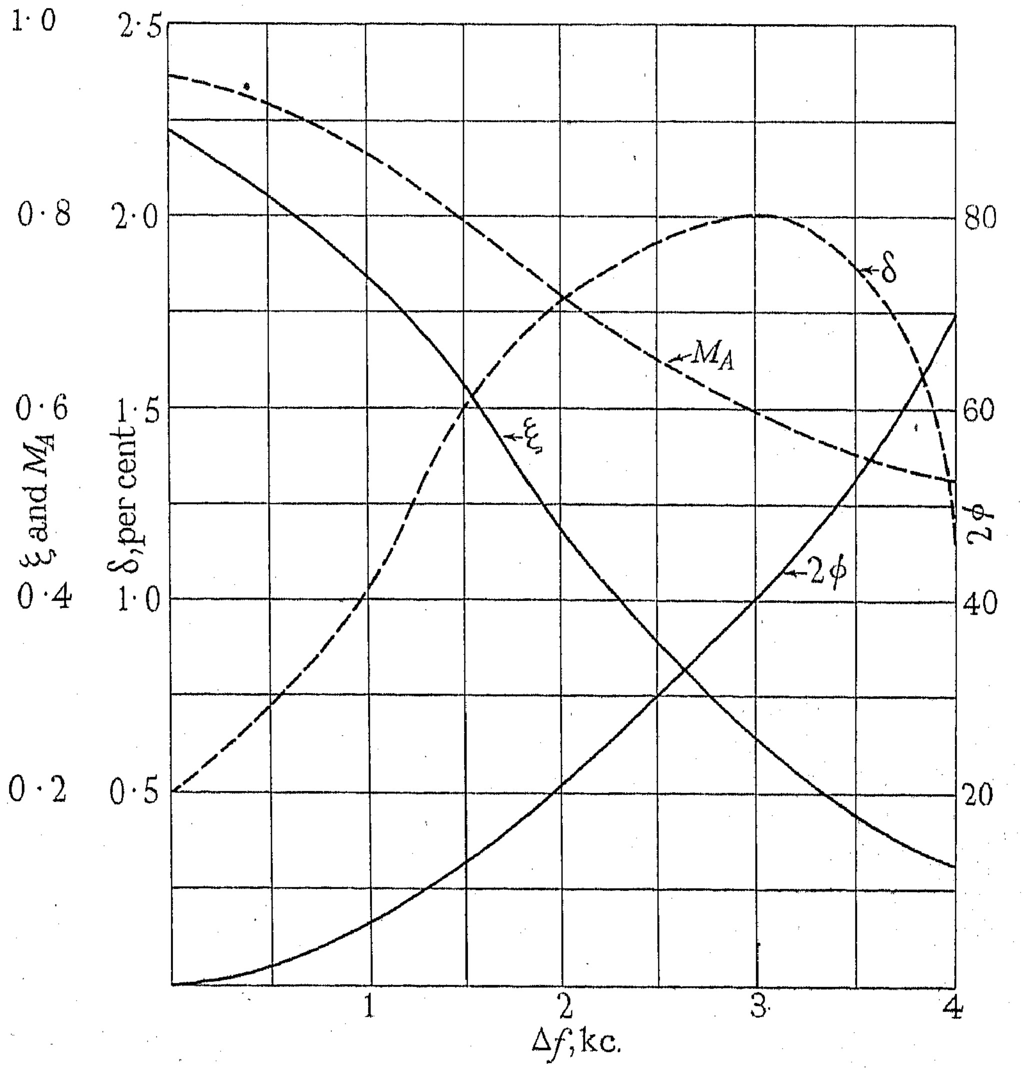

**Fig. 19.**—Constant-$k$ single-section filter. $f_0 = 25$ kc.

**Table 2**

CONSTANT-$k$ SECTION HAVING CHARACTERISTICS AS SHOWN IN FIG. 1 ($f_0 = 25$ KC./SEC.)

| $\Delta f$ (cycles/sec.) | $M_{Np}$ (Fig. 14) | $\xi$ (Fig. 1 or Fig. 18) | $2\phi$ (degrees) (Fig. 1) | $M_A$ (Fig. 12) | $M_{Nx}$ | $\delta$ | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | Chart | Chart reading | Actual value |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | | |
| 0 | 0·8 | 0·89 | 0 | 0·945 | 0·845 | Fig. 11A | 0·25 | 0·5 |
| 1 000 | 0·8 | 0·744 | 6 | 0·87 | 0·92 | Fig. 11C | 1·0 | 1·0 |
| 2 000 | 0·4 | 0·47 | 21 | 0·72 | 0·555 | Fig. 11A | 1·6 | 1·775 |
| 3 000 | 0·17 | 0·26 | 41 | 0·605 | 0·278 | Fig. 11A | 3·6 | 2·0 |
| 4 000 | 0·07 | 0·125 | 69 | 0·525 | 0·101 | Fig. 11A | 5·5 | 1·1 |
| 5 000 | 0·05 | 0·1 | 97 | 0·495 | 0·101 | Fig. 11A | 6·5 | 1·31 |

completed by drawing in the values of $\delta$ and $M_A$ (in broken lines). The Figure now gives all the required information.

It is seen that the distortion, using the constant-$k$ section, would never exceed 2 %, and that the demodulation effects are not so great as to make the equalizer complex. The sole disadvantage, but that a considerable one, is that the attenuation characteristic is far from ideal.

### Constant-$k$ section terminated by $f_\infty$ section.

One method of improving the filter attenuation characteristic is to associate an $f_\infty$ section with a constant-$k$ type of filter; but this introduces phase asymmetry. The network performance will, however, be examined not only because it affords an interesting example to illustrate the theory but also because the filter examined was actually used in certain experiments described hereafter. The attenuation/frequency and phase/frequency characteristics are shown in Fig. 25. Fig. 21 gives in full lines the values of $\xi$ and $\phi$. Calculations exactly similar to those already described give $M_A$ and $\delta$.

It is seen from Fig. 20, which compares the criterion attenuation curve with that obtainable from the constant-$k$ section terminated by the $f_\infty$ section, that the attenuation characteristic given by the network approaches the ideal far more nearly than if (see Fig. 18) the $k$ section is used alone. But it is also seen from Fig. 21 that the network, in achieving a better attenuation characteristic, has introduced intolerable distortions. These are remarkably concentrated between the modulation frequencies of 2 500 and 3 500 cycles per sec.

### Constant-$k$ band-pass section associated with constant-$k$ band-elimination filter.

Fig. 22 gives the attenuation/frequency and phase/frequency characteristics of the network therein illustrated, namely the combination of two constant-$k$ sections, one for band pass and the other for band elimination. The method employed to decouple the two networks is not shown because there are many alternative forms. The points are, however, taken experimentally.

It is at once seen that the phase characteristic is remarkably straight and the attenuation characteristic markedly steep. Fig. 23 compares the attenuation characteristic with the criterion. It will be shown later that the fact that the attenuation characteristic returns to only moderate values of attenuation after $f_s = 19$ kc./sec., or for a modulation frequency of $25 - 19 = 6$ kc./sec., does not greatly affect the efficacy of the network for preventing spectrum overlap. An $f_\infty$ band-eliminator section could, however, be used.

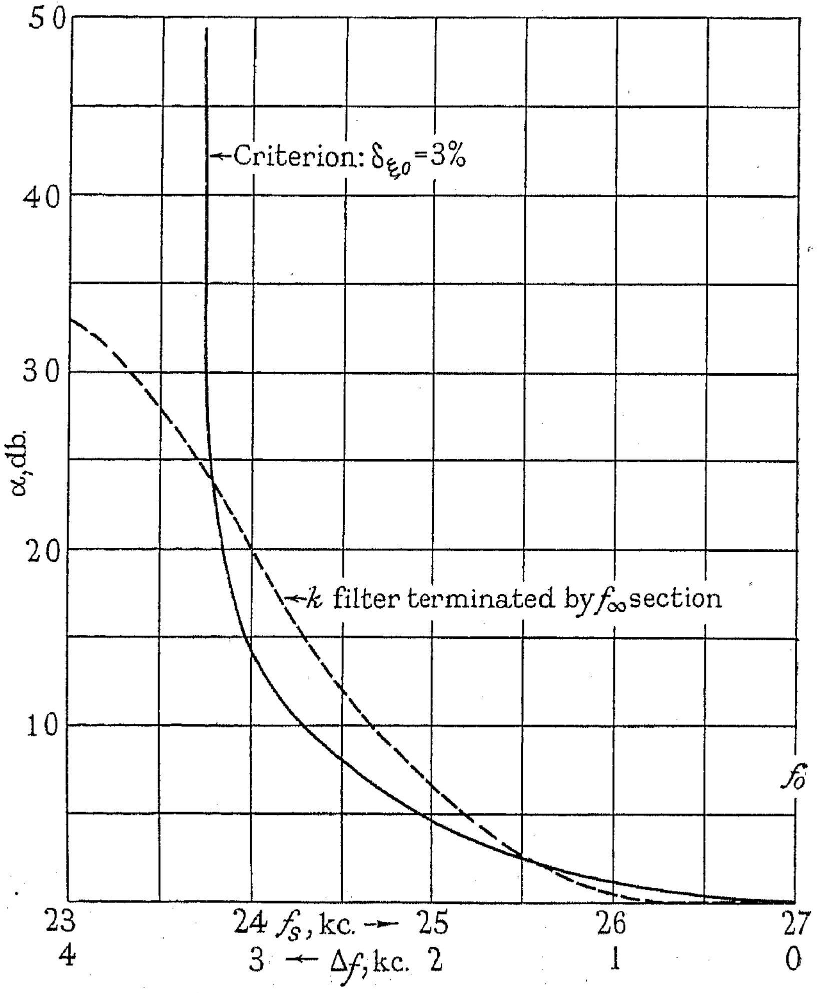

**Fig. 20.**—Filter comprising constant-$k$ section terminated by $f_\infty$ section. Points obtained experimentally.
$f_0 = 27$ kc./sec., $f_\infty = 21$ kc./sec.

Fig. 24 shows $\xi$ and $\phi$ and the fundamental quantities by which performance may be judged, namely $M_A$ and $\delta$. It is seen that the distortion never exceeds the chosen value, that demodulation never exceeds 6 db., and yet

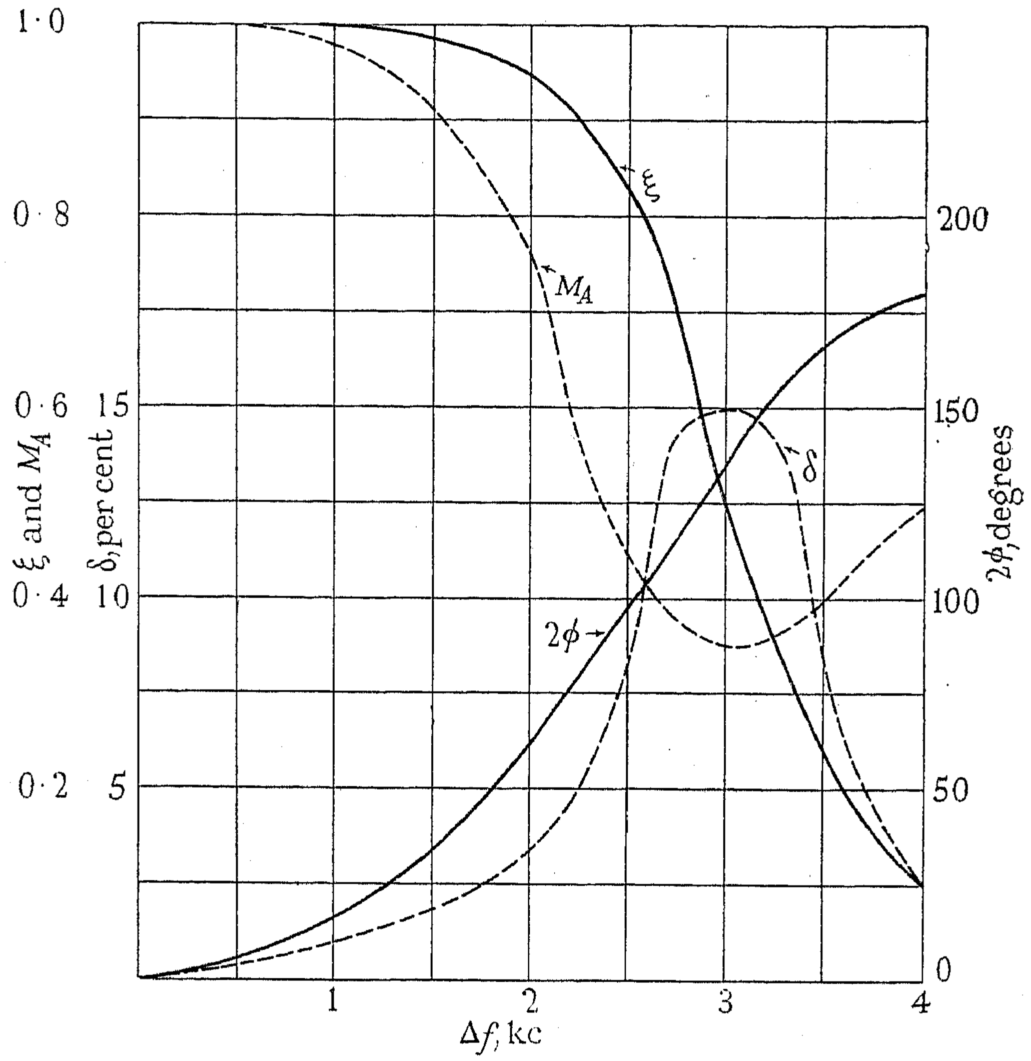

**Fig. 21.**—Filter comprising constant-$k$ section terminated by $f_\infty$ section. (Experimental values of $M_A$ shown in Fig. 27.)

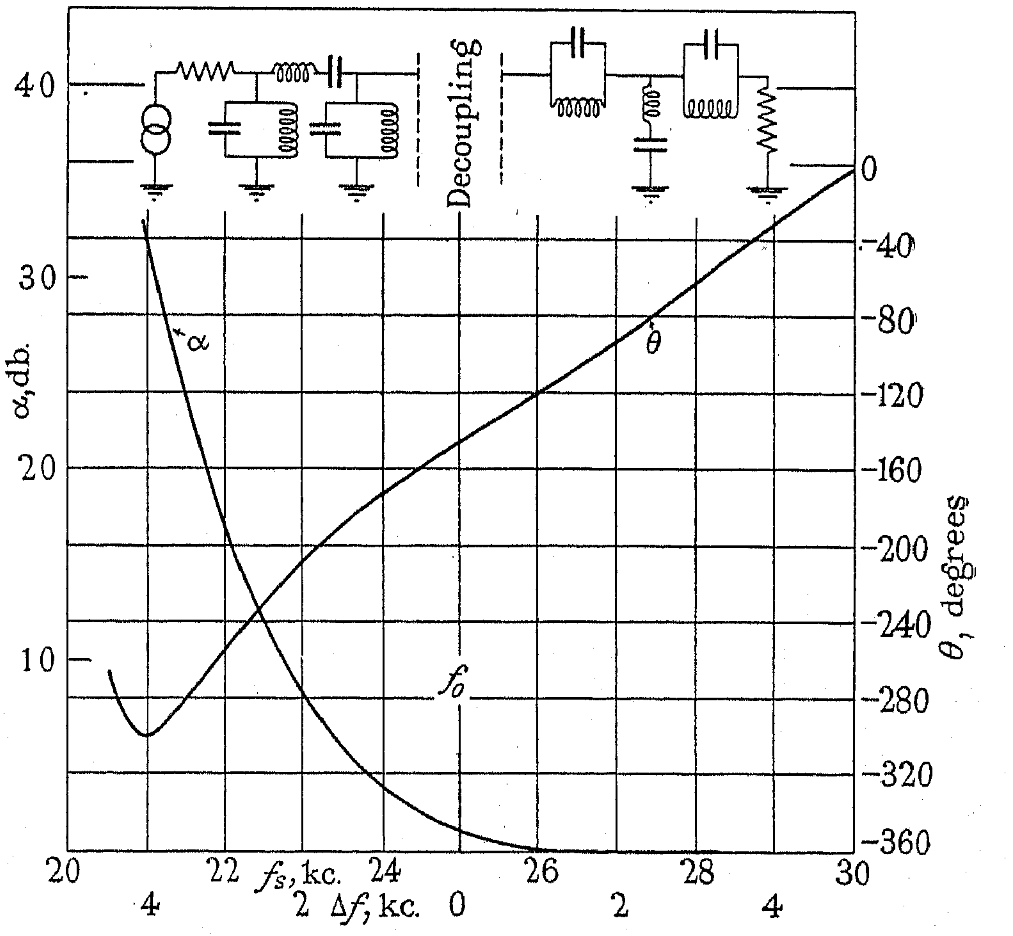

**Fig. 22**

$f_1$ (band pass) = 24 kc./sec. $f_2$ (band eliminator) = 22 kc./sec.
$f_1$ (band eliminator) = 18 kc./sec. Points obtained experimentally.

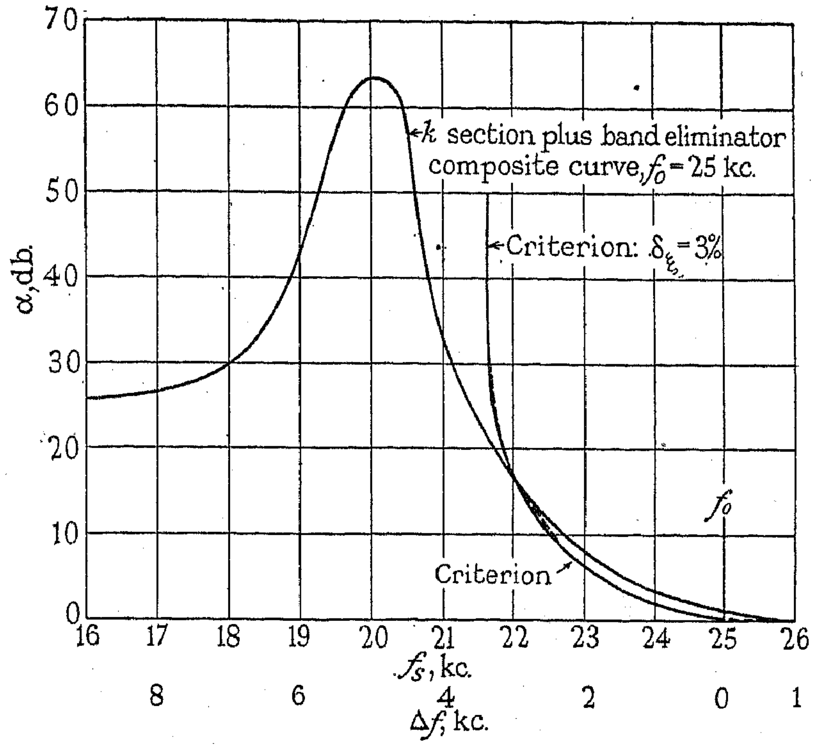

**Fig. 23.**—Constant-*k* filter plus constant-*k* band eliminator. Points obtained experimentally.

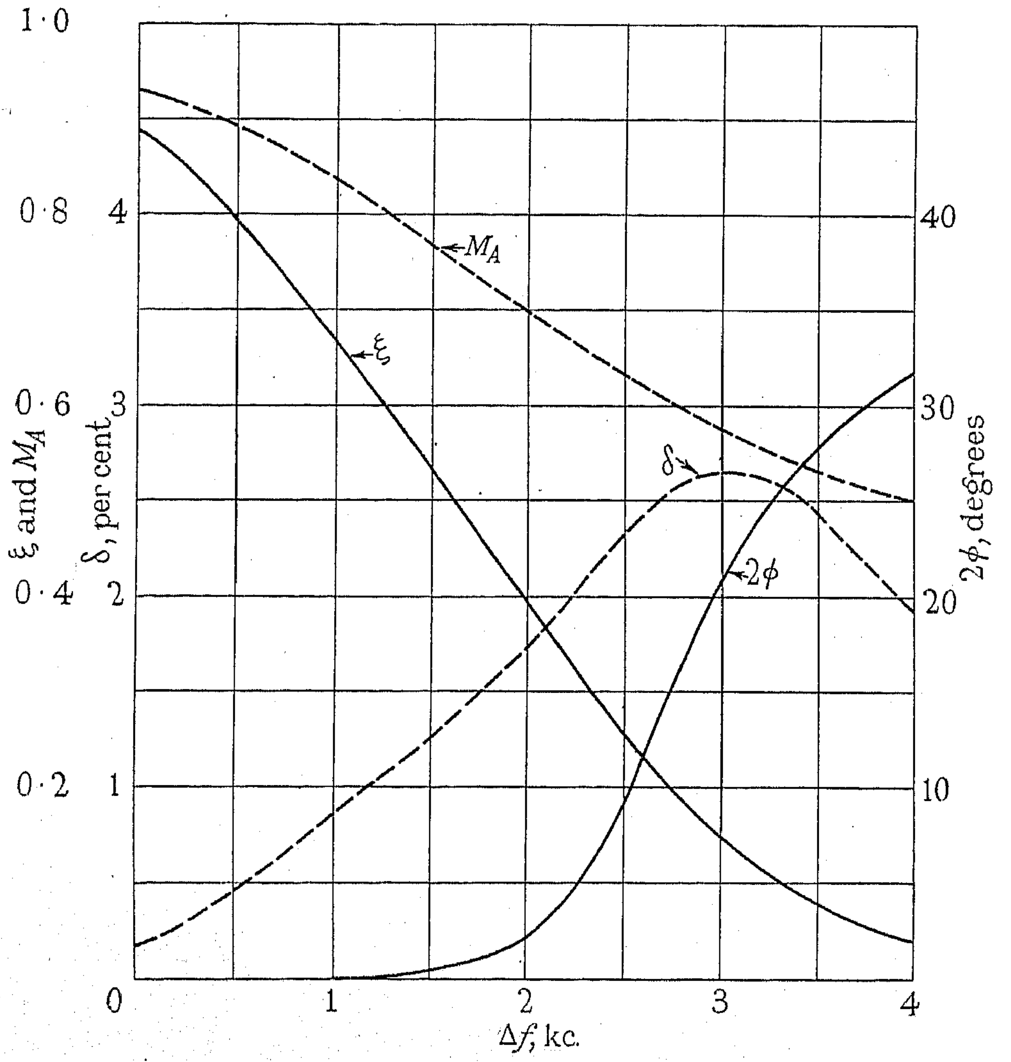

**Fig. 24.**—Constant-*k* section plus constant-*k* band eliminator.

that the network gives an attenuation of the order of 60 db. for sideband components 4–6 kc./sec. removed from the carrier frequency. This is a result which could not be achieved by the cascade connection of constant-*k* type band filter sections, because, although the required attenuation characteristics would be obtained from such networks, the phase characteristics would combine to give such phase asymmetry as to introduce intolerable distortions.

### Design of Asymmetric Systems: Conclusions

The quantitative analysis has been used to determine approximate designs. The word " approximate " is used advisedly. Firstly, it is fully appreciated that the sound-spectrum curve is an approximation and is representative of probability rather than being a rigid expression of what are the maximum sound-spectrum intensities regardless of the frequency of their occurrence. Secondly, the charts which show the effects of phase and magnitude asymmetry are based upon an approximate formula. Thirdly, it is impossible to shape filter attenuation/frequency characteristics to exact requirements. It is suggested that the analysis, " inaccurate " in a pedantic sense, has had at least this value—that it has guided a choice of a network which seems almost ideally suited for the purpose for which it was chosen. The test of the value of the system must finally rest with experiment, and Section (5) describes the work which has so far been done in this connection.

## (5) EXPERIMENTAL VERIFICATION OF THEORY

### Receiver Tests

The author's original paper[*] dealing with the subject of asymmetric-sideband transmission gave accounts of experiments devised to find out, in general terms, whether it was possible to cut off part of one sideband and yet retain reasonably good quality.

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[*] *Loc. cit.*

A superheterodyne receiver was designed in which the intermediate carrier frequency could be adjusted to lie on various points on the attenuation/frequency characteristic of the intermediate-frequency band filter. It is apparent, now that the analytical work has revealed the way in which distortions may be introduced, that the filter used, a simple 2-section 3-element impedance transforming type, did not introduce phase-asymmetry distortion to any marked degree. On the other hand, it did not cut away as much of the unwanted sideband as has been shown can be removed without the introduction of serious distortion. Correction of the receiver frequency characteristic was made in the audio amplifier stages following the detector.

The quality resulting, even when a considerable portion of one sideband was cut away, was so good that the author felt it worth while to publish his results and invite official consideration of the possibilities of the asymmetric-sideband system.

## Transmitter Tests

### Test transmitter.

As a result of representations made by the author late in 1936, the B.B.C. supplied funds for the construction of apparatus designed to give an immediate comparison between the highest standard of reproduction possible, according to present-day knowledge, and the quality obtainable from an asymmetric-sideband transmission system.

The apparatus, designed by the author, and constructed in the laboratories of R.M. Radio, Ltd., embodied means for modulating the amplitude of a carrier and routing the modulated currents either through a filter, wherein the carrier frequency was located at $f_m$, giving symmetrical treatment of sideband components, or through an asymmetric filter. The change-over could be made almost instantaneously by the operation of an external hand switch controlling relays mounted internally on the apparatus panels. The output from each filter was passed alternatively to the same rectifier, which in turn fed its output into an audio amplifier raising the level of the audio component to a value sufficient to operate a rather large high-quality loud-speaker. Other relays, also operated from the external hand switch, served to cut in and out the equalizer necessary to compensate for the demodulating effect of the sideband-cutting filter. Since there is a loss of 6 db. in the equalizer, compensated for by extra magnification in the modulation amplifier, a resistance pad, giving a 6-db. attenuation, was substituted for the equalizer when the symmetric system was in circuit; and so the audio output level, whichever filter was in circuit, remained the same without it being necessary to change the modulation magnifier constants. The carrier frequency, arbitrarily chosen so far as these experiments were concerned, was 27 kc./sec., and the filters were designed, in both cases, to give full reproduction up to modulation frequencies of 10 000 cycles per sec. The steady carrier power was of the order of 1 W. An input modulation level of 1 mW amplified to give a modulation of the carrier of 100 % produced a power of 7–8 W at the loud-speaker terminals.

All conventional circuits for amplification modulation and rectification were designed with a large factor of safety, and, in its final form, either system, symmetric or asymmetric, for frequencies of modulation so low that no sideband asymmetry could occur in the asymmetric filter, gave no more than 2 % 2nd harmonic and negligible 3rd harmonic at the loud-speaker terminals when the carrier was modulated to 90% from an input of 1 mW, the power output into the loud-speaker being of the order of 7 W.

It is clear that everything was done to make the standard quality as pure as possible and that a direct comparison could be instantaneously made between this standard and that obtainable from the asymmetric system.

All the circuits were boxed and panel-mounted.

### Test with constant-*k* band-pass section terminated by $f_\infty$ section.

At the time when the first asymmetric filters were designed the author had not appreciated that severe distortions might be produced by phase asymmetry. The theory of amplitude-asymmetry distortion was, however, appreciated and it appeared obvious that the ideal attenuation curve could be most nearly approached by using a constant-*k* section joined to and terminated by an $f_\infty$ section. It has been shown that this combination, from the point of view of the introduction of phase-asymmetry distortion, is, to say the least, an unfortunate choice, but perhaps if the mistake had not been made and its effects observed the solution, now proved so efficacious, would not have been so quickly found.

The results of the experiments made to test the degree of distortion and demodulation caused by the use of the network described give a confirmation of the theory previously set out in this paper.

Fig. 25, already referred to, shows the attenuation and phase characteristics of the network, and Figs. 20 and 21 have previously given the analysis of these characteristics in a form convenient for calculating distortion. An experiment was performed in which the audio-frequency harmonics were measured for a modulation frequency of 3 000 cycles per sec., the carrier being located at 27 kc./sec. The $\xi$ and $\phi$ values being known for this modulation (and hence sideband), the frequency distortion may be read off from Fig. 11A (in the case considered) and compared with the values found by test. The results are shown in Fig. 26. Fig. 27 shows the frequency characteristic of the asymmetric system without correction, and is again compared with the theoretical values found from the charts of Fig. 12, $\xi$ and $\phi$ being known from Fig. 21. The agreement is good, considering that the audio-frequency generator used was a rather old model and its calibration not very reliable. Shifting the experimental curve, at about 2 000 cycles per sec., 200 cycles per sec. (8 to 10 %) to the left would give almost perfect agreement. It must further be realized that the phase and attenuation characteristics from which Curve (*b*) in Fig. 27 are calculated were also found experimentally; and so, taking these factors into consideration, one may be very satisfied with the reliability

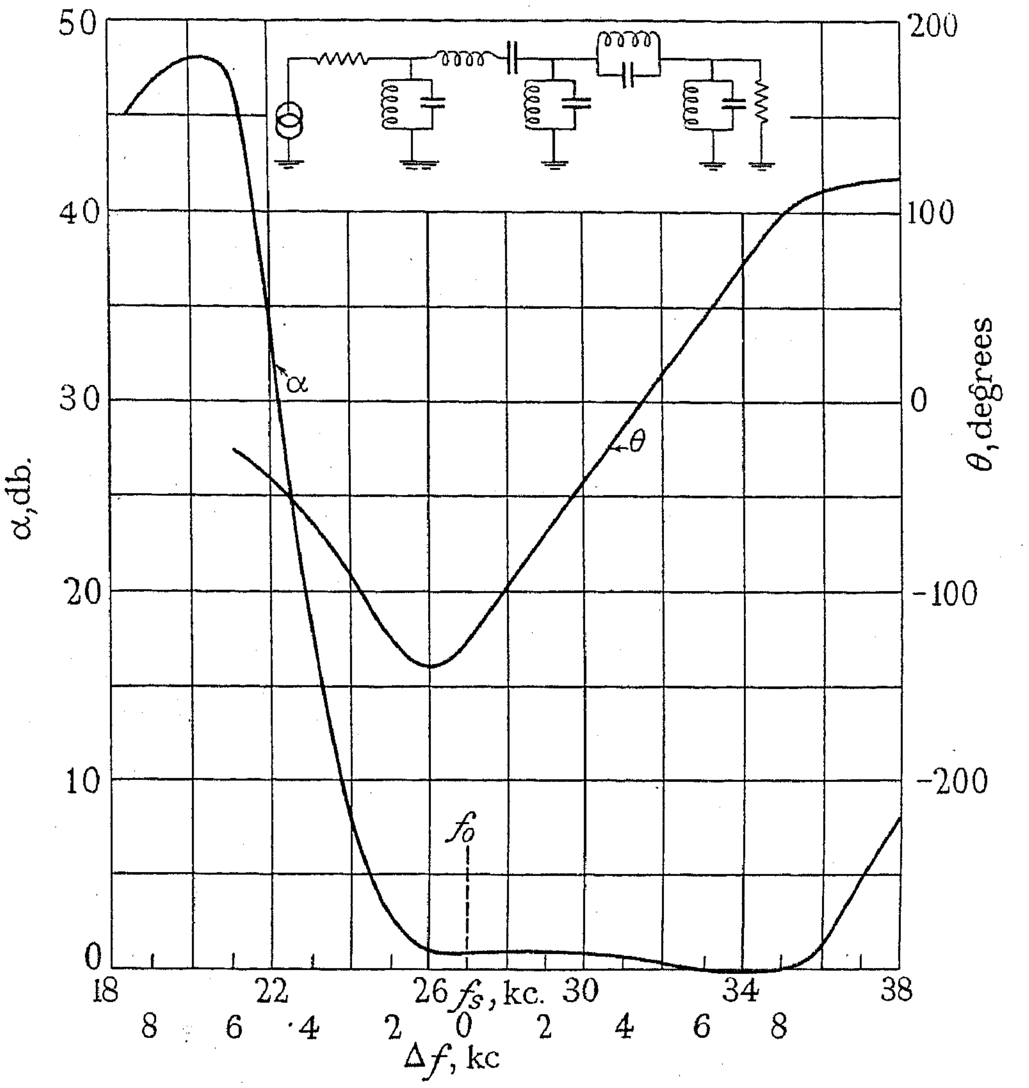

**Fig. 25**

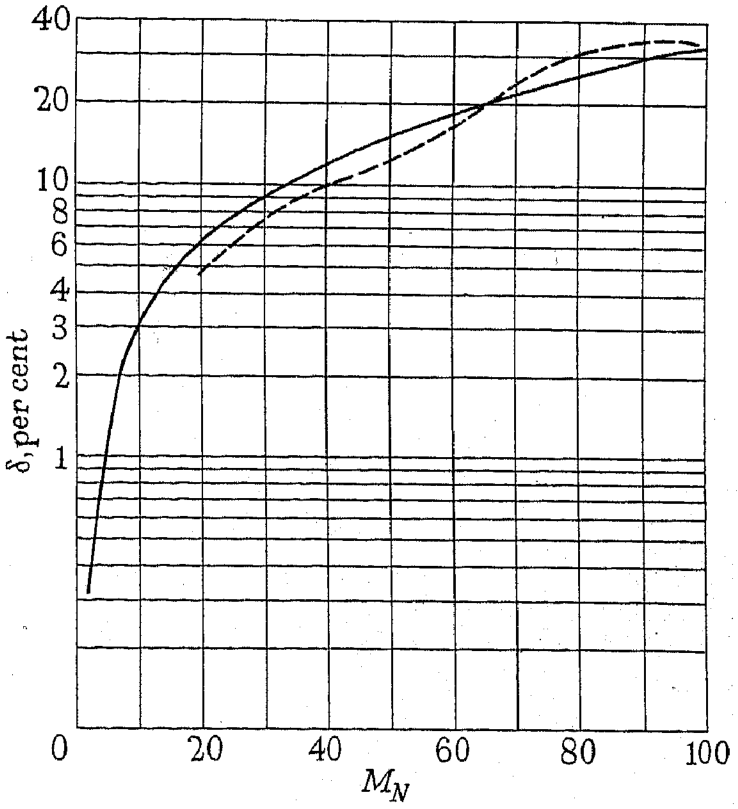

**Fig. 26**

Modulation frequency = 3 kc./sec. $k + f_\infty$ filter carrier at 27 kc./sec. $\delta$ represents percentage distortion of 2nd harmonic.

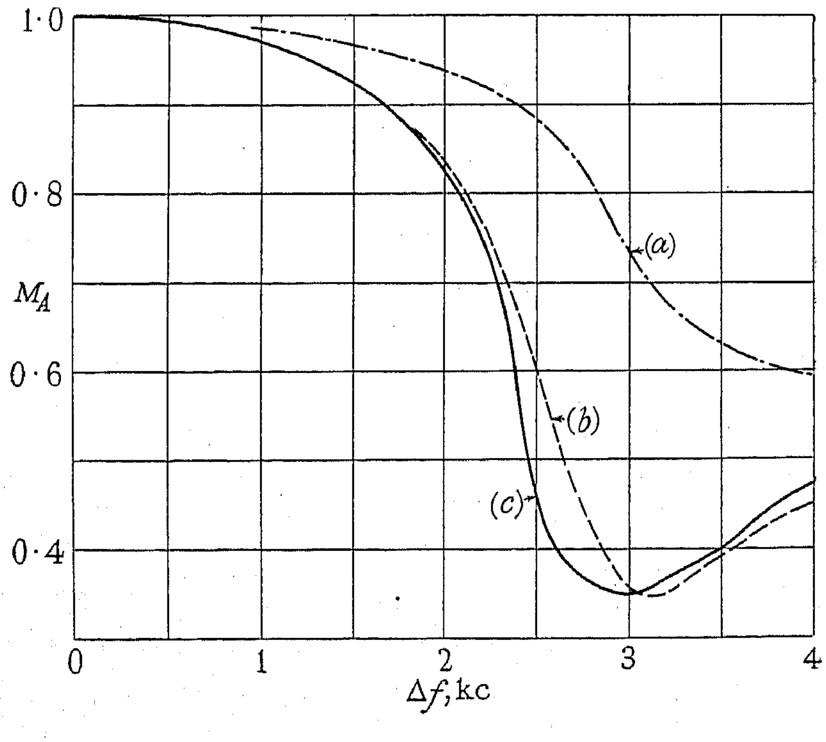

**Fig. 27**

$k$ filter + $f_\infty$ section. $f_0 = 27$ kc./sec. (see Figs. 20, 21, and 26).
(a) $M_A$ calculated from $(1 + \xi)/2$, i.e. neglecting phase asymmetry.
(b) $M_A$ calculated from Fig. 12.
(c) Experimental values of $M_A$.

of the analysis and its usefulness in guiding a choice of circuit values.

**Design of equalizer for test transmitter (first design).**

In spite of the fact, revealed by the tests, that a fundamental error had been made in neglecting phase asymmetry in the design of the filter, it was decided nevertheless to make comparative tests of quality between the orthodox and the asymmetric system. It was therefore necessary to design an equalizer to correct the demodulation produced by phase and magnitude asymmetry.

It is seen from Fig. 27 that the phase asymmetry demodulates the carrier by more than half and that therefore, as has been shown previously, the equalizer system would have to embody a passive network and a frequency-discriminative amplifier. This would have introduced greater complication in the change-over switching arrangements than was warranted considering the unsuitability of the filter; and so the passive network shown in Fig. 17B was designed in combination with extra amplification to give the best compensation possible in the circumstances.

The overall frequency characteristic of the asymmetric transmitter is given in Fig. 28, showing that the compensation was by no means perfect.

**Result of comparative tests of quality (first design).**

A receiver containing fixed filters, a high-frequency magnifying valve, and a detector, and designed to give the least possible distortion, was made responsive to the transmission from the London Regional station and its output was fed into the modulation terminals of the test transmitter. This formed the programme material for the comparative tests. The loud-speaker was placed in a room adjoining that in which the change-overs were made, and switching was so arranged that the two reproductions were distinguished by the showing of a green or red light on the loud-speaker baffle. Further switching was arranged so that either the green or the red might apply to either the orthodox or the asymmetric transmission. It was found that a different " click " sound might be heard unless the loud-speaker was short-circuited during the change-over, and the precaution to eliminate this distinctive sound was accordingly taken. There was thus no chance that the observer could be distracted by any differences between the two conditions other than those inherent in the electrical systems employed.

The author does not claim to be able to detect slight differences in quality so easily as can certain rare people. He would be inclined to say that he was unable to detect any difference between the reproduction of the orthodox and of the asymmetric transmissions, except very occasionally, and even then with no real certitude, even though the design of the circuits of the asymmetric transmitter was far from ideal. Observers with ears more acute to distinguish fine differences of quality (two from the B.B.C. Research Department) were therefore asked to judge whether they were able to detect any differences. Several hundreds of change-overs were made during two series, one in the early and the other in the late evening. The programme material, during the first session, combined speech, solo singing, and a light orchestral combination; in the second session dance music formed the bulk of the entertainment. It appeared easier for the observers to distinguish between the two types of transmission during the second session, but this might be explained because modulation is usually deeper during the relaying of dance music. The extreme similarity between the two reproductions may, however, be gauged by recording that the observers, of whom the author was then one, voted, during one set of change-overs, with considerable confidence, for a preference for the quality when one light showed rather than the other, when, in fact, they were listening to the same reproduction, the lights having been changed and not the circuits. It is, however, fully admitted that on balance the observers were more often right than wrong, and that differences could be detected.

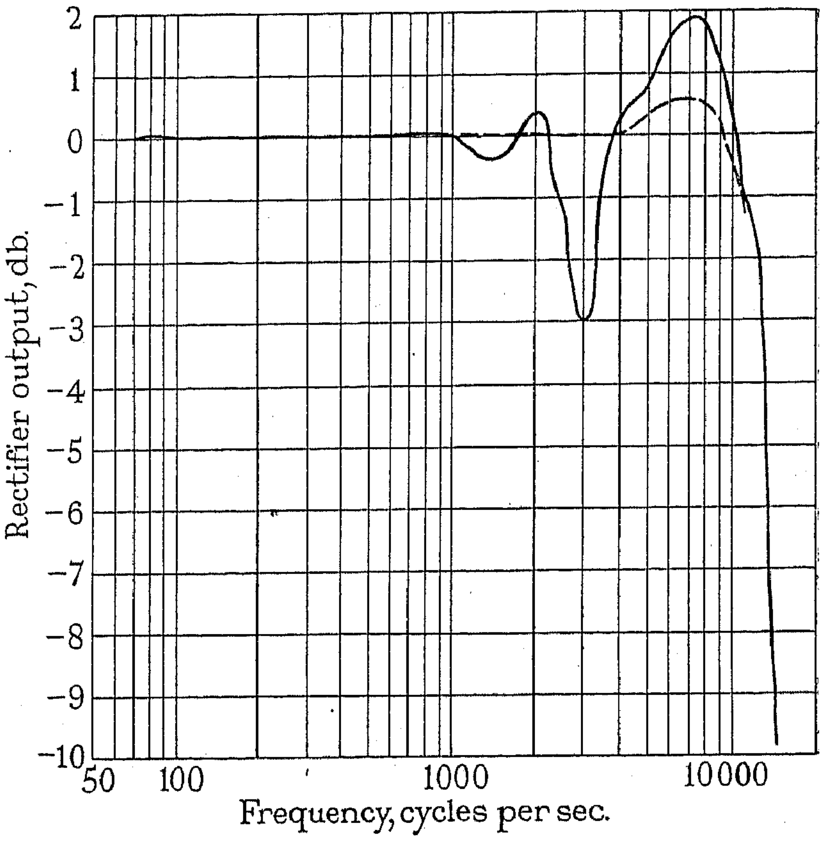

**Fig. 28**

Constant voltage of modulation. $k + f_\infty$ filter carrier at 27 kc./sec.
———— Output from asymmetric filter with equalizer in.
– – – – – Output from symmetric filter.

Taking 100 change-overs as a basic figure, it is estimated that in 50 cases no differences could be observed or the two observers cancelled each other's preferences, but that out of the remaining 50 the observers were right 35 and wrong 15 times.

This result was very encouraging, considering that it was known by experimental tests that the asymmetric system introduced 30 % distortion at 3 000 cycles per sec. for 100 % modulation (see Fig. 21), showing there would have been maxima of the order of 15 % if equalization had been perfect, but, say, 10 % without the full compensation; and considering that the demodulation effects gave an uneven frequency characteristic (see Fig. 28). The author feels certain that, had frequency multiplication been applied and the two outputs been taken to the input terminals of a typical commercial set, it would have been impossible to distinguish any differences. It is also stated with confidence that none but highly trained observers could have remarked the quality change, because tests were made with a few persons not expert in judging quality and in no case were they able to tell the difference when the circuits were changed.

The results of these first tests were so encouraging that a study was made with a view to devising a circuit which would eliminate the phase distortion but give large attenuation of the unwanted sideband components. The idea of associating the inverse filters was thus conceived, but owing to pressure of other work it was not until the autumn of 1937 that the apparatus was again put into commission and experiments continued.

### Tests with constant-*k* band pass and constant-*k* band eliminator (second design).

A filter was built containing the separate networks as described, and the band-pass and band-eliminator circuits were coupled by valves, not resistance networks as shown in Fig. 16. Opportunity was taken to test the findings of the quantitative analysis, and good agreement was proved after certain circuit aberrations had been removed.

Two B.B.C. observers were again asked to make a comparative test of quality between the symmetric and asymmetric systems, the latter being designed, according to the foregoing ideas, to eliminate phase asymmetry. Extremely exhaustive tests were undertaken to make sure that the symmetric system was, so far as is humanly possible, quite free from distortion. These tests included an examination of the wireless receiver which provided the programme input. It was proved that the system, from aerial input to loud-speaker output, gave no more than 1·5 % 2nd-harmonic and negligible 3rd-harmonic distortion. The frequency characteristics of the two transmitters, symmetric and asymmetric, were substantially the same, the maximum departures being of the order 1 db. at frequencies above 6 000 cycles per sec. The bass response was 0·6 db. less for the asymmetric system at 50 cycles, and this difference decreased until it was zero between 1 000 and 6 000 cycles.

The tests themselves were perforce of limited duration and were to a point, therefore, inconclusive. It has been decided that far more exhaustive tests must be carried out in the future.

Nevertheless, the following conclusions are fairly definite.

(1) It is agreed that the system wherein the band-pass and band-eliminator networks are used in combination is a great deal less liable to give audible distortion than that in which an $f_\infty$ section is associated with the constant-*k* filter. Thus in the previous tests the observers were able to detect the difference in quality between symmetric and asymmetric transmission with some degree of certainty, given certain conditions of modulation and programme item. In the tests on the new transmitter the observers were, over a limited period, more often wrong than right in their choice, believing the asymmetric system to be actually more pleasing and less " rough " than the symmetric.

(2) The observers remarked that they thought the quality on both transmissions inferior and that this fact was inclined to vitiate any firm conclusion. The author feels that, while this may conceivably have been due to some fault in the test apparatus or, more probably, in the loud-speaker used, nevertheless the very critical analysis of distortion and frequency characteristics would suggest that in this instance the source itself was not up to standard.

(3) The most remarkable fact adduced was that the observers were inclined to distinguish differences of frequency characteristic between the transmissions, e.g. there appeared to them to be more bass in one than the other. This, in view of the measurements, is very surprising although a quite definitely established fact. The only conclusion one may draw is that a decibel of difference is distinguishable by acute ears, provided one supposes that this difference extends over a fairly wide frequency-range.

The author, in any case, feels justified in believing that the solution of the problems of obtaining a very large attenuation of one sideband without the introduction of phase-asymmetry distortion has been achieved. Whether or not amplitude asymmetry introduces detectable distortion remains to be proved, but first tests indicate that it is far from easy, given even the most acute ears and a standard for comparison, to do so.

Another experiment, relevant to the following Section which deals with the advantages of asymmetric transmission in eliminating " sideband splash " or spectrum-overlap interference, was also demonstrated. In this experiment the modulated carrier-frequency output from the test transmitter having a carrier frequency of 25 kc./sec. was applied to a frequency multiplier, and the multiplied frequency of 250 kc./sec. was fed in turn into a typical commercial wireless receiver. An unmodulated carrier, having a difference frequency from the multiplied output from the test transmitter of 9 kc., was also fed into the receiver. The receiver was then tuned to the unmodulated carrier and conditions were arranged so that typical sideband interference caused by the test transmitter working on symmetric modulation was heard. Evidently the unmodulated carrier represented a wanted station while the output from the test transmitter was typically that of a powerful unwanted station on the frequency-contiguous channel, causing the well-known " sideband splash " interference.

Pressing the button which changed the output of the test transmitter from symmetric to asymmetric transmission greatly changed the degree of interference heard, because the sideband causing the interference was (partly) cut away. It was estimated, by the ear, obviously a very crude instrument for measuring effects quantitatively, that the interference was reduced tenfold. It was not, however, wholly eliminated, nor, as will be seen from theory, could it have been with only a 9-kc. separation and a modulation up to 10 000 cycles.

Lastly, means were arranged to create a background noise, and when this contained high-frequency audio-components the change-over to asymmetric transmission showed a considerable amelioration of the noise, particularly as regards its high-frequency components; demonstrating that, in the asymmetric system, the signal/noise ratio is superior to that obtainable with symmetric transmission.

Great advantages are seen to accrue from the cutting-away of part of one sideband, and these, in the author's opinion, outweighed the introduction of so negligible a harmonic distortion.

## (6) REQUIRED CARRIER DIFFERENCE FREQUENCIES

### General

The stated object of the preceding analysis was to find the answer to certain questions. It was required to find, *inter alia*, what economies could be effected in the frequency band width normally occupied by the radio-frequency spectrum of a broadcasting transmission, assuming that negligible harmonic distortion was introduced by removing part of a sideband. The analysis might have proved that so little of the sideband could be cut away without introducing more distortion than could be tolerated that it was not worth while to make any changes to existing technique; on the other hand, theory might have shown that by far the greater part of the sideband could be removed while still maintaining good-quality reproduction, thereby proving the advisability of using the system. This Section is therefore devoted to finding the required frequency-separation between the carrier frequencies of broadcasting stations, when different types of transmission technique are employed. It is necessary to make a classification of the variables so that the conclusions may not be ambiguous.

There is an obvious distinction between two methods of expressing the advantages of the asymmetric system. Thus we may either make a study to find out what is the minimum frequency-separation between carriers to ensure that, in no circumstances, can the radiated spectra overlap (and thus make sources of potential interference); or we can fix upon some frequency-difference established by *force majeure*, and find out what diminution of inevitable interference will be brought about if the asymmetric system is used.

This distinctive classification can be related either to a broadcasting system which is set up *de novo* and where the choice of carrier frequencies is uninfluenced by political and economic factors which have little to do with purely technical criteria, or to an existing system of broadcasting so established that it would be impracticable to suggest any diminution of the number of working stations, even though this diminution would be far less, using the asymmetric system, than that necessary to give complete freedom from spectrum overlap if double-sideband transmission were employed.

In sum, it is necessary to find: (1) What economies in the use of channels can be made with the asymmetric system applied to any new system which does not impose any other criteria than those demanding good quality and freedom from inter-channel interference. (2) What diminution of inevitable inter-station interference can be expected if the asymmetric system has to be applied to an existing set-up wherein carrier-frequency separations are less than those demanded by the criteria given in (1), i.e. when it is an essential of the application of the system that the existing service conditions shall not be disturbed. There is, lastly, a further subdivision of (2) which is concerned with the type of transmitter at present existing and whether or not it may be economically and practically adapted to make full use of the asymmetric principle. It will be shown in this connection that, if modulation of the carrier takes place at a level approximating to its maximum level (high-power modulation), it is probable that the ideal network, consisting of a constant-$k$ band filter and a constant-$k$ band eliminator in conjunction, cannot be used. If this be the case we could not expect a better result than that given by a constant-$k$ band filter alone. If the existing transmitter employs low-power modulation, wherein, as the term implies, modulation of the carrier takes place at a level greatly below the radiated carrier level, modification is comparatively simple and the full advantages of the ideal network may be counted upon.

Therefore, in studying the effects of the introduction of the asymmetric system and attempting to assess its benefits, we shall make a classification of the possible variations as follows:—

Classification (1). Minimum carrier difference frequency to prevent spectrum overlap.

Classification (2). The degree of interference experienced with a given carrier difference frequency. Thus we must find:—

(1) (A). Minimum carrier-wave separation, assuming the use of double-sideband transmission.

(1) (B). Minimum carrier-wave separation, assuming the use of asymmetric transmission, and a constant-$k$ band-pass section plus a constant-$k$ band-eliminator section with demodulation correction.

(1) (C). Minimum carrier-wave separation, assuming the use of asymmetric transmission, a constant-$k$ section only, and demodulation correction.

(1) (D). Minimum carrier-wave separation, assuming the use of asymmetric transmission, a constant-$k$ band-pass section plus a constant-$k$ band-eliminator section, and no demodulation correction.

Dealing with the last four items, (1) (A) applies to existing conditions; (1) (B) applies to existing space broadcasting transmitters having low-power modulation, but assumes that existing carrier differences can be increased (probably an impracticable suggestion); (1) (C) applies to existing space broadcasting transmitters having high-power modulation, but assumes that existing carrier differences can be increased (probably an impracticable suggestion); (1) (D) is theoretical.

(2) (A). Amount of interference expected in existing conditions (9-kc. carrier-wave separation).

(2) (B). Amount of interference expected using asymmetric transmission, a constant-$k$ band-pass section plus constant-$k$ band eliminator, and demodulation correction (9-kc. carrier-wave separation).

(2) (C). Amount of interference expected using asymmetric transmission, constant-$k$ band-pass section only, and demodulation correction (9-kc. carrier-wave separation).

(2) (D). Amount of interference expected using asymmetric transmission, a constant-$k$ band-pass filter plus a constant-$k$ band-eliminator section, and no demodulation correction (9-kc. carrier-wave separation).

It should be remarked that (2) (A) applies to the existing condition; (2) (B) applies to the condition which will apply if existing carrier-wave separations are maintained but the low-power-modulation transmitters are adapted; (2) (C) is the same as (2) (B), except that it applies to existing high-power transmitters; (2) (D) is theoretical.

## Method of Analysis

A description of the method by which the required frequency-separation between two carriers surrounded by any spectrum can be calculated was first published by the author some 5 years ago.* Its re-statement here is not redundant, partly because the results obtained are a vital part of the analysis of the asymmetric-sideband system, and partly because it may be possible to give a clearer picture of the implications of the findings than was possible 5 years ago.

The method involves first the determination of boundaries enclosing an area, having the vertical dimensions of power level and horizontal dimensions of radio frequency, within which it may be said that the spectrum energy of a modulated carrier has " appreciable " magnitude and is sufficiently spread in a horizontal dimension to allow a reproducer energized by the full spectrum to give a required fidelity of reproduction. The carrier is represented by a vertical line the top of which coincides with what will be arbitrarily called zero level. In a double-sideband transmission the maximum level (corresponding to 100 % modulation) of the sideband components will be 6 db. below zero (or carrier) level. Throughout this paper 80 % modulation is chosen as a maximum, and so in a symmetric system the maximum sideband level is 8 db. below zero level. Accepting the premise that the sound spectrum intensities are distributed as indicated in Fig. 14, then the maximum sideband level of 8 db. will occur at radio frequencies of $(f_0 \pm 0\cdot2)$ kc./sec. to $(f_0 \pm 1\cdot2)$ kc./sec. The maximum levels of sideband components created by modulation frequencies greater than 1 200 cycles per sec. will be relatively distributed according to the power ratios corresponding to the modulation factors given in Fig. 14. Thus two upper boundaries may be drawn symmetrically about the carrier representing the sideband, and hence the radio-frequency spectrum, maxima. Part of one and all of the other side of the spectrum created by symmetric-sideband transmission is shown by the part 1A in Fig. 29(1).

It is now necessary to find a lower boundary to delineate the levels at which it may be said the spectrum has " appreciable " intensity. Assume that the spectrum is received rectified and made to energize a loud-speaker, that no distortion whatsoever is introduced in the process, and that the receiver is perfectly selective and will not appreciate energy lying outside the frequency limits occupied by the spectrum. The spectrum-intensity variations are then evidently converted to audio-frequency variations, which, in turn, are made to create, through the agency of a loud-speaker, pressure variations in the air which are detected by the ear as audible sound. Presumably the receiver will be so adjusted that the maximum sounds (corresponding to the condition when the sideband components in the spectrum touch the boundary level drawn as described) are of a level to satisfy the tastes of the listener. Now if any unwanted or parasitical wave energy, having frequencies which lie between the frequency limits occupied by what we may call the " wanted stations spectrum," had a sufficient intensity, it would create audible and unwanted sounds in the loud-speaker output. On the other hand, if the intensity of the interfering-wave energy was less than a certain amount it would not create audible interference. Presumably, however, assuming a " perfect " reception system, it would create some corresponding air-wave energy, however feeble. The only basic reason, therefore, why very feeble interference is not heard is because the human ear itself is incapable of detecting it. This means that the lower boundary of the area within which energy levels may be described as " appreciable " is the ear threshold-of-hearing curve. If the ear were infinitely sensitive, then, apart from the effect of the masking of small sounds by the louder, it would appreciate in the postulated system the effects of all noise of whatever level. A " perfect " crystal set tuned to a local station is in one sense very selective but only because the ear is, by its limited sensitivity, also selective.

It is now necessary to choose the relative levels between the minimum of the ear threshold-of-hearing boundary and the maximum sideband intensity so that a complete area may be enclosed within which we may say wave energy can be appreciated in ideal conditions of reception.

The choice of this (decibel) scalar distance is a matter of the choice of the " contrast " level necessary to give as faithful a rendering of the original as possible. If we attempt to imitate too nearly the conditions of the concert hall, background noise becomes too loud to be eliminated by any but the strongest signals. It is said that a symphony orchestra has a contrast level between *ppp* and *fff* of 80–90 db. The ear has a range between sensation level and ear threshold-of-hearing of 130 db. Allowing for the many difficulties inherent in the design of high-fidelity systems, and allowing for a reasonable signal/noise ratio, it is proposed to choose 60 db. as a contrast level desirable for good-quality reproduction. Choosing this value permits the area to be enclosed as shown by the part 1A in Fig. 29(1).

It will be clear that if reception is to remain, in all circumstances, unspoiled by unwanted interference, then the area must be considered sacrosanct and must never be invaded by energy components created by unwanted stations or by sources of parasitical waves. We shall only consider hereafter the possibilities of invasions of energy due to spectrum components created by other transmissions located on frequency-contiguous channels, i.e. we shall not consider questions of " noise " invasions.

The method of determining the minimum required carrier difference frequency can now be more fully explained. We distinguish first between a wanted station, always labelled " W," and an unwanted station, distinguished on the diagram by the letter " U." A shape is cut out, preferably from thin celluloid, which is the shape of the sacrosanct wanted-stations frequency-decibel area that must never be invaded if interference is to be negligible. Another shape is similarly prepared, the upper limits of which are drawn to represent the carrier and sideband intensities and therefore the upper limits of the energy created by an unwanted station's transmission. The combination of cut-offs introduced in the apparatus of transmission and the fact that the sound energy itself is so relatively feeble at high modulation frequencies allows the upper boundary to become substantially a vertical line at carrier difference frequencies of 8 000 cycles per sec. But the area representing the invading energy extends theoretically infinitely downwards from zero level. Since we are only interested in its value in relation to a limited vertical scale, it is not necessary, except in certain cases where one sideband is attenuated, to consider any limits to the lower boundary. Thus in the part 1A in Fig. 29(1) the unwanted station's energy is only delineated as regards its maxima.

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* " The Required Minimum Frequency Separation between Carrier Waves of Broadcast Stations," *Proceedings of the Institute of Radio Engineers*, 1933, vol. 21, p. 193.

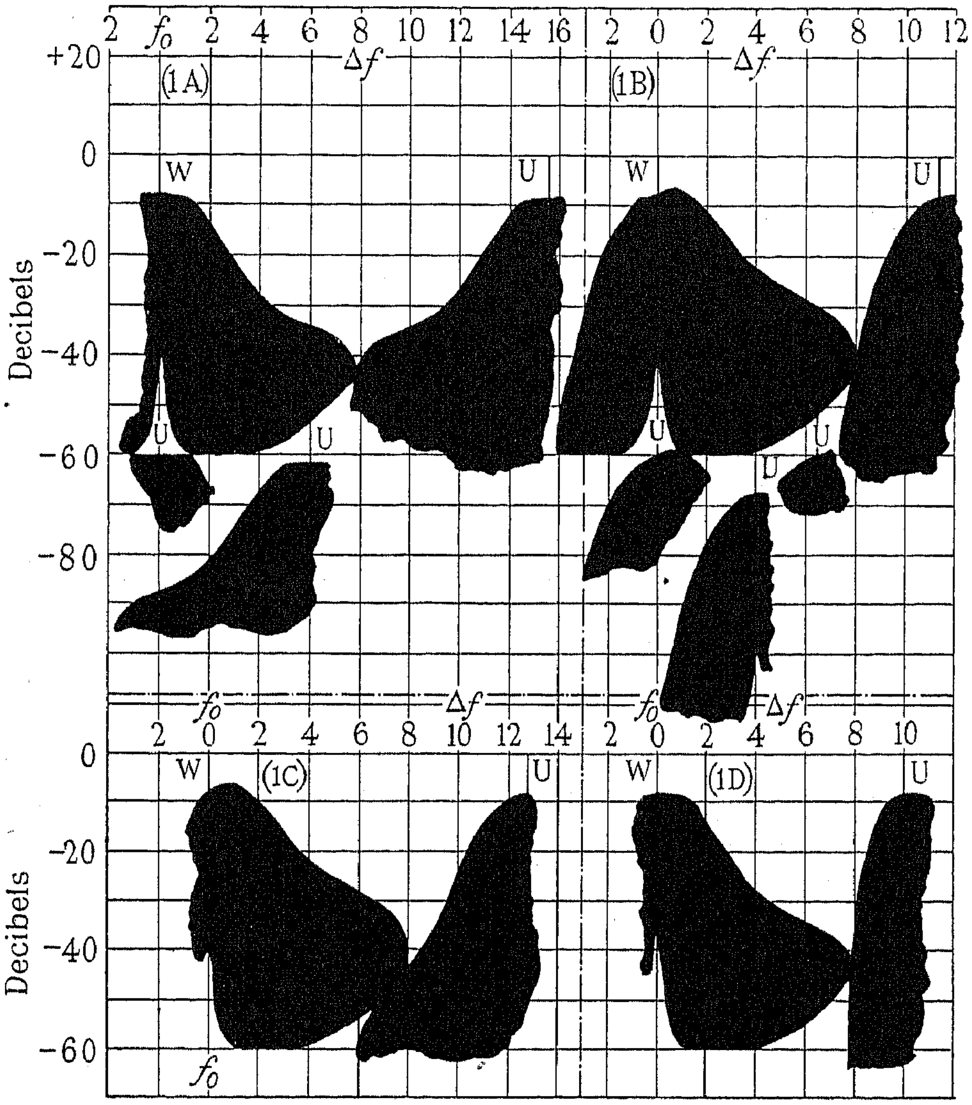

**Fig. 29(1).**—Minimum required carrier difference frequencies.

1A: Double sideband. 1B: Asymmetric; *k* filter plus band eliminator, demodulation correction. 1C: Asymmetric; *k* filter only, demodulation correction. 1D: Asymmetric; *k* filter plus band eliminator, no demodulation correction.

The wanted-station shape is anchored on scaled squared paper with its carrier components vertical; and the unwanted-station shape is held, also with carrier vertical, in various relative positions, it being always ensured, however, that the shapes shall not overlap. The horizontal scalar distance between the lines representing the carriers is the frequency separation, and the vertical scalar distance between the carrier tips is the relative level. This indicates the required minimum carrier difference frequency for interference-free reception, given different relative fields at any one reception location.

If now the wanted-station area is again fixed but the unwanted is separated by sub-optimum carrier difference frequencies, then there is an area of overlap. This area, varying with a change of relative levels, is a measure of the interference produced when the carrier-frequency separation is smaller than that necessary to prevent interference. The determination of the conditions can be performed with different shapes representing the different spectra created by various types of transmission.

## Preparation of Required Spectrum Areas

### Double sideband.

In Fig. 29(1) the part 1A shows the required spectrum areas for double-sideband transmission and has been prepared according to methods previously described. The lower-frequency sideband spectrum of an unwanted station is also shown occupying the next higher-frequency channel to that occupied by the wanted station, which has its upper-frequency sideband fully represented. The two transmissions are of equal level and their spectra touch but do not overlap.

### Asymmetric.

The parts 1B, 1C, and 1D in Fig. 29(1) show different asymmetric spectra. The conditions are given in the Figure, and the figures and letters chosen are in accordance with the schedule of conditions previously set out under " General " in this Section. The following is the method of preparation of the required shapes.

For the fully transmitted sideband: If no demodulation correction is supposed to take place then the fully transmitted sideband spectrum shape is the same as that representing the double-sideband case. If, however, correction is introduced, obviously the level corresponding to that of the sideband maxima when double-sideband transmission is represented is multiplied by $1/M_A$ (see Fig. 12).

For the attenuated sideband: This is first drawn as for the fully transmitted sideband. The decibels of attenuation given by the curve representing the attenuation/frequency characteristic for the circuit supposed to be used are then subtracted from the curve drawn to represent the fully transmitted sideband. The resulting boundary gives the maxima of the attenuated sideband.

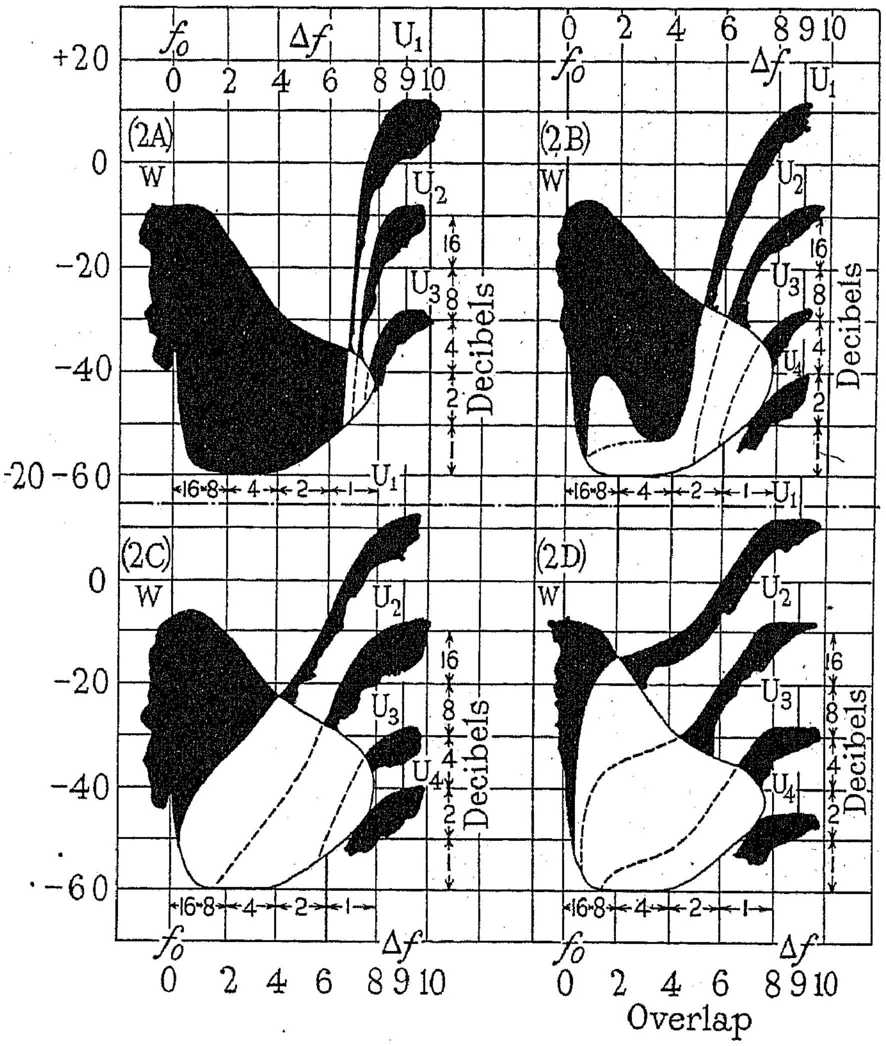

**Fig. 29(2).**—Spectrum-overlap interference.

2A: Asymmetric; *k* filter plus *k* eliminator, no demodulation correction.
2B: Asymmetric; *k* filter plus *k* eliminator, demodulation correction.
2C: Asymmetric; *k* filter only, demodulation correction.
2D: Double sideband.

## Explanation of Fig. 29(1)

**Minimum separation.**

The black areas in the parts 1A, 1B, 1C, and 1D in Fig. 29(1) represent some typical positions of the sliding unwanted-station spectrum, but in order to economize space neither all the positions that may be used nor the full extent of the spectra are shown. The conclusions given hereafter were obtained by measuring the relative positions of the various shapes representing the spectra; the diagrams only exist to show how these were typically placed.

## Explanation of Fig. 29(2)

**Interference.**

The degree of interference is assessed in the parts 2A, 2B, and 2C, in Fig. 29(2), representing conditions corresponding to the chosen labelling. The degree of interference must be a function of the amount of the wanted-station's frequency-decibel area which is occupied by the energy components of the frequency-contiguous unwanted station. But the interpretation of the results is not made in terms of the direct ratio of the areas invaded (*a*) when double-sideband, (*b*) when asymmetric technique is used, because this would not give a true assessment of the economies effected by cutting away part of one sideband.

Thus the listener adjusts his receiver, when sideband interference takes place, to cut off the upper parts of the spectrum in order to get rid of extraneous sounds. This process becomes increasingly destructive to good-quality reception as more and more " top " is cut off. If conditions are such that the sideband invasion becomes very pronounced, the listener, rather than forgo all intelligibility, tolerates, albeit grudgingly, some interference. This means that the onslaught of unwanted energy which brings the invading frequency close to the carrier is far more dangerous to peaceful reception than one that is staged in the remoter outer regions of the sideband.

For these reasons the amount of interference is more truly assessed by giving in the invaded areas different " weight " factors than by taking their direct ratio as a measure of annoyance. These weight factors have been chosen as follows: between 8 000 and 6 000 cycles per sec., 1; between 6 000 and 4 000, 2; between 4 000 and 2 000, 4; between 2 000 and 1 000, 8; between 1 000 and 0, 16. Similarly, weight factors are chosen for the vertical scale. These are shown in Fig. 29(2). The black areas are those in which the wanted spectrum is never invaded, while the conditions for different relative levels between wanted and unwanted stations are shown by dotted lines in the white spaces.

It will be remarked that in the part 2B in Fig. 29(2) the attenuation-curve shape using the band eliminator is such that a larger part is carved out of the wanted spectrum than would be the case if an $f_\infty$-section band eliminator were used. The condition has not been studied; it might be that phase-asymmetry distortion would become serious if a more complex network were employed, but this seems unlikely since the tendency to inverse flexures of the phase characteristics of the inverse types of filter sections would still exist.

## Increase of Available Channels

Let us assume that it may be argued that, after all, existing interferences are not serious and that the great majority of people listen to the local station and are not bothered as much by spectrum-overlap effects as an inspection of the diagrams would lead the reader to believe. Taking what is, to the author's way of thinking, this false premise as a basis, it is interesting to work out how much closer stations could be moved, if the asymmetric technique were adopted, to give the same degree of interference as exists to-day. Choosing the condition of equal level and the asymmetric conditions 2D and 2B in Fig. 29(2), we find that a separation of 6 kc. could be tolerated, giving an increase of about 30 % in the number of available channels.

### Summary of Results

The results are summarized in Table 3.

In considering the "gains" we take, as a criterion, carrier and single-sideband working. It is clear that in a 1 000-kc. gamut, the maximum modulation frequency transmitted being 7 800 cycles per sec., this system would allow 128 free channels. A figure of merit is given to each system, this figure being the ratio of the free channels available for that system assuming no spectrum overlap. The percentage gain is the number of extra channels over and above those given by the double-sideband system divided by the number given by the carrier and single-sideband system, multiplied by 100. The interference figure of merit is the interference factor of a given system divided by the interference factor of the double-sideband system, multiplied by 100.

Table 3*

| Condition | Reference in Fig. 29(1) or 29(2) | Practical case | Minimum separation of carriers to prevent overlap (kc.) | Gain of asymmetric over double-sideband in number of available channels for no overlap | | | Ratio of interference in given system to that in double-sideband system with 9-kc. carrier separation: equal fields at point of reception |
|---|---|---|---|---|---|---|---|
| | | | | Number of channels per 1 000 kc. | Figure of merit | Gain (%) | |
| Double sideband .. .. | 1A | Existing conditions | 15·6 | 64 | 0·5 | — | — |
| Asymmetric filter and eliminator, demodulation correction | 1B, 2B | Results obtainable by modifying existing low-power-modulated transmitters | 11·4 | 88 | 0·686 | 19 | 0·12 |
| Asymmetric filter only, demodulation correction | 1C, 2C | Results obtainable by modifying existing high-power-modulated transmitters | 12·8 | 78 | 0·61 | 11·9 | 0·234 |
| Asymmetric filter and eliminator, no demodulation | 1D, 2A | Theoretical | 10·0 | 100 | 0·78 | 28·1 | 0·011 |
| Carrier and single sideband .. | — | An eventual possibility | 7·8 | 128 | 1·00 | 50 | 0·00 |

* If, with the ratio of the fields of the wanted and unwanted stations equal to unity, it was decided that we could support the same degree of interference as is suffered to-day [see 1A, Fig. 29(1)], then the carrier-wave separation for Condition 2B would be 6 kc., giving 150 channels in a 1 000-kc. band, an increase of 40 channels over and above those available with a 9-kc. separation.

Table 3 is supplemented by Figs. 30 and 31, which show some of the information in curve form.

Summarizing, the investigation proves that the asymmetric system makes for great economy in the use of any available frequency gamut. Its application would be a great help in minimizing the all-too-common sideband interferences set up because of the insufficient carrier-wave separations commonly employed. A point of further interest not brought out quantitatively in the analysis is that the signal/noise ratio is increased by the adoption of the system, assuming, of course, that the receiver has also a narrowed band width of response enabling it to benefit from the asymmetric transmission.

## (7) RECEPTION PROBLEMS

### General

The preceding analysis has assumed that the receiver introduces no distortion and is sufficiently selective to be quite unaffected by any wave energy lying outside the boundaries which enclose an area within which, it has been postulated, wave energy is "appreciable." This Section treats the problems of reception in terms of actualities: no receiver is perfect in the sense postulated, and its departure from perfection must to some extent modify the conclusions.

It is suggested, however, at the outset that it is hardly the business of the transmission authorities to attempt to solve reception problems introduced by changes in transmission technique, provided they can be satisfied that it is not in principle impossible or impracticable to design the circuits required to make full use of the changes. Transmission is to reception what environment is to the evolution of species; it is "up to" the species to modify themselves to the changed environment. It is, however, the business of the transmission authorities to ensure as far as possible that the spectra of stations occupying contiguous channels shall either not overlap or that the overlap is small. If this is not done then the receiver, even if it has an asymmetric response in relation to its response to the carrier component, cannot *per se* eliminate inter-station interference. Thus if a wanted station lies between two neighbours both of which create fields comparable with that of the wanted station at the point of reception, both sidebands of the wanted station will be equally interfered with and single-sideband or asymmetric-sideband *reception* cannot eliminate such interference. On the other hand, it would be of no use in reducing inter-station interference if, assuming asymmetric transmission, the receiver gave a symmetrical response about the carrier and of sufficient frequency-

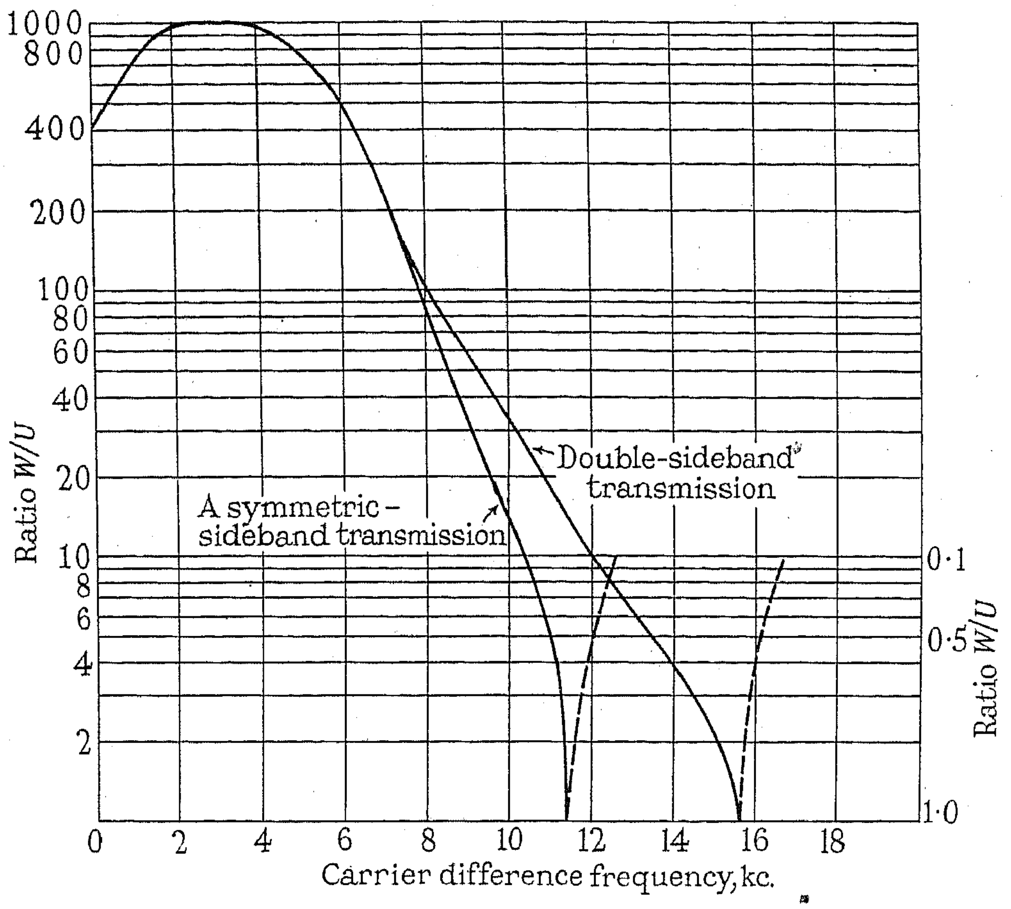

**Fig. 30.**—Required frequency-separation for no spectrum overlap.

Asymmetric demodulation corrected in transmitter. Contrast level, 60 db. $W$ = wanted station, $U$ = unwanted station. Right-hand scale refers to broken-line curves.

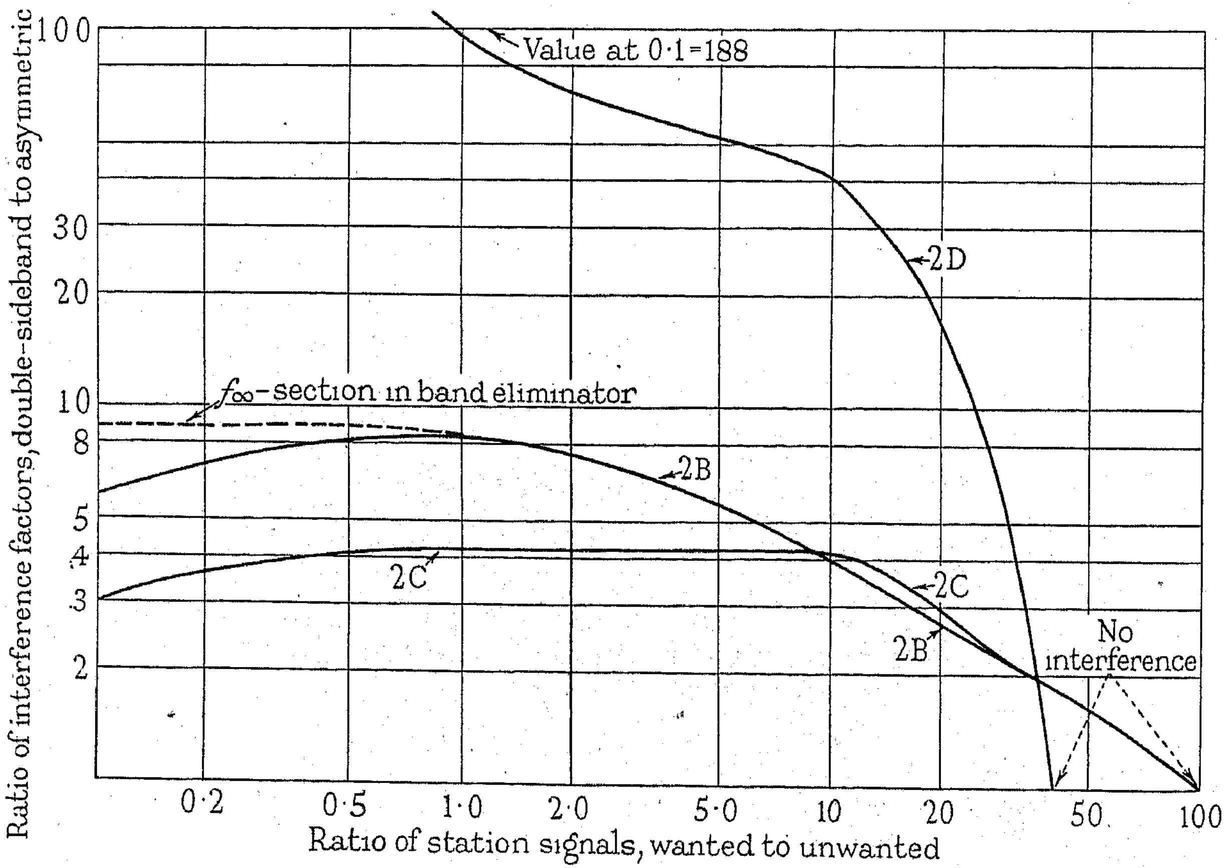

**Fig. 31**

width to include not only the wanted but inevitably the unwanted sideband. The asymmetric system postulates co-operation between receiver and transmitter, and this is one of the difficulties in applying it to space broadcasting.

So far we have postulated " ideal " reception, i.e. the receiver has been supposed to be responsive only to the energy components of the wanted station and completely unresponsive to any other. But evidently this is a theoretical postulate; the receiver cannot embody filters with infinitely sharp cut-offs nor filters which, with asymmetrically-disposed characteristics in relation to the received carrier, do not also introduce distortion. It is clear that the wave-envelope distortion, due to magnitude and phase asymmetry between sideband components introduced by a sideband-cutting filter, can be produced equally in the receiver and the transmitter. In other words, it is the combined characteristics of all the filters which are interposed between the circuits creating a normal modulation of the carrier and a detector which must be analysed before the degree of distortion and sideband-cutting may be assessed. It would be unnecessarily laborious to work out an example here, but in general the modifications to previous conclusions introduced by the practical receiver demand that the carrier separations should be a little greater than those given by the former analysis.

### Modifications of Space-Broadcasting Receivers for Asymmetric Transmission

If asymmetric technique were applied in to-day's conditions of space broadcasting to a given transmitter, it is doubtful whether its introduction would be remarked by any of the possibly millions of listeners who are accustomed to tune in that station. The distortions produced would be negligible compared with those existing in typical receivers, and the apparent strength of signals would be the same as before the changes were made. Presumably also in certain conditions of reception the interference from a powerful neighbouring station would be the same as it was before the changes were made.

This interference could only be eliminated by narrowing the band width of reception of the receiver and disposing the carrier frequency asymmetrically within this band. This process would eliminate the interference even if the other neighbouring station created a relatively strong field at the given point of reception. The interference would, however, only be eliminated at the risk of an introduction of added harmonic distortion in the receiver. This increase of distortion would not, it is suggested, be remarked by the ordinary listener.

Turning from this robust consideration of what could undoubtedly happen to the tuning of the ordinary set by the ordinary listener who is automatically finding the best way to benefit from the new system, we may discuss what would happen to the high-quality set in the field of an asymmetric spectrum. Firstly, no high-quality set can be so described when to-day it seeks to pick up any but the local station; to eliminate sideband jamming during distant and some local-station reception necessitates the introduction of increased selectivity, which robs quality of its prime necessity, namely " top " reproduction. Secondly, if the high-quality set tuned to the local station with the carrier frequency symmetrically placed in relation to its filters is to be so good as to appreciate, without a reference standard, the deterioration of quality due to asymmetric transmission, it will be a comparatively expensive and a very rare article indeed. It is therefore obvious that the problem, if indeed it can be said to exist, of the high-quality receiver is one concerning a very small minority.

During the discussion of the author's original paper a number of persons gave it as their opinion that the problems of eliminating spectrum-overlap interference could be solved by designing the intermediate-frequency filters of the receivers so that they would accept only the carrier and one sideband, and by vastly intensifying the carrier component at the detector, transmission being meanwhile modified to a carrier-and-single-sideband system. This would be an ideal solution, of course, if some 30 million receivers could be replaced on the same day. Since this would be impossible, obsolescent receivers would, according to this suggestion, suffer distortions from the modified transmitters. These distortions would be intolerable, amounting to 10 % for 80 % modulation at every modulation frequency; not, as in asymmetric technique, to some 2 % to 3 % only at modulation frequencies between 2 500 and 3 500 cycles per sec.

Another suggestion made was that double-sideband transmission should be continued but that receivers should be adapted to cut off one sideband and intensify the received carrier so as to eliminate the distortion introduced by the receiver filters in cutting off one sideband. This, however, would be no help when the sideband chosen was interfered with by a neighbouring station. It was suggested that a solution to this difficulty might lie with a frame aerial to eliminate the neighbouring station prone to jam the sideband selected for reception, and a sideband-eliminating filter to get rid of the jamming from the other. This method attempts to turn the listener into a skilled wireless operator, a process which would be widely resisted.

It is important to note that in respect of both these suggestions authoritative opinion showed that it was perfectly feasible to design receivers which would in fact augment the carrier component in relation to the sideband component. The use of such types of receivers for high-fidelity local reception would therefore reduce even what are still by the author considered to be negligible harmonic distortion components in the asymmetric transmission to values acceptable to the most discriminating ears. In sum, the ordinary receiver seeking distant-station reception could not remark distortion in comparison with that already existing. The high-quality receiver, useful as such only when tuned to the local station, would not introduce any more distortion than that existing in transmission and proved to be negligible. The high-quality and eventually all receivers could eliminate *all* distortion if means were adopted to intensify the carrier component at the detector. This point is stressed in the Conclusion to the paper.

## (8) ADAPTATION OF EXISTING TRANSMITTERS

The experiments so far conducted have been concerned with carrier frequencies of the order of 25–27 kc./sec., partly because the system was primarily devised for a carrier-frequency wire broadcasting system, partly because it is naturally far easier to shape filter characteristics to requirements for sideband attenuation at these frequencies than at " broadcasting " frequencies.

### Low-Power-Modulated Transmitter

In adapting a low-power-modulation transmitter, the low-power stages, where modulation takes place, would be substituted by a low-power asymmetric-sideband modulator using a frequency of the order of 30–50 kc./sec., and this frequency would be multiplied to the value of the carrier frequency used by the station and would then be fed into the high-frequency magnifier. Naturally all the filters in this magnifier would have the carrier frequency $f_0'$ located at their mean pass frequency $f_m'$, and no further phase or magnitude asymmetry between sideband components would be introduced.

### High-Power-Modulated Transmitter

In transmitters using the high-power system of modulation, filter networks giving the required asymmetric treatment to counterpart sideband components would have to be interposed between the high-power output from the modulated magnifiers and the input to either the aerial or, if used, further magnifying valves. Such filters would presumably have to perform their functions at frequencies coincident with or near to the carrier frequency, and, unless the proper types of network were chosen to prevent it, excessive and uncontrollable voltages might be developed across the terminals of the filter elements. Thus two problems would arise in adapting the high-power-modulated transmitters: first, how to shape the attenuation/frequency characteristics when the ratio $\Delta f/f_0$ was relatively very small; and, second, how to avoid the development of very high voltages across the filter elements. The author has not been able to give more than a superficial consideration to these problems and his existing conclusions may therefore not be verified, but it would appear that suitable networks could be devised which would approximate in their performance to the ideal and yet be practicable in use.

The great majority of transmitters in use in the world to-day employ low-power modulation, and could therefore be cheaply and efficiently modified to embody the best form of the asymmetric system, assuming that demodulation correction must take place in the transmitter.

## (9) CONCLUSION

In spite of discouragement from some quarters, the public insists that the wireless set exists to give pleasure; the listener chooses this or that programme because it pleases him, rejects another because it does not. The attention that the wireless-set user gives to a programme is also partly determined by the quality of reproduction but probably more by the freedom it enjoys from interference from extraneous sounds. It is primarily programme interest which pleases, but interference is a bugbear however good the programme. If proof were needed that considerations of quality of reproduction are not paramount in the mind of the ordinary listener one would have but to listen to the sounds emanating from his set and compare them, not necessarily with the original, nor even with the quality obtainable from a high-fidelity set, but rather with the results given by a receiver costing only a little more than the most popular and which is well within the purchasing power of the majority. If proof were needed that interference is a greater bugbear to the ordinary listener than the combination tones created by harmonic distortion, an analysis of correspondence and of the efforts of national committees to cure it would furnish such proof. This is not said in disparagement of the designers and retailers of wireless sets, who rightly cater for public demand, nor need it be inferred that because public interest centres round programme content, the first cost of a set, and convenience, listeners would be indifferent to improvement in quality of reproduction.

If these observations be accepted as truly representing the situation, then it seems to be hardly justified to forgo the advantages of asymmetric-sideband broadcasting because it may introduce some small extra measure of harmonic distortion. This point is raised because in the discussion on the author's previous paper there appeared to be a considerable body of official opinion which opposed the scheme on account of its alleged failure to give an acceptable standard of quality. The quantitative analysis and the results of experiments presented in this paper should, it is hoped, prevent a repetition of this criticism.

But if the objections of that esoteric but influential few who hold that quality is of paramount importance are enough to cause the rejection of the scheme as so far promulgated, then it is hoped that they will accept it when it is put forward as a step in an evolutionary policy. No one will deny that ultimately the ideal solution is to use a carrier and single-sideband system for the broadcasting service, provided flawless quality can be obtained. Therefore the scheme proposed is one which aims to achieve this ideal gradually but which asks for some courage in facing a temporary and theoretical failure to produce a completely distortionless transmission. The first step would be to introduce some form of asymmetric transmission. The slight distortions, noticeable perhaps by a very few listeners using very rare and expensive sets and possessing highly-discriminative ears, would soon be cured by the technique of carrier intensification in the receiver. The luxury of yesterday becomes of wide application to-day; one might well see, therefore, the introduction of the asymmetric system as the stimulus towards wireless-set evolution in terms of what some call homodyne reception, namely carrier-component intensification at the detector. If a majority were so equipped then the transmitters could safely narrow their radiated spectra and increase a distortion eliminated in the receiver. In the end, and assuming a sufficient number of receivers properly designed to make use of the changed technique, single-sideband transmission might be adopted and the proper solution achieved. But apart from this suggested solution, and dealing only with to-day's problems, it would appear to the author better to risk the introduction of minor distortions than to continue with the existing and abominable interferences which are set up whenever one seeks to listen to distant stations and which are all too audible when one is listening to local stations in B and C service areas.

## (10) ACKNOWLEDGMENTS

The author is indebted to the directors of David Allen and Sons, Ltd., who have allowed him to spend considerable time on the preparation of the paper, and to the British Broadcasting Corporation for providing the funds for the construction of test apparatus. Sincere thanks are due to Countess Von Zeppelin for her work, detailed in Appendix II, in evolving a most useful expression for the distortion component when both $\xi$ and $\phi$ are variable. Mr. R. E. H. Carpenter has given pertinent criticism and advice during the progress of the experiments, and Mr. G. B. Ringham has greatly aided the evolution of the theories herein set out by his careful conduct of the experimental design and construction work; efficient help has been rendered in the latter connection by Chief Petty Officer G. V. Burchell.

## APPENDIX I

The analysis given in this paper has purposely treated the simpler cases, the only variable being either $\xi$ or $\phi$, without deriving an expression for the distortion. This procedure is logical inasmuch as a method of attack is thereby illuminated and expressions for the simpler cases can be derived from the formula for the general case by giving one or other of the variable fixed values. It is, however, shown on pages 42–46 that the general formula has a limited although highly practical application. It is therefore proposed in this Appendix to derive expressions for the distortion in the special cases in order that the application of the general formula may be verified or modified.

$\xi < 1$, $\phi = 0$.—The resultant vector OR has a magnitude given by equation (7) as

$$\mathrm{OR} = (b^2 \sin^2 \Delta\omega t + a^2 \cos^2 \Delta\omega t + 1 + 2a \cos \Delta\omega t)^{\frac{1}{2}}$$

It is further shown that the harmonic has its maximum amplitude when $\Delta\omega t = 90°$. Thus, at this instant,

$$\mathrm{OR} = (b^2 + 1)^{\frac{1}{2}}$$

Consider Fig. 32, in which the full-line curve represents a pure sine wave. The effect of magnitude asymmetry is always to raise the value of OR beyond what it would be if no magnitude asymmetry existed. This value we call $D$, and it is seen that the broken-line curve, which is the distorted-wave envelope, is formed by the combination of a pure sine wave (full line) and the 2nd harmonic (shown as a chain-dotted line having an amplitude wholly above the true mean line of the rectified currents). Thus we can see that the peak amplitude of the harmonic is $D/2$, where

$$\frac{D}{2} = \frac{\mathrm{OR} - 1}{2} = \frac{(b^2 + 1)^{\frac{1}{2}} - 1}{2}$$

But if $\xi = 0$, $M_N = 1$, and $b^2 = 0{\cdot}25$; then an error no greater than 1 % (and that in exceptional cases) is introduced if we write

$$\frac{D}{2} = \frac{\dfrac{b^2}{2} + 1 - 1}{2} = \frac{b^2}{4}$$

The distortion factor is given by dividing the peak harmonic amplitude by the peak carrier amplitude, or

$$\delta_\xi = \frac{b^2}{4a}$$

Substituting the values of $b$ and $a$ in terms of $M_N$ and $\xi$ gives

$$\delta_\xi = \frac{M_N}{8} \frac{(1 - \xi)^2}{1 + \xi}$$

which is the expression given on page 44.

$\xi < 1$, $\phi$ *finite*.—If $D_{\phi 1}$ and $D_{\phi 2}$ are the differences when $\Delta\omega t = 0$ and $\Delta\omega t = 180°$ respectively between the amplitude of the distorted wave and the amplitude of the undistorted wave, it can be seen that the percentage distortion if not too large can be expressed as

$$\delta_\phi = \frac{D_{1\phi} + D_{2\phi}}{2[1 + M_N - (D_{1\phi} + D_{2\phi})/2]}$$

We may furthermore write, from equations (15) and (16),

$$D_{\phi 1} = (M_N^2 - 2M_N \cos \phi + 1)^{\frac{1}{2}} - (1 - M_N)$$

$$D_{\phi 2} = (1 + M_N) - (M_N^2 + 2M_N \cos \phi + 1)^{\frac{1}{2}}$$

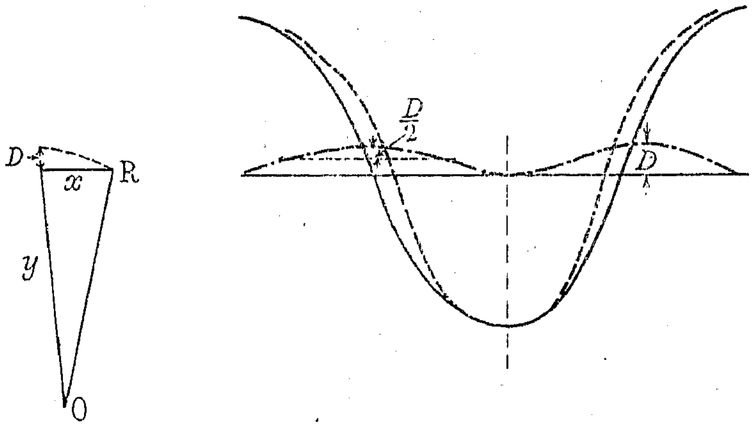

Fig. 32

This can be shown to give, for the percentage distortion,

$$\delta_\phi = \frac{(1 + M_N)^2[1 + (1 - Q)^{\frac{1}{2}}] - 2(M_N \cos^2 \phi/2 + 1)}{(1 + M_N)^2[1 - (1 - Q)^{\frac{1}{2}}] + 2(M_N \cos^2 \phi/2 + 1)} \times 100$$

where $$Q = \frac{2M_N}{(1 + M_N)^2}(1 + \cos \phi/2)$$

If $\phi$ is very small,

$$\delta_\phi = \frac{(1 + M_N)\,[1 + (1 - Q)^{\frac{1}{2}}] - 2}{(1 + M_N)\,[1 - (1 - Q)^{\frac{1}{2}}] + 2} \times 100$$

Points derived from this expression, which must not be used unless $2\phi$ is greater than about 90°, are shown in Fig. 13B and agree well with the graphical analysis.

## APPENDIX II

The case where $\xi$ and $\phi$ are both variables is treated here with reference to Fig. 33, which in general is representative of the same conditions as Fig. 5 and only differs in that the attenuated sideband component in the one case is the fully represented one in the other and vice versa, and that a different instant of time is chosen in Fig. 33 from that represented in Fig. 5. Obviously the expression derived will be the same whichever Figure is used.

From Fig. 33 it is obvious that

$$OR = (OC^2 + CR^2)^{\frac{1}{2}} = OC\left(1 + \frac{CR^2}{OC^2}\right)^{\frac{1}{2}} \quad . \quad (30)$$

Two approximations are now made to render the algebra less cumbersome than if analysis of a complete expression for OR were attempted. These approximations are:—

(1) It is assumed that the ratio CR/OC will never exceed a value such as to make unjustifiable the approximation

$$\left(1 + \frac{CR^2}{OC^2}\right)^{\frac{1}{2}} \simeq \left(1 + \frac{CR^2}{2OC^2}\right)$$

(2) It is assumed that FC may be neglected in comparison with OF, in the denominator of the ratio $CR^2/(2OC^2)$.

The implications of these approximations are dealt with in the text of the paper. Using the approximations enables (30) to be written

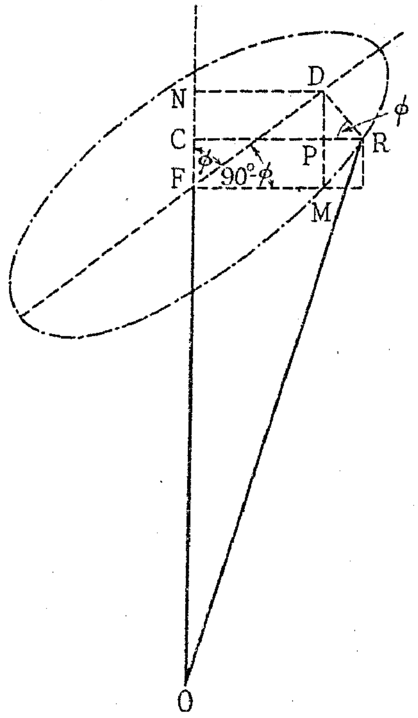

Fig. 33

-·-·- Locus of R.
- - - - - Construction lines.
——— Vectors.

$$OR = OF + FC + \frac{CR^2}{2OF}$$

Or, letting OF = 1,

$$OR = 1 + FC + \frac{CR^2}{2} \quad . \quad . \quad . \quad . \quad (31)$$

It has been shown on page 41 that the magnitude of any axis of the ellipse traced by R is given by

$$(a^2 \cos^2 \Delta\omega t + b^2 \sin^2 \Delta\omega t)^{\frac{1}{2}}$$

indicating that the two components at right angles forming this resultant axis are $a \cos \Delta\omega t$ and $b \sin \Delta\omega t$.

Evidently, therefore,

$$DR = b \sin \Delta\omega t \quad . \quad . \quad . \quad . \quad (32)$$

$$FD = a \cos \Delta\omega t \quad . \quad . \quad . \quad . \quad (33)$$

From Fig. 33 also,

$$FC = FN - NC$$

$$\therefore\ FC = FD \cos\phi - DR \sin\phi \quad . \quad . \quad (34)$$

$$CR = CP + PR$$

and, since CP = FM,

$$\therefore\ CR = FD \sin\phi + DR \cos\phi \quad . \quad . \quad (35)$$

Substituting in (34) and (35) the values of FD and DR given in (33) and (32) respectively,

$$FC = a \cos \Delta\omega t \cos\phi - b \sin \Delta\omega t \sin\phi \quad . \quad (36)$$

$$CR = a \cos \Delta\omega t \sin\phi + b \sin \Delta\omega t \cos\phi \quad . \quad (37)$$

and, combining (31) with (36) and (37),

$$OR = 1 + a \cos \Delta\omega t \cos\phi - b \sin \Delta\omega t \sin\phi + \tfrac{1}{2}(a \cos \Delta\omega t \sin\phi + b \sin \Delta\omega t \cos\phi)^2$$

Expanding the expression in the bracket multiplied by $\frac{1}{2}$ gives

$$\tfrac{1}{2}(a^2 \cos^2 \Delta\omega t \sin^2\phi + 2ab \cos \Delta\omega t \sin \Delta\omega t \cos\phi \sin\phi + b^2 \sin^2 \Delta\omega t \cos^2\phi)$$

or

$$\tfrac{1}{4}\left[a^2 \sin^2\phi(1 + \cos 2\Delta\omega t) + 2ab \cos\phi \sin\phi \sin 2\Delta\omega t + b^2 \cos^2\phi(1 - \cos 2\Delta\omega t)\right]$$

The value for OR can now be rewritten in terms of the sum of three different forms, namely (i) those in which $\Delta\omega t$ does not occur, (ii) those containing $\Delta\omega t$, and (iii) those containing $2\Delta\omega t$, as follows:—

$$OR = \left[1 + \tfrac{1}{4}(a^2 \sin^2\phi + b^2 \cos^2\phi)\right] + \left[a \cos \Delta\omega t \cos\phi - b \sin \Delta\omega t \sin\phi\right] + \left[\tfrac{1}{4}(a^2 \sin^2\phi - b^2 \cos^2\phi) \cos 2\Delta\omega t + 2ab \sin\phi \cos\phi \sin 2\Delta\omega t\right]$$

Evidently the first expression within square brackets is a constant; the fundamental is represented by the second expression within square brackets; and the last expression within square brackets is that for the harmonic, being in terms of $2\Delta\omega t$. The amplitude of the fundamental is given by

$$(a^2 \cos^2\phi + b^2 \sin^2\phi)^{\frac{1}{2}} \quad . \quad . \quad . \quad (38)$$

and the amplitude of the harmonic by

$$\tfrac{1}{4}\left[(a^2 \sin^2\phi - b^2 \cos^2\phi)^2 + 4a^2b^2 \sin^2\phi \cos^2\phi\right]^{\frac{1}{2}}$$

which simplifies to

$$\tfrac{1}{4}(a^2 \sin^2\phi + b^2 \cos^2\phi) \quad . \quad . \quad . \quad (39)$$

The harmonic ratio is given by dividing expression (39) by expression (38), or

$$\delta = \frac{a^2 \sin^2\phi + b^2 \cos^2\phi}{4(a^2 \cos^2\phi + b^2 \sin^2\phi)^{\frac{1}{2}}}$$

Substituting the values of $a$ and $b$ in terms of $M_N$ and $\xi$ gives

$$\delta = \frac{M_N}{8}\,\frac{1+\xi^2-2\xi\cos 2\phi}{(1+\xi^2+2\xi\cos 2\phi)^{\frac{1}{2}}} \quad . \quad . \quad (40)$$

and (40) is the equation used, with certain reservations, in the text of the paper, to determine the distortion.

Note that from (38) we may write

$$M_A = \frac{M_N}{2}(1+\xi^2+2\xi\cos 2\phi)^{\frac{1}{2}} \quad . \quad . \quad (41)$$

which is again used in the text of the paper.

## DISCUSSION BEFORE THE WIRELESS SECTION, 2ND MARCH, 1938

**Mr. H. Bishop:** The B.B.C. has followed the author's work with great interest both from the purely technical point of view and from the point of view of its importance to broadcasting in this country and abroad. The question of the future use of asymmetric-sideband broadcasting or any other specialized type of transmission is, in fact, rather more international than national in character. Nothing is truer than the first sentence of the Summary to the paper, and it is clear that any decisions which the B.B.C may make must be bound up with what is done abroad. The wavelength situation is being debated at the five-yearly Telecommunications Conference which is at present taking place in Cairo, and we shall know at the end of the conference whether any more wavebands have been allotted for broadcasting in the face of the eager demands of practically all the other wireless services. We should indeed be foolish if we were not willing to examine any system of the kind described in the paper which might hold out some possible hope of alleviating the present wavelength congestion.

The paper refers solely to broadcasting, but it might be a good idea to examine its application to other services if there is hope of the more efficient use of wavebands for these services. So far as the present situation is concerned, the amount of interference which the broadcast listener suffers in areas of high field-strength is not serious, but, of course, in areas which it is not possible to serve so well it is a different story. On the medium wavelengths the Lucerne Plan of distribution is being adhered to fairly well by the European countries. Interference is acute in the long waveband, and in the last few years it has been particularly severe on the short waves, where no internationally agreed plan is in force.

In examining the author's system it is a question of balancing the deterioration of quality from which we now suffer, due to the attenuation of the upper frequencies of modulation to avoid interference from frequency-contiguous channels, against the deterioration which is likely to be observed with his system. In this connection I would mention the use of variable selectivity, which was raised in the discussion on the author's previous paper.

Referring to page 66, I am rather shocked by the statement that it is not the business of the transmission authorities to attempt to solve reception problems introduced by changes in transmission technique. Any change the B.B.C. proposes to introduce must take into account the receiver position, as in fact the author said in his verbal résumé of the paper. When one considers the money locked up in receivers in use in this country, it is clear that any change in transmission technique which involves changes in receivers must not be introduced without a great deal of thought and preparation.

At the Bucharest Conference of the C.C.I.R. a year or so ago Great Britain recommended that research should continue on special systems of transmission, with the intention that, when such a system had been proved to be of practical value, it might help to reduce interference and improve quality with the present frequency-separation.

**Mr. H. L. Kirke:** The author mentions that the receiver filter will introduce distortion, and he suggests that this distortion will be small. I should like to ask him whether he thinks that the distortion would be any smaller than the distortion introduced by a similar filter in the transmitter, and therefore why the same care has not to be taken in the receiver as in the transmitter in order that the same degree of smallness of distortion shall be achieved. He also mentioned the question of the desirability of eventually working with a single sideband and a largely suppressed carrier; I agree that that is the system to aim at. If we can get a step nearer to that ideal by the introduction of the asymmetric-sideband system or some modification of it, then I think that that may be a good thing to do, but I am not certain whether the asymmetric-sideband system by itself and for itself is good. Great difficulties lie ahead of any organization which attempts to get all countries to do one and the same thing with one and the same object; until that can be done, the asymmetric-sideband system will only be of use where two countries come to an agreement independently of the rest.

I hope that the idea of closer spacing of carriers will not be adopted. In the last few years the technique of receiver design has gone forward by leaps and bounds. It is now possible to buy a radio receiver at a not very extravagant price which will receive well up into the region of 8 000–9 000 cycles per sec., and it makes a vast difference to the enjoyment of broadcasting programmes if that band of frequencies can be received without distortion. Most of the receivers which are in use to-day have their tone controls turned to what is usually called "mellow," chiefly because of the distortion which is introduced.

I agree with Mr. Bishop that the transmission authorities must study receiver technique, whether it is a question of changing over to the single-sideband and suppressed-carrier system or, for example, of changing over from amplitude modulation to phase or frequency modulation.

I find the paper very confusing, and I feel that if the author could have referred to such quantities as $M_A$ and $M_N$ a little more in words it would have made it easier even for the mathematician to follow what is a very complicated subject.

**Dr. E. K. Sandeman:** The first point which occurred to me in looking at this paper was that it was not essential to use a band-eliminator filter for the purpose of obtaining phase compensation: other types of phase-compensating network could be used equally well.

A point arising out of one of Mr. Kirke's remarks is that asymmetric-sideband transmission demands the introduction of correction in the transmitter for the distortion introduced by the filters in the receiver. This means that it would be necessary that the same type of receiver should be used everywhere, a restriction which would impose a serious limitation on the manufacturers of receiving sets.

I feel that the difficulties of straight single-sideband working are largely resident in cost considerations rather than in the technical problems outlined in this paper, which are all sidetracked by the introduction of a large local carrier.

I also think that the practical difficulties of any system of asymmetric-sideband working are appreciable. For those reasons, and also because single-sideband working has definite advantages over asymmetric-sideband on the grounds of channel spacing, I should prefer to wait until the progress of receiving-set design and manufacture has further reduced costs to the point where the community as a whole can afford straight single-sideband working.

**Mr. P. G. A. H. Voigt:** When the author read his last paper before this Section he assured us that the distortion introduced by his system was negligible, and I was very hopeful of its success. This paper appears to prove that about 2 % is the maximum distortion which is to be expected if his system is treated carefully. The big difference which has occurred in the interim is shown by the fact that whereas in the previous paper the carrier wave was running halfway up the skirt of the sideband, in this paper it is on the top.

I am able to confirm to some extent the author's curve for energy in the spectrum. A long time ago, before frequency standards were available for the testing of microphones, I came to the conclusion that it was possible to derive some relevant information by striking a piano mechanically at a constant strength, and measuring the output with a microphone. If the efficiency of the piano were the same on each note a level response would be obtained, and I expected that if I found great discrepancies the trouble was likely to lie in the microphone rather than in the piano. The results of that test showed a large difference between one note and the next, but if all the results were plotted, and especially if the microphone was tried in various positions and the average value determined, a fairly level response was obtained over the middle part of the scale, and a falling-off at the extreme top. I have since been able to check that the microphone used was substantially linear to within 3 db.

Reverting to the question of distortion, the figure of 2 % is very reassuring, especially if the interference can be cut down by the amount stated. At present, high-quality reception with very flat tuning even of the London National station in certain areas in South London is completely spoilt by the fact that London National fades at times so that it is only a slight signal which interferes with Trieste. The resultant sideband splutter, especially if Trieste is over-modulated, is very bad.

For reception on the system described in the previous paper I gathered that the standard receiver would be satisfactory. It now appears that an addition will have to be made to the receiver, and that addition will in turn produce some additional distortion. Might it happen that the distortion produced in the receiving corrector would be the inverse of that in the transmitting corrector, and so compensate for it?

**Mr. C. E. G. Bailey:** The author's demonstration by means of his celluloid vector was a valuable illustration of frequency modulation; thus, in the case where $\phi$ was 90° the vectors were of equal magnitude, and the resultant hardly varied at all. It is interesting to reflect that the amount by which that resultant did vary represents the difference between a system with first-order sidebands and one with true frequency modulation. This difference would be equalized by a suitable set-up of additional vectors, and the model made to give, say, a second-order approximation of frequency modulation.

As regards the method of eliminating phase distortion, better results might have been obtained with an $m$-derived section, which would only have meant one extra element as compared with the band-stop filter. It might in fact be found that if the original filter were turned from a constant-$k$ into an $m$-derived type the slope would be made steeper, and it would be possible to dispense with the extra cell.

I am interested in the question of the phase characteristics of the filter in the receiver. In the conventional superheterodyne receiver this filter simply consists of pairs of linked circuits. The phase therefore varies widely, and if the filter is arranged, as it frequently is in the case of sets of expanding selectivity, to have a double hump, this phase characteristic may even have a kink in the centre. When the receiver has been in use for some time and the capacitance arms of the filter have altered in value, the amplitude characteristic becomes very asymmetrical. With double-sideband transmission some of this symmetry cancels out, but it seems likely that the phase characteristic may prove worse still, and may in effect annul many of the advantages of the elaborate filter put up at the transmitter in a single-sideband system. This point should not be neglected, especially as the paper is one in which the practical difficulties of marrying a new transmission system to an existing collection of receivers have been carefully considered.

As regards the author's demonstration, I noticed a certain amount of distortion throughout his experiments.

**Dr. L. E. C. Hughes** (*communicated*): We know that the standard of reproduction tolerated by the public is deplorably low, and the recent advances in receiver design which make broadcast transmission more pleasant to listen to are not reflected in the mass-produced receiver. It is true that the amplitude distortion permitted in the author's system is low in comparison with that normally tolerated in radio receivers. At the same time he is retarding progress in good-quality reception by making it impossible for receivers to be used which aim at a lower degree of amplitude distortion. Such a procedure cannot be defended on technical grounds, while the complete elimination of one sideband of frequencies, no difficult task, establishes at once an implied improvement in every demodulator, and therefore is worthy of support, although it may be many years before it is practicable to re-allocate international frequencies to permit the operation of more channels. It is nearly impossible to contemplate the elimination of the carrier, in spite of the power savings at the transmitter. Engineers cannot defend giving the public what it is alleged to want, because the public has no possible means of judging what it might want if it knew what was available. The task of enlightening the public to an appreciation of good-quality reproduction, and what it is worth, might be considered a legitimate activity of the new Engineering Public Relations Committee, of which The Institution is a supporter. If the public could be persuaded to take broadcast programmes more seriously, broadcasting, as a system, would become more valuable and so justify its great expense. One of the major factors is the acoustic fact that good-quality reproduction, involving crisp speech and clear-cut acceleration of notes, means a clean bass and good high-note response without a sharp cut-off, and is far more attention-compelling than the " mellow " tone which the public has been trained to accept, by the simple process of not offering it anything else. The great majority of the public has not been trained to listen to original music, and does not attend orchestral concerts or appreciate high-grade violin playing at first-hand. I have the opportunity of listening to between 50 and 100 new broadcast receivers and loud-speakers every year under carefully standardized conditions, and I find ample evidence that the general quality of the mass-produced broadcast receiver is deteriorating. The trade is not passing technical benefits to the public, and from this point of view any advocacy of further degradation is retrograde, whatever technical merit the author's scheme otherwise possesses.

**Mr. W. T. Sanderson** (*communicated*): The paper has justified, by analysis, the faith the author expressed some 2 years ago, in the practicability and advantages of his system of asymmetric-sideband transmission. Whether it is desirable at this stage to press for its adoption depends on one's opinion of the future of broadcasting as a whole.

There is in many minds a belief that all high-quality broadcast reception will eventually be achieved by the use of wires and that wireless broadcasting will continue principally to serve large, thinly-populated areas. For this purpose and for all the commercial services occupying the other available channels, a distortion of a few per cent is unimportant and the asymmetric system offers most valuable advantages.

Even if broadcasting were to continue on the present lines, I should hesitate to support the immediate adoption of the asymmetric system only because of the fear that, as soon as the change had become effective, station separation would be reduced to 5 kc. and all chance of getting good reproduction wiped out for ever.

It is claimed that the public do not want the reproduction of top notes because if a tone control is provided the user always sets it at " mellow." Surely this is a false deduction. Much as one may dislike " mellow " reproduction, it is more pleasant than the harmonic distortions, resonances, and sideband splash, which contribute so largely to top-note response to-day. Further technical advances, particularly in loud-speaker design, should remove present difficulties, when the demand for a wider frequency spectrum would probably arise. Asymmetric-sideband transmission would provide a means of satisfying the requirements if a 9-kc. carrier separation still existed. A large proportion of music is written to stimulate the hearer, and the higher frequencies produce such stimulation. The average listener may, however, want to be lulled, and may use the tone control to this end.

**Mr. P. P. Eckersley** (*in reply*): The asymmetric-sideband system was designed to eliminate sideband-splash interference, not, as would appear from the discussion, solely to introduce harmonic distortion.

The decision as to whether the system merits adoption would be more easily taken if some organization would spend the few hundreds of pounds necessary to make a trial.

In the absence of large-scale practical tests and continuing discussion instead of much-needed action the merits are debatable, as Mr. Bishop states, in terms of a balance of factors. The discussion has revealed a quite understandable reluctance on the part of responsible people to introduce even more distortion into broadcasting systems, but has not shown any desire to consider the merits of asymmetric technique in regard to its ability to eliminate interference. As a matter of principle it is right to attempt, so far as is practically possible, to eliminate harmonic distortion. As a matter of principle it is equally right to attempt, so far as is practically possible, to eliminate interference. In principle, therefore, these two factors cancel and it is the harder task, once the safe shelter of principle is abandoned, to decide the question quantitatively rather than qualitatively.

No one has challenged the quantitative conclusions set out in the paper and they can therefore be re-stated with more confidence since they have passed unscathed the critical examination of well-informed technicians.

(1) The harmonic distortion introduced in transmission by asymmetric technique amounts to between 2 % and 3 %. This maximum figure of distortion occurs only when modulation is of the order of 80 %, namely it occurs during about 10 % of the time of transmission. The distortion of this value occurs at a modulated frequency of 3 000 cycles per sec. (roughly) and falls to negligible values at 2 000 and 4 000 cycles per sec. and is thus concentrated in a narrow band of frequencies.

(2) For the introduction of the negligible amount of distortion in transmission defined above, sideband splash may be reduced 10-fold, but receivers, when tuned asymmetrically to do this, may introduce more distortion than the 2 %–3 % existing in transmission.

As a reassurance of (1) above it has been proved that even highly trained ears have found considerable difficulty in distinguishing between orthodox and asymmetric technique even in terms of a standard the public could never expect unless they were allowed a service of wire broadcasting.

Thus, in the first place, high-quality reception need not be disturbed by the introduction of the asymmetric-sideband system, because high-quality reception can in any case only take place when the field from the wanted station is very strong and when therefore sideband splash is not heard and when therefore the receiver may be tuned symmetrically. The symmetrically tuned receiver does not introduce further distortion due to asymmetrical treatment of sidebands and that is why, in answer to Mr. Kirke, it is necessary to take so much care in designing the transmitter filters, since they alone determine distortion in conditions for high-fidelity reception.

But the point made by several speakers is that the receiver must be tuned asymmetrically if it is to benefit from the introduction of asymmetric transmission in eliminating sideband-splash interference, and it must, in being tuned asymmetrically, introduce more distortion than is present in transmission.

The question is then: " Is it worth while introducing more distortion for the sake of eliminating interference ? " This question is resolved by knowing how much extra distortion will be introduced, or more exactly by knowing what proportion of the total distortion existing in typical receivers will be added by asymmetric tuning and asymmetric transmission. In the latter question we are involved with Dr. Hughes's point that, bad as receivers are, we should encourage improvement rather than, as he implies, discourage it by introducing more distortion.

Taking these points in order of their statement, I think that, provided it can be proved that the extra distortion, in spite of asymmetric tuning where sideband splash is otherwise inevitable, is negligible, then the asymmetric system should be used.

Mr. Voigt has brought out a point which I think is highly relevant in asking if in fact one must always refer to " added distortions." In any broadcasting system we must presuppose several sources of harmonic distortion. Is it conceivable that the phases of all these distortions will be coincident? Surely not. Indeed, experiments prove that the amplitudes of harmonics seldom add arithmetically. I have attempted at different times to prove experimentally the findings of my analysis, and seldom have I been able on a first experiment to get as much harmonic as would be indicated by theory to exist, given certain measured degrees of asymmetry between counterpart sideband components. Only after a rigid examination of the rest of the system to eliminate all sources of harmonic distortion have the quantitative results agreed with theory, and, as said above, the sum of a number of harmonics is a vector not an arithmetic addition.

Mr. Bailey observes, and I see no reason to deny his contention, that phase asymmetry in typical receiver filters may introduce vastly more distortion than is spoken of as a part of asymmetric-sideband technique. But I have lately made experiments which prove that the addition, in the chain of filters between modulation output and detector input, of two filters consisting each of two tuned circuits coupled through the mutual inductance between their inductance elements does little to change the $M_A$ versus $\Delta f$ characteristic, showing that the extra phase distortion is very small.

The proper adjustment for the receiver filter is that it shall be substantially flat over a range of frequencies $f_0 - \Delta f_\infty$ to $f_0 + \Delta f'$, where $\Delta f'$ is the highest modulation frequency fully represented.

If this adjustment is made there is a moderate degree of phase-characteristic symmetry, and that is probably why the extra distortion introduced by a receiver filter so properly adjusted is slight.

Dr. Sandeman and, to a degree, Mr. Kirke miss this point when the former argues that every receiver will not only introduce extra harmonic distortion but extra demodulation as well, making it necessary to apply demodulation correction to every receiver or to use the same type of receiver " everywhere." If the receiver filter is flat over the range $f_0 - \Delta f_\infty$ and only introduces an attenuation, which tends to eliminate a sideband component already substantially removed in the transmitter filter, at frequencies lower than $f_0 - \Delta f_\infty$, no extra correction is needed in these conditions since the full 6-db. correction required has already been supplied in the transmitter.

My own feeling is, that, seeing that there is a good deal of distortion present in transmission in any case, believing that distortions do not necessarily add but often cancel, and knowing that there is a large amount of distortion existing in receivers in any case, the extra distortions, due to asymmetric tuning, would pass unnoticed, whereas the elimination of interference would not; the former, even if present in a small degree, is fully compensated for by the absence of the latter in large degree.

We come now, however, to a point implicit in a statement made by Mr. Bishop and clearly set out by Dr. Hughes. This is the question as to whether we should not leave things alone and attempt rather to encourage the production of better receivers than to recognize distortion as inevitable and to introduce more than is absolutely necessary in transmission. The shock administered to Mr. Bishop by my statement " that it is hardly the business of transmission authorities to solve reception problems introduced by changes in transmission technique " rebounded upon its author when I realized that this was a quotation outside its full context and Mr. Bishop failed to state that the sentence he quoted was completed by saying " provided they can be satisfied that it is not impossible or impracticable to design circuits to make full use of the changes." I would remind Mr. Bishop that he was as impatient as I was when permission to proceed with the Regional Scheme was delayed because it was alleged that it was impossible for a typical receiver to select between two transmissions of equal strength having an average of 100 kc. carrier-difference frequency. My statement, when taken in its full context, seems justifiable unless all of what used to be called " private enterprise " is to be managed by the State.

In spite of Mr. Kirke's revelation that for a moderate price one can buy receivers " which will receive well up into the region 8 000–9 000 cycles per sec." (i.e. into the region of sideband splash), I am inclined to agree with Dr. Hughes " that the general quality of the mass-produced receiver is deteriorating." Mr. Kirke's receiver would seem to be chiefly useful for local station reception, but I would add that it has to be a very good receiver even for that purpose if, as Mr. Sanderson so rightly observes, its top reproduction is not to be more irritating than pleasing. The average receiver wisely cuts off top to avoid sideband splash and, on local station reception, that horrible hard grating noise that is so prone to accompany a receiver with a " good " frequency characteristic.

Turning, however, to Dr. Hughes's contribution, one is forced to be drawn into seemingly irrelevant discussions of economics and sociology if one is to prove that, admirable as are his intentions to elevate public taste, the means to do so in the present state of affairs are non-existent. While it is no doubt deplorable that the standard of reproduction tolerated by the public is very low, it is equally sad that the cost of good reproduction, both in money and in bulk, is very high.

I agree that it is the duty of engineers to attempt to " pay the public the compliment of offering them something slightly better than the public thinks it wants," but it is no good doing this unless, as the salesman says, " the price is right." Reforming zeal seems often more pronounced as the problem to be solved is more removed from reality. The way to give the public good reproduction is first to create a public which demands it, and secondly to give them the money to pay for it. Deploring the fact that private enterprise produces a cheap article which makes a nasty noise but which is the only article the average public can buy, gets us no farther. Is it, moreover, realized that besides being costly the apparatus of good quality is extremely bulky? People tend to live in smaller and smaller rooms, while a wireless set of the high-fidelity type is relatively massive. The " bijou set " was sold in such vast numbers mainly because it was so " bijou," cost very little, and gave an adequate result. It supplied a public demand. Before, therefore, the engineer can do his duty he must face the facts, which are (1) that the public wants to be amused, (2) the public either does not find broadcasting sufficiently amusing to justify a large expenditure, or (3) has not much money to spend on it, or (4) even if an individual has a great deal of money he nevertheless does not want to lumber up his rooms with great boxes emitting even the most crisp top.

In sum, assuming the continuance of the present economic and political regime and assuming the continuance of wireless rather than wire broadcasting, distortion in the majority of receiving sets is inevitable. Desirable as it would be to reduce it, I cannot see how it can be done without State control, regimentation, and subsidy. Assuming, therefore, that distortion, in large amounts, is inevitable, I argue that the extra distortion, which may well be a less distortion, due to asymmetric transmission and reception will not be noticed. Which then is worse, a distortion which would not be noticed or an interference that is? Is quality alone defined in terms of harmonic distortion? Does not the ridding of interference constitute a gain in quality? Is not the possibility of extending the frequency range an increase of service?

Nor is this all. The asymmetric principle, according to my proposals, is the first step in an evolution towards the commonly desired end of carrier and single-sideband working. I cannot accept the " reasons " which led Dr. Sandeman to deny the relevance of my proposals to treat the asymmetric system as a step in an evolutionary development. Dr. Sandeman would prefer to wait until the progress of receiving-set design . . . " has further reduced costs to the point where the community as a whole can afford straight single-sideband working." Dr. Sandeman would be more convincing if he even indicated what this evolution should be, how receiver manufacturers, interested only in making money, are going to tackle the problem of producing receivers for " straight " single-sideband working, and what incentive there is to produce single-sideband receivers for double-sideband transmission.

I greatly appreciate Mr. Voigt's contribution to the discussion, which reinforces the ideas I have endeavoured to set out. The experiments he made on the sound spectrum analysis are interesting in that they confirm the general usefulness of the curve of Fig. 14.

Turning, at the conclusion of this reply, to the more technical points, I agree with Dr. Sandeman that it would be possible to use a multi-section filter in conjunction with phase-compensation networks, but would it be possible to produce an apparatus having the same performance as the one described in my paper for the same cost? It would seem to me that a merit of my system lies in its simplicity and therefore small cost. I agree with Mr. Bailey that *m* derived sections could be used, particularly in the band-elimination filter, in order to continue the full attenuation over the outer parts of the cut-away sideband, but it is perhaps proved by the spectrum-analysis diagrams that the present performance could hardly be bettered in the generality of cases without introducing either more distortion or more spectrum overlap.

I am grateful to Mr. Bailey for showing how my model, designed to show the vector analysis of the asymmetrical treatment of sideband components, could be adapted to illustrate frequency modulation—the point had escaped me.

Bad distortion was, as Mr. Bailey observes, present during the demonstration given after the meeting, a fault (later discovered) having developed in the receiver common to both transmissions. I am sorry that because I adopted the commonly-employed convention of setting out the definition of the symbols in an introductory list, rather than defining them each time they were mentioned, Mr. Kirke found more difficulty in understanding the paper than is implicit in the mastery of the subject. Mr. Kirke should sympathize with my difficulties, however, when he appreciates that several critics said the paper was far too long while he, on the other hand, would have it longer.

In conclusion it is worth repeating that the asymmetric system was developed primarily for use in connection with carrier-current wire broadcasting, wherein it has proved both practicable and useful.

As Mr. Sanderson so relevantly observes, wire broadcasting must obviously one day replace much of wireless broadcasting, and maybe it would be better to concentrate on developing this system which, for a cheap price, would give good-quality reproduction to the public, than spend more time and money on " bolstering an inefficiency " by applying asymmetric-sideband technique to space broadcasting.

# COEXISTENT THERMAL AND THERMIONIC FLUCTUATIONS IN COMPLEX NETWORKS[*]

By F. C. WILLIAMS, M.Sc., D.Phil., Graduate.

(*Paper first received* 2*nd December*, 1937, *and in final form* 16*th February*, 1938.)

## SUMMARY

This paper is concerned with the fluctuations generated in networks with non-uniform temperature distribution. Experiments are described which verify a formula relating to thermal fluctuations deduced in an earlier paper. The simplified representation adopted in that paper is fully substantiated: the fluctuations are shown to be independent of the temperature of reactive circuit elements.

Coexistent thermal and thermionic fluctuations are then discussed. They are assumed to be mutually independent, and experiments are described which verify this hypothesis. A representation of thermionic fluctuations is proposed analogous to that adopted for thermal fluctuations, and a general formula applicable to certain classes of networks containing thermionic valves is deduced.

This formula is applied in a concluding section to a wide-band photocell amplifier. Various practical recommendations are made which yield improved signal/noise ratios.

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## (1) INTRODUCTION

In a recent paper[†] the author deduced a general formula for the thermal fluctuations[‡] appearing between any two points in a network of linear conductors. The analysis was based on the hypothesis that the fluctuations can be regarded as being produced by elementary fluctuation generators supposed acting in series with each element of resistance. Thus in series with the typical element of resistance $r_x$ there was supposed to exist a generator delivering a mean-square voltage within any frequency band $df$ of value $\overline{v_x^2} = 4r_xkT_xdf$.

It was then shown that the mean-square voltage, within the frequency band $df$, appearing between any two points B and B′ of the network was

$$\overline{v^2} = Z_{BB'}^2 4kdf \sum \frac{r_xT_x}{Z_{Bx}^2} \quad . \quad . \quad . \quad . \quad (1)$$

where

$Z_{Bx}$ = modulus of the transfer impedance from the hypothetical generator in series with the typical resistance, $r_x$, to a short-circuit joining the points B, B′. That is $1/Z_{Bx}$ = current in the link joining BB′ per unit e.m.f. in series with $r_x$.
$Z_{BB'}$ = modulus of the network impedance from B to B′.
$k$ = Boltzmann's constant.
$T_x$ = absolute temperature of $r_x$.

The impedance values are relevant to the mid-frequency of the small band $df$.

With uniform temperature distribution this was shown to reduce to

$$\overline{v^2} = 4kTrdf \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $r$ is the real part of the impedance from B to B′, a result in accordance with Nyquist's earlier analysis.[†]

Equation (2) was verified experimentally in the earlier paper: one purpose of the present paper is to verify the general equation (1).

Llewellyn[‡] has used the equation

$$\overline{v^2} = \left(\frac{R_1R_2}{R_1 + R_2}\right)^2 4kdf\left(\frac{T_1}{R_1} + \frac{T_2}{R_2}\right) \quad . \quad . \quad . (3)$$

relevant to a resistance $R_1$ at temperature $T_1$ in parallel with a resistance $R_2$ at temperature $T_2$. This is a particular example of the general equation (1). Moullin and Ellis[§] have described an experimental check of this equation: their results were not in perfect agreement with it, but they ascribed the discrepancy to imperfections of their experimental method and did not then consider that it merited closer investigation.

The present paper also extends equation (1) to cover certain classes of networks generating thermionic as well as thermal fluctuations.

This extension has been made in order to facilitate the prediction of the total noise output of amplifiers: the well-known simple formulae for shot and thermal effects permit such prediction only when one or other of the effects is dominant, for the mechanism of their addition in complex networks has not been studied. The non-uniform temperature distribution is included partly for the sake of completeness, but also because variation of the temperature of a resistance modifies its thermal fluctuation without modifying the circuit parameters, thus facilitating the experimental determination of the contribution of each circuit element to the total fluctuation.

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[*] The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of The Institution not later than one month after publication of the paper to which they relate.

[†] See Bibliography, (1). (B and B′ replace A and A′ in the paper referred to.)

[‡] The term "fluctuation" is here used to refer to the spontaneous oscillations of potential which appear in electrical circuits due to "Brownian motion," "shot effect," etc.

[†] See Bibliography, (2).

[‡] *Ibid.*, (4).

[§] *Ibid.*, (3).

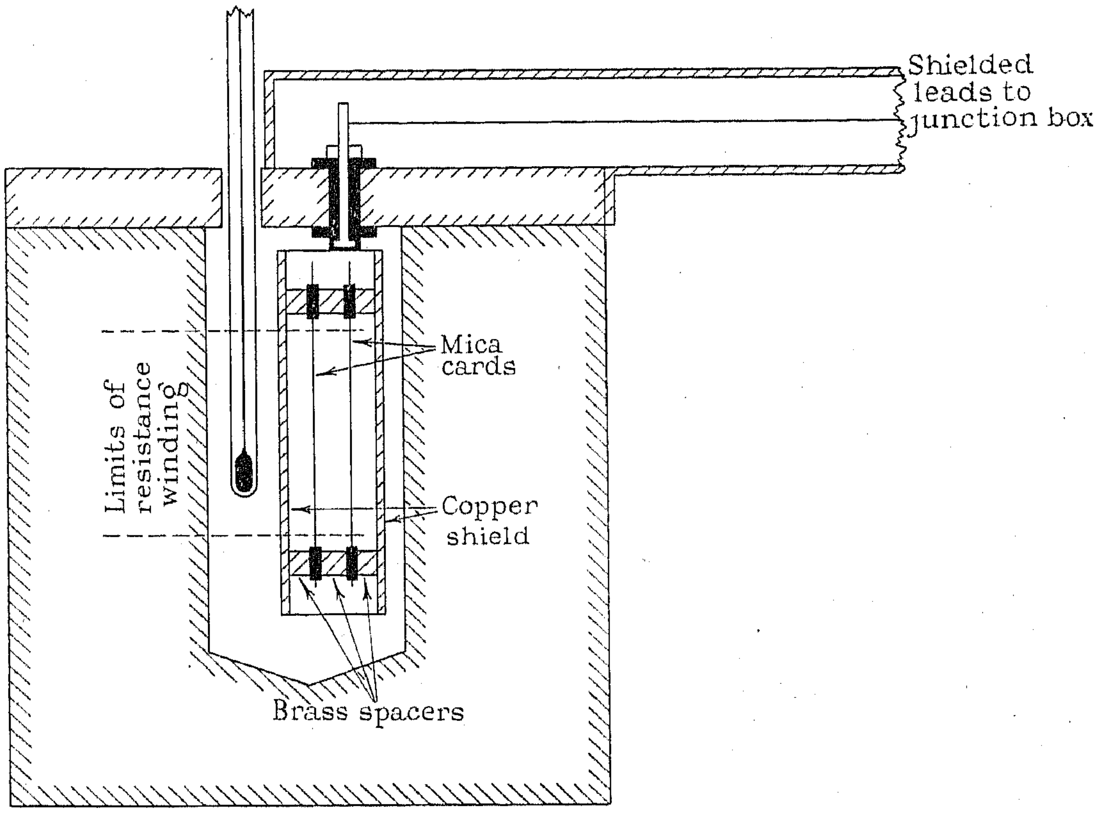

**Fig. 1.**—Sketch of box containing heated resistance. Mica washers shown black.

It may appear that equation (1) does not strictly relate to circuits with "non-uniform temperature distribution," but only to circuits having finite elements at different temperatures. However, the elements of resistance such as $r_x$ can be taken very small and the condition of continuous temperature variation approached as closely as desired.

An example of the use of the formulae is given in a concluding section, where they are used to determine the noise/signal ratio in a wide-band photocell amplifier.

### (2) EXPERIMENTAL APPARATUS

An 8-stage amplifier supplying a thermojunction was used; its design was straightforward and does not merit detailed description. The response characteristic was adjustable by the insertion of suitable filter circuits. The magnification could be varied by adjustment of screen grid potentials: at its maximum, full-scale deflection (50 cm.) corresponded to an input voltage of about $0 \cdot 1\ \mu V$ (r.m.s.). Fluctuations generated in the amplifier itself were equivalent to the thermal-agitation voltage generated in a resistance of $2 \cdot 3\ k\Omega$ connected to the input terminals.

A junction box of $\frac{1}{8}$ in. brass sheet with a flush-fitting lid contained that portion of the apparatus which was to be kept at room temperature. The output from a calibrated attenuator was fed to this box, as also were the input leads from the amplifier, and the shielded leads from the box containing the circuit elements to be heated.

The box containing the heated elements is shown in Fig. 1. It consisted of a 6-in. cube of cast iron having a 2 in. dia. hole drilled as shown; the cast-iron lid was $\frac{1}{2}$ in. thick and the leads passed out through mica bushes. This massive construction was adopted in order to obtain a substantially uniform temperature inside the box, so that the measured temperature should be that of the resistance. The box was heated externally with bunsen burners, and was allowed to attain a steady temperature before each set of observations was taken.

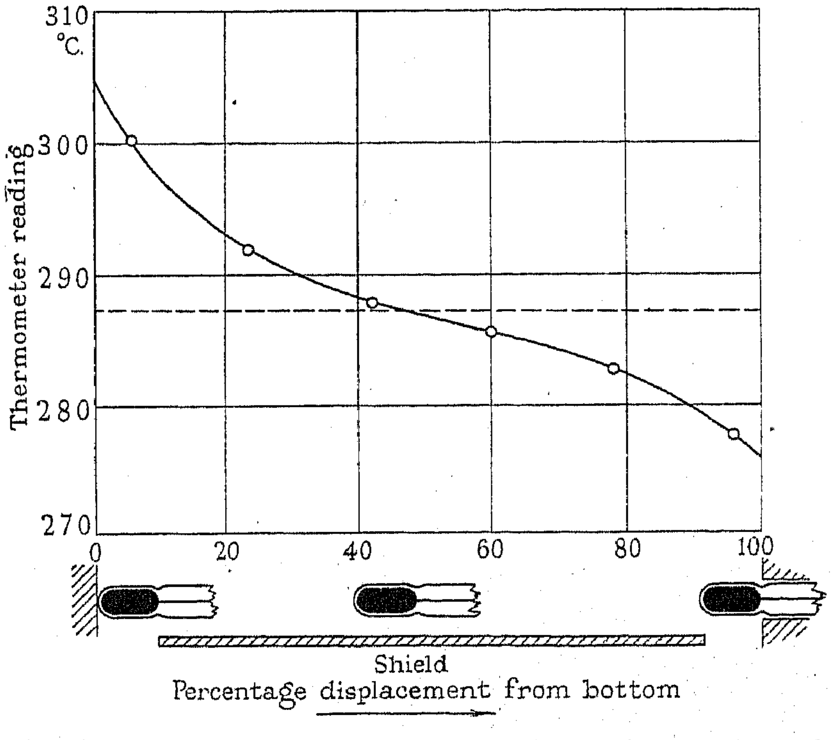

**Fig. 2.**—Vertical temperature distribution inside the heated container shown in Fig. 1.

A test of temperature distribution is shown in Fig. 2, where recorded temperature is plotted as a function of the vertical immersion of the thermometer: it can be seen that the whole range of temperature is only 29 deg. C.; that is, little more than 10 % of the excess of the mean temperature above the room temperature. It is presumed that this figure, 10 % of the excess, is sensibly independent of the excess, since the non-uniformity is due to uneven heating of the box, which was always heated from below. Thus with a maximum temperature of 500° C. the range of temperature probably had a maximum value of about 7 % of the absolute temperature (50° K. in 773° K.). The resistances to be heated were wound in two sections on thin mica cards. The wire was 0·001 in. dia. nichrome. The two cards were enclosed in a copper shield and spaced from each other and from the shield by transverse lengths of $\frac{1}{4}$ in. square brass rod. The spacers, together with the shield, gave a totally enclosed resistance (see Fig. 1). The temperature inside the shield must therefore have been very close to the average temperature over its outside surface. The positions of the shield and thermometer are shown in Fig. 1; the correct immersion for the thermometer was deduced from Fig. 2. On the basis of Fig. 2 and the foregoing discussion it is thought that errors due to uneven temperature distribution cannot have exceeded 1 % in absolute measure.

The thermometer was of quartz and was calibrated with boiling water, naphthalene, benzophenone, and sulphur. The calibration points were all within 4 % of the marked temperature.

### (3) EXPERIMENTAL PROCEDURE

The input impedance of the amplifier was of the order of 0·25 MΩ, and was believed to be largely capacitative. The effect of this on the fluctuations generated by the test apparatus was eliminated by the technique described below. In Fig. 3, $\mathbf{Z} = R + jX$ represents the complex test apparatus, while $jX'$ represents the reactive input impedance of the amplifier and its associated leads.

The sinusoidal calibrating voltage $V$ of frequency $f$ was introduced at C.

It follows from equation (1) that the mean-square fluctuation voltage between B and B′ in the narrow frequency range $df$ situated about $f$ is

$$\overline{v^2} = \left|\frac{\mathbf{Z}jX'}{\mathbf{Z} + jX'}\right|^2 4kdf \sum \frac{r_x T_x}{Z_{Bx}^2}$$

The mean-square voltage from B to B′ due to $V$ is

$$\overline{v_c^2} = \left|\frac{jX'}{\mathbf{Z} + jX'}\right|^2 V^2$$

The ratio of these expressions is

$$\frac{\overline{v^2}}{\overline{v_c^2}} = \frac{\left|\dfrac{\mathbf{Z}jX'}{\mathbf{Z} + jX'}\right|^2}{\left|\dfrac{jX'}{\mathbf{Z} + jX'}\right|^2} \cdot \frac{2kdf}{V^2} \sum \frac{r_x T_x}{Z_{Bx}^2}$$

This ratio is independent of $X'$: thus the reactive component of input impedance does not affect the measurement. A comparison of the deflection due to $V$ and that due to fluctuations generated in $\mathbf{Z}$ and $jX'$ in parallel thus measures the fluctuations which would appear across $\mathbf{Z}$ if it were isolated. This result holds only if the amplifier input impedance is purely reactive; it can be shown that in the general case the "series" method of comparison is preferable to that in which first $\mathbf{Z}$ and then $V$ are connected to the amplifier, the ratio of the two deflections being taken as a measure of the fluctuation generated in $\mathbf{Z}$ if it were isolated.

The amplifier was not calibrated in absolute measure. The fluctuations generated in the test apparatus were compared with those generated in a metallic resistance of known value. The observed fluctuations were expressed in terms of the resistance which generated a fluctuation equal to that generated in the test apparatus.

The following deflections were noted:—

$\theta_1$ . . . with the amplifier input short-circuited. This was due to amplifier noise.
$\theta_2$ . . . with the test apparatus connected to the amplifier.
$\theta_3$ . . . as $\theta_2$ but with a sinusoidal calibrating voltage $V$ in series with the test apparatus.
$\theta_1$ . . . repeated.
$\theta_4$ . . . with a metallic resistance $R'$ connected to the amplifier.
$\theta_5$ . . . as $\theta_4$ but with $V$ in series with $R'$.

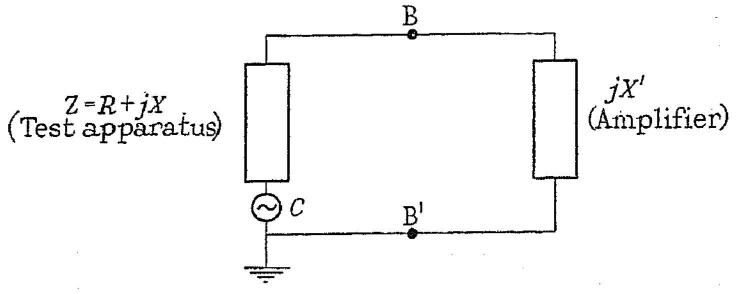

Fig. 3

The deflections were not steady, but showed a random variation of about 5%. Accordingly, 6 sets of readings $\theta_1 \ldots \theta_5$ were taken for each experimental point, values of $(\theta_2 - \theta_1)/(\theta_3 - \theta_2)$ and $(\theta_5 - \theta_4)/(\theta_4 - \theta_1)$ being derived from each set.

Any individual value of either ratio which differed more than 5 % from the mean was rejected as spurious. Such rejections averaged about 1 in 10; the bulk of the remainder were within 2 % of the mean.

By taking measurements in this order, the effect of the inevitable slow decrease of amplification with time was eliminated, for each set of measurements, $\theta_1$ to $\theta_3$ and $\theta_1$ to $\theta_5$, was taken in less than 2 minutes.

It follows from the above discussion that the measured value of the fluctuation voltage generated across the test apparatus, when isolated, expressed in equivalent kilohms at the temperature of $R'$, is

$$R_e = \frac{\theta_2 - \theta_1}{\theta_3 - \theta_2} \frac{\theta_5 - \theta_4}{\theta_4 - \theta_1} \mathrm{R}' \,\mathrm{k}\Omega \ (R' \text{ in kilohms})$$

The final value of the observed fluctuations was deduced from this equation, using the mean values of $(\theta_2 - \theta_1)/(\theta_3 - \theta_2)$ and $(\theta_5 - \theta_4)/(\theta_4 - \theta_1)$.

As a test of the method and the apparatus, several new resistances were checked against one which had been shown* to yield a pure thermal fluctuation. Those resistances which were to be heated in subsequent experiments were tested in the special container;, the remainder were placed in the junction box for test.

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* See Bibliography, (1) (Section 4).

Fig. 4 shows the results of the test. Here the equivalent resistance in kilohms determined by fluctuation measurements is plotted as a function of the d.c. resistance in kilohms. A straight line having a slope of 45° is drawn on the figure. All points lie close to the line and show a mean divergence of 0·2 % only.

It is to be noted that any appreciable grid current in the first amplifier stage would yield a parabolic curve. The experimental results show no such tendency over the range covered, and it is presumed that grid current was neglibible, i.e. less than, say, 0·05 μA.

## (4) RESULTS WITH METALLIC RESISTANCES IN PARALLEL

In this experiment the test impedance **Z** consisted of two sensibly pure resistances. The temperature of one resistance ($R_2$) could be adjusted to any value between 20° C. and 470° C.: the other ($R_1$) remained at room temperature.

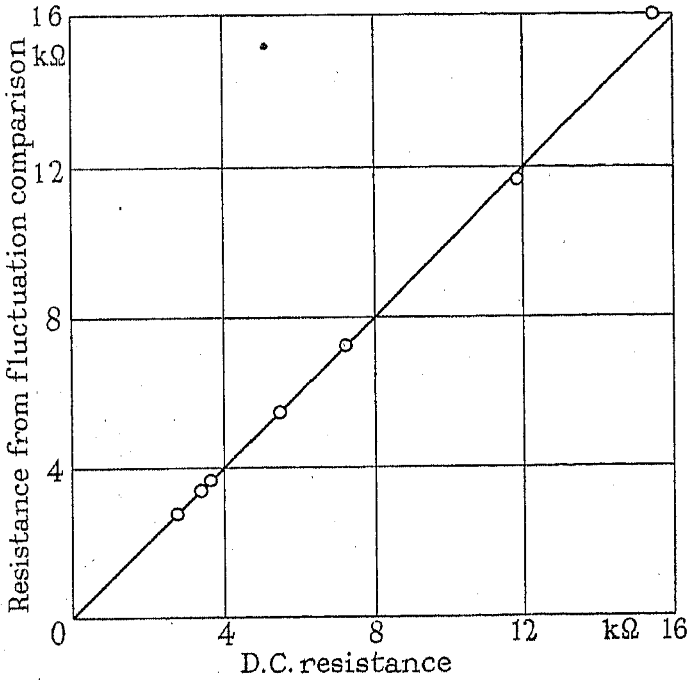

**Fig. 4.**—Comparison of resistance values obtained from fluctuation measurements with the d.c. values.

It follows from (3) that the relevant fluctuation in the range *df* is

$$\overline{v^2} = 4R_1kT_1df\left(\frac{R_2}{R_1+R_2}\right)^2\left(1+\frac{T_2}{T_1}\cdot\frac{R_1}{R_2}\right)^{*}$$

Expressing $\overline{v^2}$ in terms of the equivalent metallic resistance $R_e$ at the temperature $T_1$ required to yield an equal fluctuation, we have

$$R_e = R_1\left(\frac{R_2}{R_1+R_2}\right)^2\left(1+\frac{T_2}{T_1}\cdot\frac{R_1}{R_2}\right) \quad . \quad . \quad (4)$$

This expression is linear in $T_2$ if $T_1$ and $R_2$ remain constant. The value of $R_2$ was found to increase 2 % at 470° C.: in presenting the results its value has been taken as independent of temperature and equal to its measured resistance at 220° C. The proximity of the

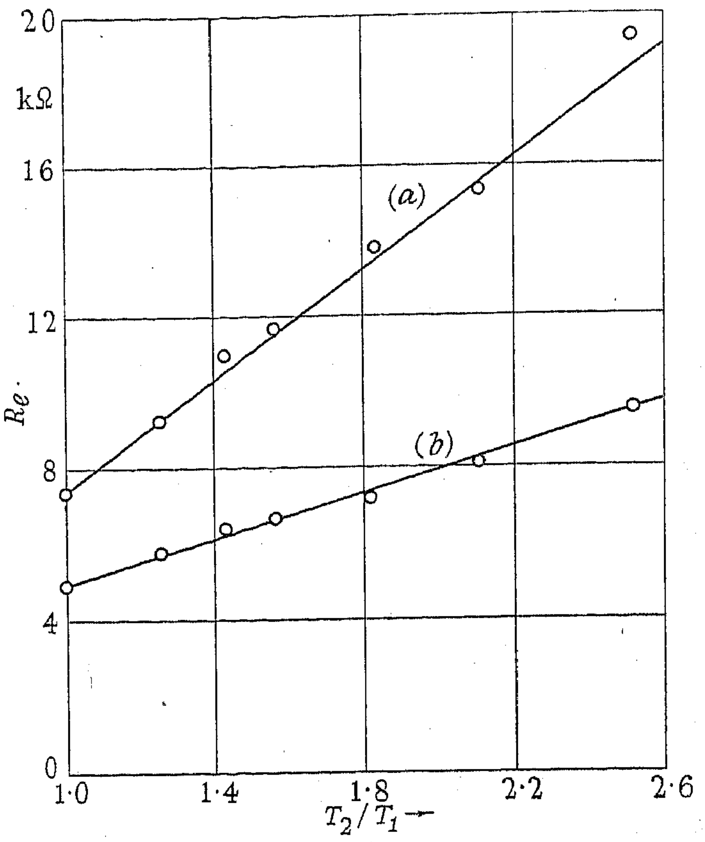

**Fig. 5.**—Fluctuations across two resistances in parallel as a function of the ratio of the temperatures of the resistances. Amplifier response limited to a 10 % band about a frequency of 8 kc.

Full lines show predicted values.
Plotted points show experimental results.

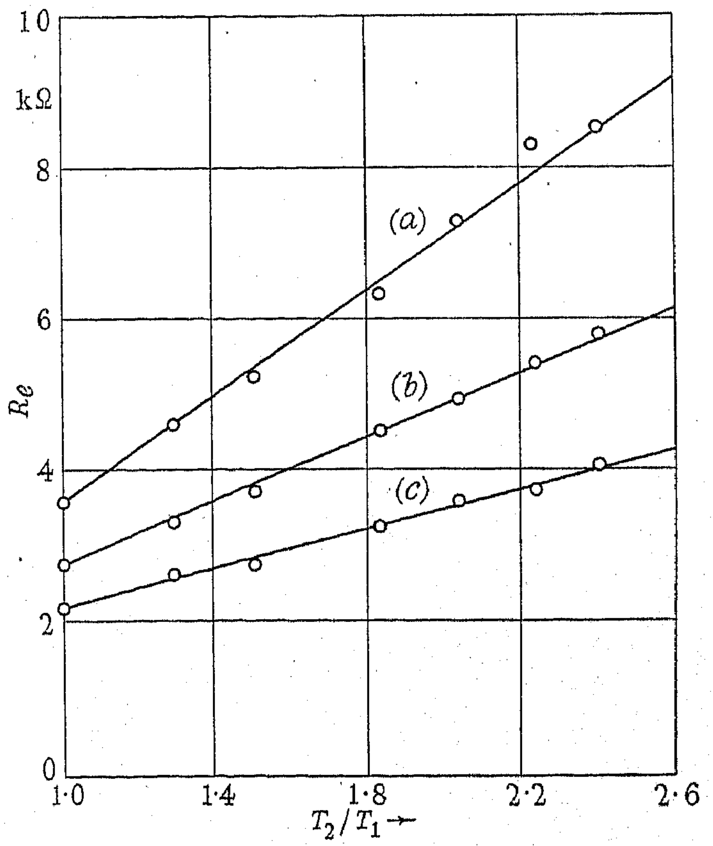

**Fig. 6.**—As Fig. 5, but with different resistances and with the amplifier response limited to a 5 % band about a frequency of 5 kc.

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* $T_2$ and $T_1$ are the absolute temperatures of $R_2$ and $R_1$, respectively.

heated apparatus caused $T_1$ to vary over a range of 5 %: this variation was taken into account in the ratio $T_2/T_1$.

The results shown in Fig. 5 were obtained with the amplifier passing a 10 % band* centred on a frequency of 8 kc. Line (*a*) on the figure is drawn according to equation (4), with $R_1 = \infty$ and $R_2 = 7\cdot41$ k$\Omega$: line (*b*) refers to $R_1 = 13\cdot55$ k$\Omega$ and $R_2 = 7\cdot41$ k$\Omega$. The observation points lie close to these lines: thus for line (*a*) the maximum discrepancy is + 4 % and the mean is + 0·4 %: for line (*b*) the figures are ± 3 % and + 0·02 % respectively.

Fig. 6 records the results of a similar test, but the amplifier now responded to a 5 % band centred on a frequency of 5 kc. Line (*a*) relates to $R_1 = \infty$ and $R_2 = 3\cdot54$ k$\Omega$, line (*b*) relates to $R_1 = 11\cdot9$ k$\Omega$ and $R_2 = 3\cdot54$ k$\Omega$, whilst for curve (*c*) $R_1 = 5\cdot485$ k$\Omega$ and $R_2 = 3\cdot54$ k$\Omega$. The maximum and mean discrepancies for lines (*a*), (*b*), and (*c*) are + 4 % and − 0·2 %, − 3·5 % and − 0·6 %, and + 3 % and − 0·3 % respectively.

These experiments appear to establish equation (3) beyond doubt, and verify the view expressed by Moullin and Ellis that the discrepancy they found was due to experimental error.

## (5) THERMAL FLUCTUATIONS IN A RESONANT CIRCUIT WITH NON-UNIFORM TEMPERATURE

In the earlier paper† a resonant circuit such as is shown in Fig. 7 was tested under conditions of uniform temperature distribution and shown to yield results supporting equation (1). With the two resistances at different temperatures, equation (1) gives

$$\overline{v^2} = Z^2_{BB'}4kdf\left[\frac{R_1T_1}{R_1^2 + L^2\omega^2} + \frac{R_2T_2}{R_2 + 1/(C^2\omega^2)}\right]$$

Hence, if $R_1$ and $R_2$ are small compared with $L\omega$ and $1/(C\omega)$, we have, by the process previously adopted,

$$R_e = Z^2_{BB'}R_1\left[\frac{1}{L^2\omega^2} + \frac{R_2T_2}{R_1T_1}\cdot C^2\omega^2\right]$$

$$= Z^2_{BB'}\frac{R_1}{L^2\omega^2}\left[1 + \frac{R_2T_2}{R_1T_1}\cdot\left(\frac{\omega}{\omega_0}\right)^4\right] \quad . \quad . \quad (5)$$

where $\omega_0 = 1/\sqrt{(LC)}$.

The curves of Fig. 8 show experimental results obtained with a circuit with $C = 0\cdot001\,\mu$F (nominal) and $L = 183$ mH. $R_2$ was heated; it had a value of 1 975 $\Omega$, whilst $R_1$ was 1 860 $\Omega$. Experiment at the resonant frequency showed that residual losses in the coil were equivalent to a series resistance of 390 $\Omega$: this was regarded as independent of frequency. The total value of $R_1$ was thus 2·250 k$\Omega$. For curve (*a*) $\omega/\omega_0$ was unity, for curve (*b*) $\omega/\omega_0$ was 1·15, and for curve (*c*) $\omega/\omega_0$ was 0·895.

All points lie close to straight lines in accordance with (5).

Values of $Z_{BB'}R_1/(L^2\omega^2)$ were deduced from the observations with $T_2/T_1 = 1$, a condition in which the expression has already been checked.* The dotted lines were then calculated from equation (5). They lie close to the experimental lines, the maximum divergence being 5 %, but they are in all cases steeper by about 10 %.

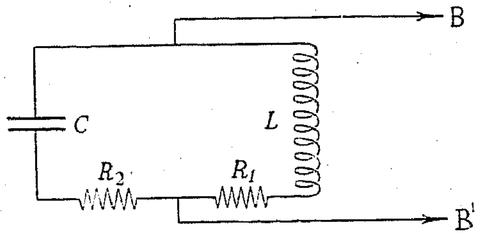

**Fig. 7**

Fig. 9 records the results of a similar test, but with the resistance (now termed $R_2$) in series with $L$ heated instead of that in series with $C$ (now termed $R_1$).† The coil remained at room temperature and thus its inherent resistance $R_2'$ (= 390 $\Omega$) remained at $T_1$. With these conditions it follows that

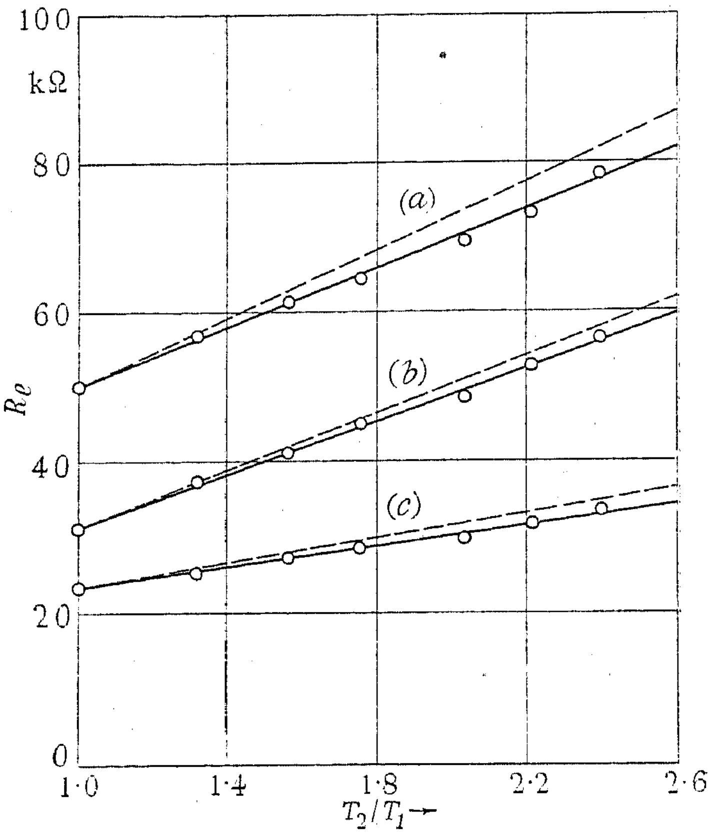

**Fig. 8.**—Fluctuations in a resonant circuit as a function of the temperatures of the resistance elements.

Curve (*a*).—$\omega/\omega_0 = 1$
Curve (*b*).—$\omega/\omega_0 = 1\cdot15$ $\Big\}\ \omega_0 = 2\pi \times 10\,450$
Curve (*c*).—$\omega/\omega_0 = 0\cdot895$
The amplifier band width was about 3 %.
The dotted lines show the expectation.

$$R_e = Z^2_{BB'}R_1C^2\omega^2\left[1 + \frac{R_2'}{R_1}\left(\frac{\omega_0}{\omega}\right)^4 + \frac{R_2T_2}{R_1T_1}\left(\frac{\omega_0}{\omega}\right)^4\right]df$$

The lines in Fig. 9 have been drawn according to this expression, absolute values again being based on the observations with $T_2/T_1 = 1$. The experimental points follow the lines very closely.

Finally the condenser alone was heated, the remainder of the circuit (i.e. $L_1$, $R_1$, $R_2$, and $R_2'$) remaining at room temperature. The amplifier response curve was centred on the resonant frequency of the circuit and had a much greater band-width. Accordingly, the amplifier was sensitive to the whole fluctuation output of the test circuit. The results, recorded in Fig. 10, show conclusively that the fluctuation output is independent of the temperature of $C$, and hence that the resistance components alone are the seat and origin of the mechanism producing the thermal agitation fluctuation.

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* The "band" is here defined as the percentage frequency interval enclosed between ordinates of the response curve having values $1/\sqrt{2}$ of the maximum ordinate: in comparisons of this kind the exact shape of the response curve is immaterial.

† See Bibliography, (1).

* See Bibliography, (1) (Section 5).

† Thus the definitions of $R_1$ and $R_2$ implicit in Fig. 7 are interchanged: this has been done so that $T_2$ remains the adjustable temperature.

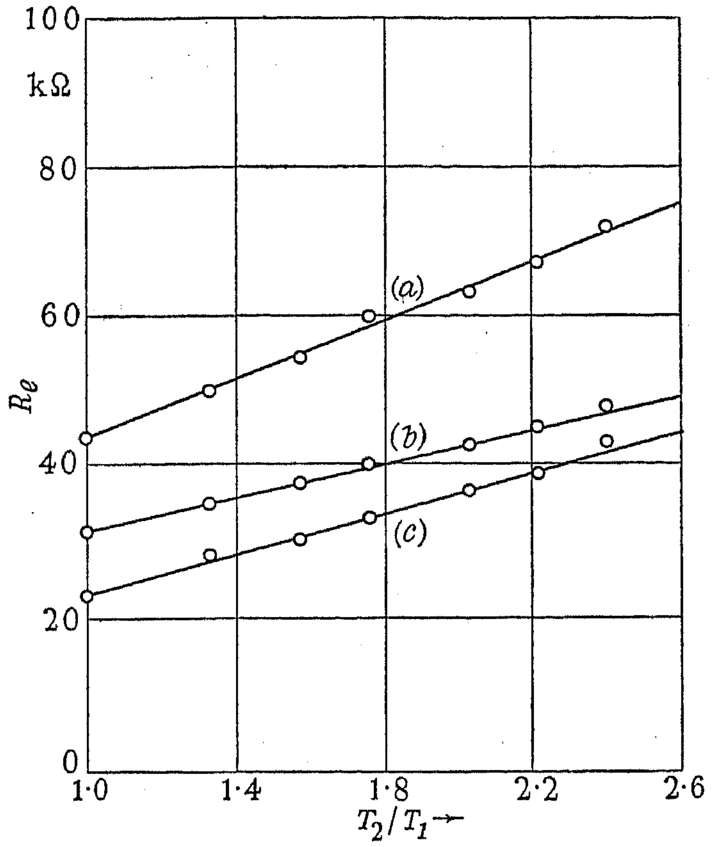

**Fig. 9.**—As Fig. 8, but with $R_1$ and $R_2$ interchanged (see Fig. 7).

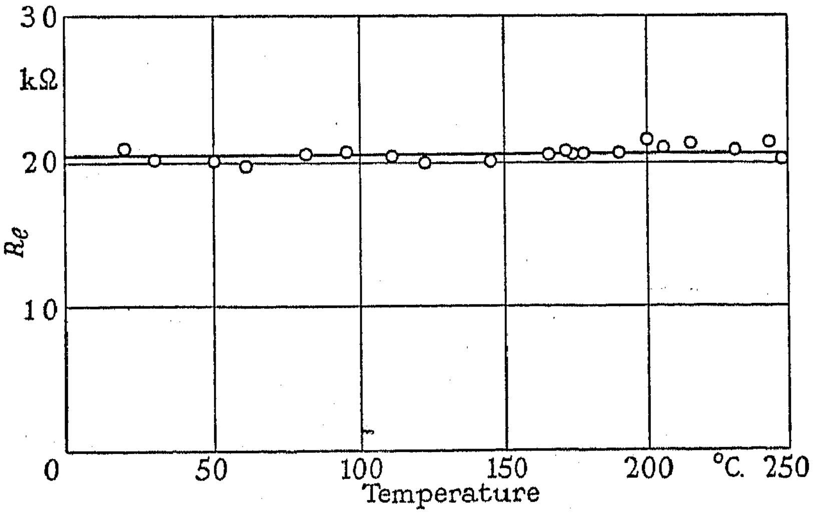

**Fig. 10.**—Fluctuations in a resonant circuit as a function of the temperature of the condenser.

This result is of much interest when it is remembered that the total fluctuation voltage (i.e. in the frequency range 0 to ∞) of a circuit at uniform temperature is expressed by the equation*

$$\overline{v^2} = \frac{kT}{C}$$

and is independent of $R$. It follows also that this expression is valid with non-uniform temperature, provided $T$ is the temperature of *all* resistive elements. [Note:—It is now clear that the mere form of equation (2) is insufficient evidence that the fluctuation can be ascribed to $R$ alone, a fact which this last experiment has finally established.]

## (6) COEXISTENT THERMAL AND THERMIONIC FLUCTUATIONS

In most analyses of circuit fluctuations the preponderant contribution to the fluctuation can be ascribed to either thermal or thermionic effects: when both are present to a comparable degree the evaluation of the resulting fluctuation presents considerable difficulty. Thus, referring to Fig. 11, there will exist across $R$ a thermal fluctuation due to Brownian motion of the electrons and a further fluctuation due to the passage of the thermionic current. If the current is of suitable value [of the order of $2kT/(eR)$, where $e$ is the electron charge] the magnitudes are comparable. Such low currents must be used if recent advances in the theory of the shot effect are to be investigated experimentally.*

If the current is temperature-limited the two effects are evidently independent, and therefore additive, hence we may write

$$\overline{v^2} = R^2\left(\frac{4kT}{R} + 2Ie\right)df\dagger \quad . \quad . \quad . \quad . \quad (6)$$

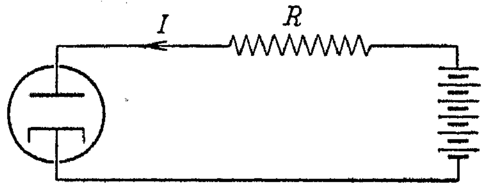

**Fig. 11**

as the total fluctuation across $R$ in the range $df$, where $e$ is the electron charge and $I$ is the steady value of the temperature-limited current.

If the current $I$ is space-charge-limited the valve will exhibit a finite slope resistance $\rho$. Two difficulties are at once encountered. Firstly, fluctuations of anode potential due to thermal agitation in $R$ may dictate the release of electrons from the potential barrier inside the valve, thus vitiating the condition of mutual independence of the fluctuations. Secondly, the mechanism of the addition of the effects is rather obscure. It is possible to argue that mutual dependence of the thermal and thermionic effects will be small since the fluctuations of anode potential are small. This paper will proceed on the assumption that the effects are completely independent, the suitability of the assumption being later verified by a rigorous experimental investigation. The mechanism of the addition has been briefly outlined‡ by the author in an earlier paper, the resulting expression being

$$\overline{v^2} = \left(\frac{R\rho}{R+\rho}\right)^2\left\{\frac{4kT}{R} + 2AIe\right\}df\S \quad . \quad . \quad (7)$$

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* See, for example, Bibliography (3).

* See Bibliography, (7), (8), and (9).

† The second term, $2IeR^2df$, measures the shot contribution.

‡ See Bibliography, (6), Section (3).

§ $A$ is an arbitrary constant fully defined in item (6) of the Bibliography.

This equation can usefully be interpreted with reference to the equivalent circuit of Fig. 11 shown in Fig. 12, in which the valve is represented by the resistance $\rho$.

Here the thermal fluctuation in $R$ is represented by a hypothetical generator as already described. An analogous generator is shown in series with $\rho$: this generator must yield an output $\overline{v_s^2}$ such that it represents the thermionic fluctuations generated in the valve. Comparison of (7) with (3), which relates to an identical circuit, shows that the relevant value of $\overline{v_s^2}$ is

$$\overline{v_s^2} = 2AIe\rho^2 df^* \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This representation of the source of thermionic fluctuations as concentrated in a hypothetical generator has even less physical significance than the similar representation adopted for thermal fluctuations: its adoption, however, permits of the formulation of general equations which simplify the computation of fluctuations in complicated circuits. Thus in any network containing thermionic valves which are replaceable by equivalent resistances (that is, valves operating in the so-called "linear" regime†), the fluctuation appearing between any two points B and B′ can be shown by the process used in deriving equation (1) to be

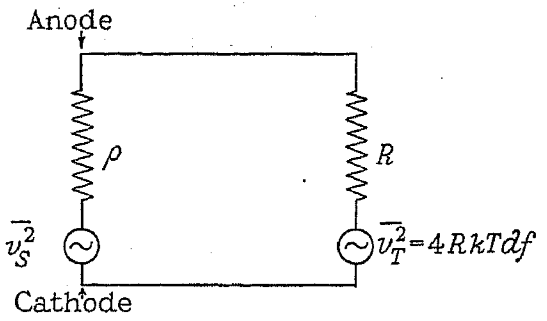

**Fig. 12**

$$\overline{v^2} = Z_{BB'}^2\left\{4k\sum\frac{R_xT_x}{Z_{Bx}^2} + 2e\sum\frac{A_yI_y}{Z_{By}^2}\rho_y{}^2\right\}df \quad . \quad (9)$$

where $A_y$, $I_y$, and $\rho_y$, are the values of $A$, $I$, and $\rho$ relevant to the typical valve denoted by the subscript $y$: the remainder of the notation is that of equation (1).

If any of the valves are triodes or multi-electrode valves normally represented as resistances in circuit calculations, there may exist networks in the grid circuits which generate fluctuations. In such cases the grid fluctuation must be separately calculated, multiplied by $\mu^2$ ($\mu$ = amplification factor of valve), and added to the fluctuation given by (8). This process follows the usual representation of grid voltages: it is valid only when the grid and anode fluctuations are mutually independent, and the analysis does not therefore apply to retroactive circuits.

The total output of the hypothetical generator in series with $\rho$ is then

$$\overline{v^2} = 2AIe\rho^2 df + \mu^2\overline{v_g^2}df$$

$$= (2AIe + g^2\overline{v_g^2})\rho^2 df \quad . \quad . \quad . \quad . \quad (10)$$

where $\overline{v_g^2}df$ is the grid fluctuation in the range $df$, and $g$ is the mutual conductance of the valve. The final representation is shown in Fig. 13. The modification of (9) to allow for $\overline{v^2}_g df$ is self-evident and will not be discussed.

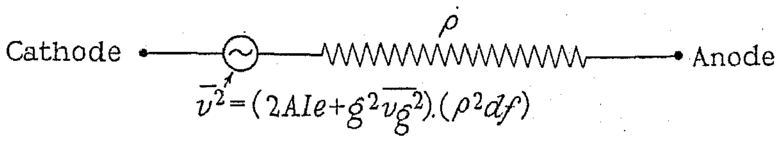

**Fig. 13**

It may be noted that $\rho$ is often sensibly infinite: no difficulty is experienced, however, provided $\rho$ is first treated as finite and only put equal to infinity when the expression has been cast into a suitable form.

## (7) EXPERIMENTAL INVESTIGATIONS OF CO-EXISTENT THERMAL AND THERMIONIC FLUCTUATIONS

The results of Section (6) depend entirely on the assumption of mutual independence of thermal and thermionic effects: the present Section describes experiments designed to test this hypothesis. The first experiments relate to a test of equations (6) and (7); the circuit used is shown in Fig. 14.

$R_2$ and $C$ are decoupling components, the impedance of $C$ being negligible compared with $R$ at all frequencies to which the amplifier is sensitive. The method of evaluating the fluctuation across $R$ in terms of that across the metallic resistance $R'$ was precisely that described in Section (3). All the apparatus except $R$ remained at $T_1$ (it was situated in the junction box); $R$ was heated in the special container.

It follows from (7) that the fluctuation across $R$ expressed as equivalent resistance at temperature $T_1$ is

$$R_e = R\left(\frac{\rho}{R+\rho}\right)^2\left\{\frac{T_2}{T_1} + \frac{2AIeR}{4kT_1}\right\}$$

$$= R\left(\frac{\rho}{R+\rho}\right)^2\left\{\frac{T_2}{T_1} + 20AIR\right\} \quad . \quad . \quad (11)$$

if $T_1$ = 290° K. In the experiments $T_1$ varied slightly: a correction—always less than 1 %—was added to the results to compensate for the observed variation.

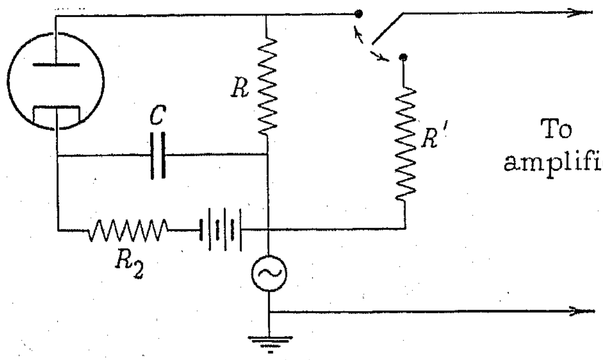

**Fig. 14**

Fig. 15 shows the results of the test. Curve (*a*) relates to a directly heated Osram D.E.H.410* valve passing a temperature-limited current of 16 $\mu$A: $R$ had a value of 3·54 k$\Omega$ and $\rho$ was sensibly infinite. The amplifier passed a 10 % band centred on 8 kc., flicker effect should therefore be negligible and $A$ may be taken as unity. Line ($a$) is plotted from (11); the plotted points are the experimental results and show a maximum discrepancy of $-2$ % with a mean discrepancy of 0·8 %. Curve ($b$) relates to the more important case of a space-charge-limited current. The valve was an indirectly heated Mazda V914 double diode, one anode only being used. It was chosen because it yielded a low anode current without resorting to a retarding field. The value of $[\rho/(R + \rho)]^2$ as determined by the direct method explained in an earlier paper* was 0·423. $R$ was again 3·54 kΩ. The value of $A$ was deduced from the observation at $T_2/T_1 = 1$; it was 0·665. Agreement is again good, the maximum error being $+2{\cdot}2$ % and the mean $+0{\cdot}7$ %.

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* This formula can also be deduced from Pearson's representation of thermionic fluctuations in terms of a resistance $\left(=\frac{1}{2}\cdot\frac{AI}{g^2}\cdot\frac{e}{kT}\right)$ connected in the grid circuit of a triode [see Bibliography, (5)].

† With non-linear operation fluctuations are no longer additive [see Bibliography, (10)].

* Oxide-coated filament.

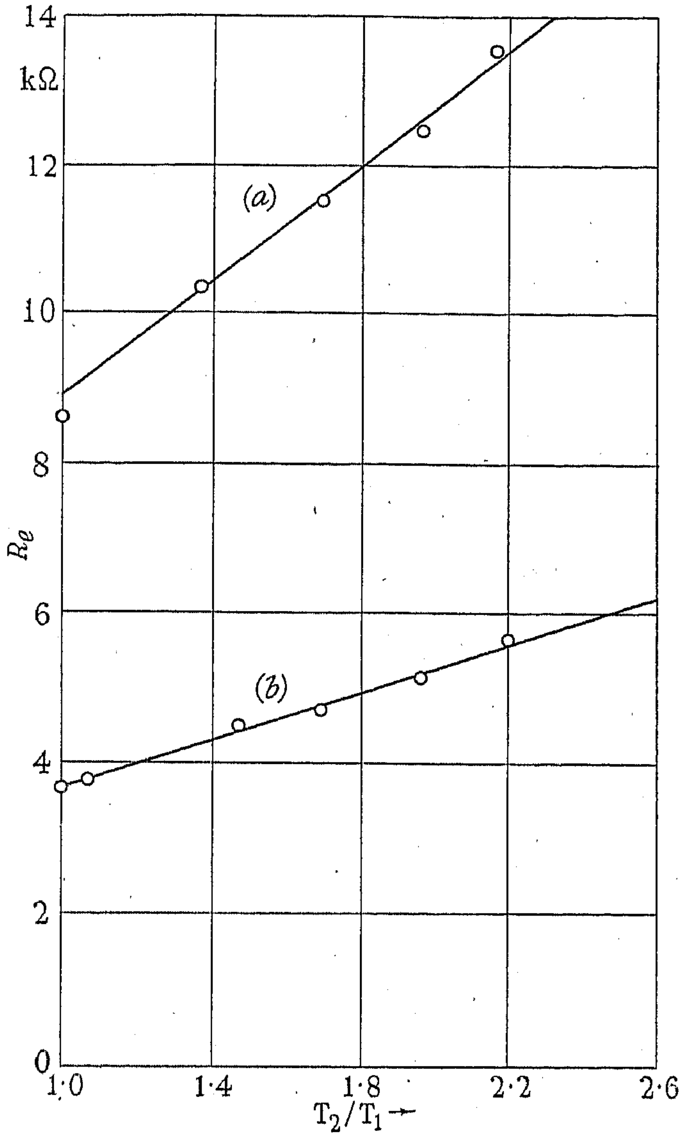

**Fig. 15.**—Thermal and thermionic fluctuations in the circuit of Fig. 14 as a function of the temperature of $R$.

Curve ($a$).—Temperature-limited current in Osram D.E.H. 410 valve, oxide-coated filament.

Curve ($b$).—Space-charge-limited current in Mazda V914 valve, oxide-coated cathode.

The straight lines show predicted values. The amplifier passed a 10 % band about a frequency of 8 kc.

Some part of this discrepancy may be due to error in the evaluation of $A$. Perhaps a preferable value of $A$ would be the mean of the values determined from all the points. This value has been taken in curve ($c$) of Fig. 16, which relates to a similar experiment using a different valve of the same type. In this case the anode current was 20 $\mu$A, and $[\rho(R + \rho)]^2$ was 0·435. The deduced value of $A$ varied between 0·89 and 0·81 and had a mean value of 0·865. All points lie close to the calculated line. Curve ($a$) corresponds to ($a$) in Fig. 15, except that the valve was an Osram D.E.8.L.F. with a directly heated thoriated tungsten filament: the current was 10·6 $\mu$A, temperature-limited.

Curve ($b$) shows the expectation when both the above currents flow simultaneously through the anode resistance $R$: this is a final test of the mutual independence of three fluctuation sources. It follows at once from equation (9) and the process used in deriving equation (11) that the expected fluctuation, expressed in terms of equivalent resistance, is given by

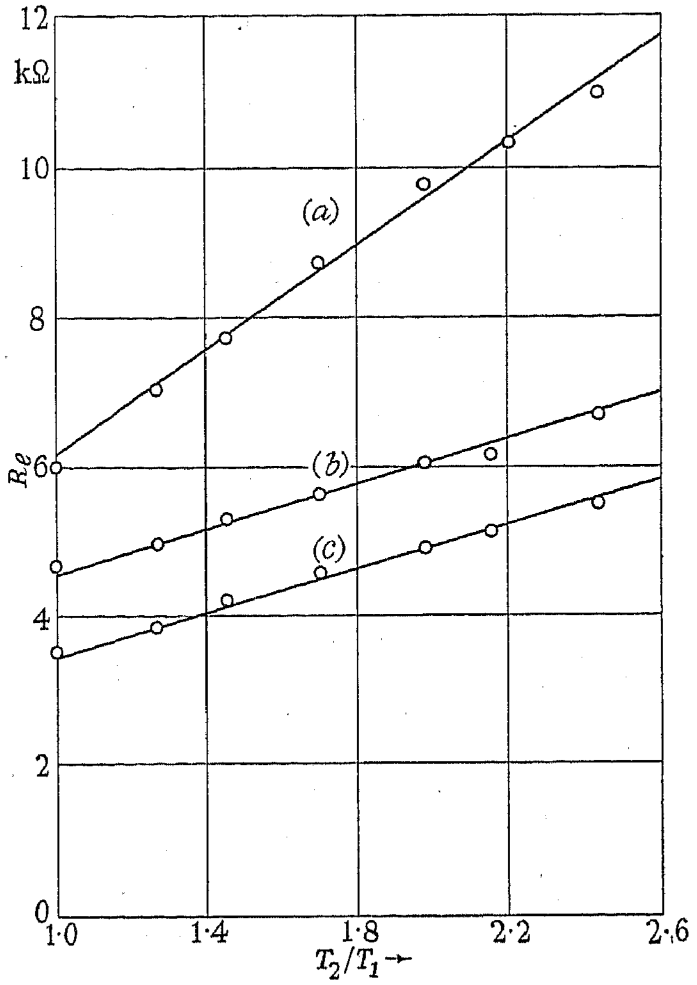

**Fig. 16.**—As Fig. 15, but with different valves.

Curve ($a$).—Temperature-limited current in Osram D.E. 8 valve, thoriated filament.

Curve ($c$).—Space-charge-limited current in Mazda V914 valve, oxide-coated cathode.

Curve ($b$).—The above temperature- and space-charge-limited currents flowing simultaneously.

The straight lines are predicted values.

$$R_e = R\left(\frac{\rho}{R + \rho}\right)^2\left\{\frac{T_2}{T_1} + 20R(AI_S + I_T)\right\}$$

where $I_S$ and $I_T$ are the space-charge-limited and temperature-limited currents respectively. The straight line ($b$) has been calculated from this equation, using the data relevant to curves ($a$) and ($c$); the small corrections for variation of $T_1$ were again applied to the experimental results.

The experimental points show a maximum discrepancy of $-3{\cdot}5$ % and a mean discrepancy of $+0{\cdot}5$ %.

These experiments are felt to establish the assumed independence of thermal and thermionic effects beyond reasonable doubt: further, they augment earlier evidence* in showing that the equation

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* See Bibliography, (11), Section (2).

$$\overline{v^2} = 2AIe\left(\frac{R\rho}{R+\rho}\right)^2 df$$

is a suitable representation of thermionic fluctuations, whether in fact they are to be ascribed to a thermal or a shot mechanism. They also show that, when this representation is adopted, $\rho$ can safely be regarded as free from fluctuations in value, and that $\rho$ is not then to be regarded as the seat of a coexistent thermal fluctuation, as suggested by Llewellyn.† These results also add to the evidence showing that the doubts very properly expressed by Aldous and Campbell‡ regarding the mathematical adequacy of such a formula as an expression for the fluctuations generated by a space-charge-limited current do not, in practice, limit its application.

## (8) APPLICATION TO WIDE-BAND PHOTOCELL AMPLIFIERS

Fig. 17 shows a typical circuit for the first stage of a photocell amplifier. $C_1$ and $C_2$ represent the inter-electrode and stray capacitances to earth of grid and anode respectively. The values of the coupling condenser and leak will be supposed such that they can be omitted from the analysis. If the leak is comparable with $R_1$ the relevant value of $R_1$ in the analysis is $R_1$ in parallel with the leak. Following the process outlined in Section (6), the grid fluctuation voltage $\overline{v_g^2}df$ is first calculated by consideration of circuit elements to the left of the dotted line. It follows from equation (9) that

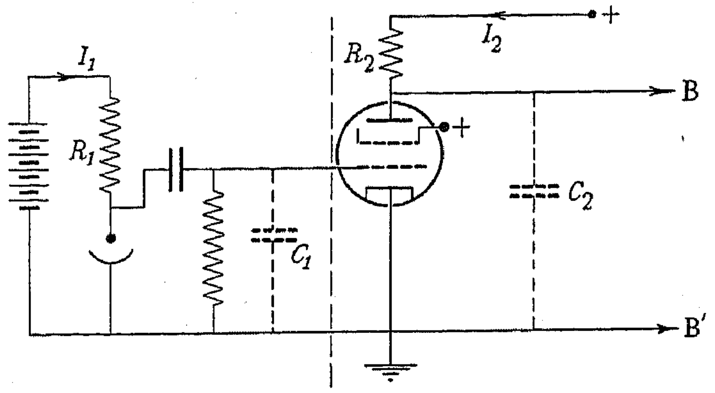

Fig. 17

$$\overline{v_g^2}df = \frac{R_1^2}{1+R_1^2C_1^2\omega^2}\left[\frac{4kT}{R_1} + 2I_1e\right]df \quad . \quad (12)$$

$df$ being supposed situated about the frequency $\omega/(2\pi)$, and it is supposed that $\rho = \infty$ and $A = 1$ for the photocell.§ The equivalent circuit of the elements to the right of the dotted line is shown in Fig. 18, the fluctuation derived from components to the left being included. Application of formula (9) to this figure yields a fluctuation between B and B′ given by

$$\overline{v^2} = \frac{R^2}{1+R^2C_2^2\omega^2}\left[\frac{4kT}{R_2} + g^2\overline{v_g^2} + 2AI_2e\right]df$$

where $R = R_2\rho/(R_2+\rho)$. In general $\rho >> R_2$, and hence, from (12),

$$\overline{v^2} = \frac{R_2^2}{1+R_2^2C_2^2\omega^2}\left[\frac{4kT}{R_2} + \frac{g^2R_1^2}{1+R_1^2C_1^2\omega^2}\left(\frac{4kT}{R_1} + 2I_1e\right) + 2AI_2e\right]df$$

If now a sinusoidal signal of frequency $\omega/(2\pi)$ and mean-square value $I_S^2$ be superposed on $I_1$, there will result a signal voltage across $R_2$ given by

$$V_S^2 = \frac{R_2^2}{1+R_2^2C_2^2\omega^2}\left[\frac{g^2R_1^2}{1+R_1^2C_1^2\omega^2}\right]I_S^2$$

Hence the (noise/signal)² ratio over the band width $df$ is

$$N_\omega^2 = \frac{\overline{v^2}}{V_S^2} = \frac{1}{I_S^2}\left[\frac{4kT}{R_1} + 2I_1e + \frac{1+R_1^2C_1^2\omega^2}{g^2R_1^2}\left(2AI_2e + \frac{4kT}{R_2}\right)\right]df$$

which, taking $T$ as 290° K., can be rewritten as

$$N_\omega^2 = \frac{2I_1edf}{I_S^2}\left[1 + \frac{0\cdot05}{R_1I_1} + \frac{1+R_1^2C_1^2\omega^2}{g^2R_1^2}\left(\frac{AI_2}{I_1} + \frac{0\cdot05}{I_1R_2}\right)\right] \quad . \quad (13)$$

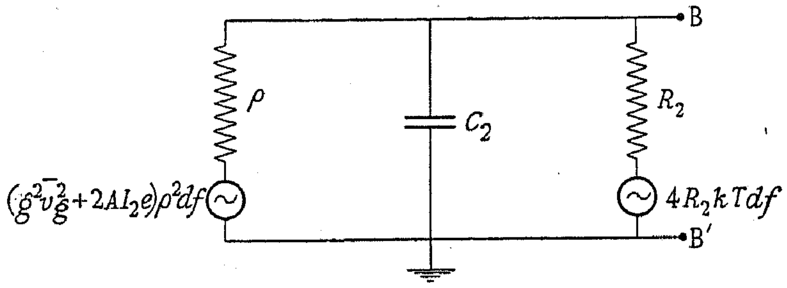

Fig. 18

$2I_1edf/I_S^2$ is the (noise/signal)² ratio in the actual photocurrent and is the lower limit of $N_\omega^2$. The limit is approached only if each of the second two terms in the bracket is much less than 1.

At this point it may be noted that the last term, representing thermal agitation in the anode resistance $R_2$, can be omitted if $R_2 >> 0\cdot05/(AI_2)$. Since $AI_2$ will be of the order of $10^{-3}$, this condition will always be satisfied.

If now an amplifier is connected across $R_2$ which has a frequency characteristic such that there results a signal output voltage $v_S = mI_S$, where $m$ is supposed independent of the frequency of $I_S$ over the range $0 - f_{max.}$ and zero outside that range, there will result in the range $df$ an output fluctuation

$$\overline{v^2} = m^2N_\omega^2I_S^2$$

neglecting fluctuations generated in the amplifier itself. The total fluctuation output will be

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* See Bibliography, (6). † *Ibid.*, (4). ‡ *Ibid.*, (14). § *Ibid.*, (13).

$$\overline{v^2} = m^2 I_S^2 \int_0^{f_{max.}} N_\omega^2$$

yielding an output (noise/signal)$^2$ ratio

$$N^2 = \int_0^{f_{max.}} N_\omega^2 = \frac{2I_1 e}{I_S^2} \left[ 1 + \frac{0\cdot 05}{R_1 I_1} + \frac{AI_2}{I_1 g^2 R_1^2}\left(1 + \frac{R_1^2 C_1^2 \omega^2{}_{max.}}{3}\right) \right] f_{max.} \qquad (14)$$

where $\omega_{max.} = 2\pi f_{max.}$

$2I_1 e f_{max.}/I_S^2$ is the (noise/signal)$^2$ ratio existing in the photo-current, and represents an absolute minimum value for $N^2$, say $N_0^2$. Further, $N^2$ decreases to a limiting value greater than $N_0^2$ with increasing $R_1$, for, with $R_1$ infinite,

$$N^2 = N_0^2 \left[ 1 + \frac{AI_2 C_1^2 \omega^2{}_{max.}}{3 I_1 g^2} \right] \quad . \quad . \quad . \quad (15)$$

In this expression the second term represents shot effect in the first stage. It follows that the best valve to use for the first stage is that which yields the lowest value for $AI_2 Cg^2/g^2$, where $C_g$ is the grid-earth capacitance of the valve and is supposed a dominant component of $C_1$. The valve should be operated in a condition giving a minimum value of $AI_2/g^2$, a condition which has been found before* and whose attainment has already been discussed. Values as low as 50 are readily attainable. Hence, taking $C_1$ as 10 $\mu\mu$F and $\omega_{max.} = 12\cdot 56 \times 10^6$,

$$N^2 = N_0^2 \left[ 1 + \frac{0\cdot 3 \times 10^{-6}}{I_1} \right] \quad . \quad . \quad . \quad (16)$$

Turning now to finite values of $R_1$, this limit will be approached only if $R_1 >> 1/C_1\omega_{max.}$ and

$$0\cdot 05/R_1 << 0\cdot 3 \times 10^{-6}, \text{ i.e. } R_1 >> 0\cdot 17 \times 10^6$$

Both conditions will be sufficiently satisfied if $R_1$ is equal to or greater than 1 megohm.

Hence, if $R_1$ exceeds 1 megohm the (noise/signal)$^2$ ratio is governed by shot effect of the photo-current and shot effect in the first stage, shot effect in the first stage being dominant or negligible according as $I_1$ is much greater or much less than $0\cdot 3\ \mu A$ $\left[AI_2 C_1^2 \omega^2{}_{max.}/(3g^2)\right.$, see equation (15)$\left.\right]$.

It is believed that television photo-currents are of the order of $10^{-10}$ amp.; amplifier shot effect is then dominant and the noise/signal ratio is of the order of 55 $N_0$ (see equation 16). If, under similar conditions, $R_1$ were chosen so as to avoid the necessity for subsequent correction of the frequency characteristic, say $R_1 = 1$ k$\Omega$, we have, from (14), $N > 1\,000\,N_0$. The corrected system utilizing high $R_1$ is therefore extremely advantageous.

It seems impossible to reduce $C_1$ appreciably below 10 $\mu\mu$F, and values of $N$ approaching $N_0$ can only be obtained by either increasing the photo-current to about $10^{-7}$ amp., or by developing valves for which $AI/g^2$ is of the order of $0\cdot 5$.

It is interesting to note from equation (14) that with sufficiently great photo-currents the signal/noise ratio approximates very closely to the limiting value obtaining in the photo-current. This limit cannot be approached with electron multipliers, for secondary emission will augment the initial fluctuations of the photo-current. It follows that there must exist some value of photo-current below which the electron multiplier is preferable and above which the thermionic amplifier is preferable. This critical value depends on how much secondary emission in the multiplier augments the initial photo-current fluctuations. It seems likely, however, that the critical current will be so high as to lie in a region where background noise is comparatively unimportant.

In the foregoing analysis fluctuations generated in stages subsequent to the first are neglected. Application of the same process to the second stage with the anode current of the first valve replacing the photo-current considered, shows at once that conditions can easily be chosen justifying this procedure. It is necessary only to ensure that the stage gain of the first few stages shall exceed, say, 2 at a frequency of $\omega_{max.}/(2\pi)$. Further, in the integration leading to (14), $A$ was assumed independent of frequency. Such is in fact untrue on account of flicker effect, which increases $A$ at low frequencies. But the influence of flicker effect is confined almost entirely to frequencies less than, say, 2 kc. At these frequencies the third term in equation (13) will be negligibly small compared with its values at higher frequencies, since $R_1 >> 1/(C_1\omega_{max.})$. (In other words equation (13) shows that the relative importance of valve noise in a "corrected" amplifier increases rapidly with the frequency $f$ about which $df$ is situated.) Flicker effect can therefore be neglected and does not invalidate the analysis. It may be noted that this does not hold with "uncorrected" amplifiers, for then $R_1 \simeq 1/C_1\omega_{max.}$ It is therefore a further advantage of the corrected system that flicker effect is effectively suppressed, the fluctuation output of the amplifier due to valve noise being concentrated towards the higher-frequency end of the spectrum.* Also grid current has been neglected in the discussion; it will yield an entirely negligible fluctuation provided it is less than, say, $10^{-8}$ amp., and will not become dominant in the typical case considered till it exceeds $0\cdot 34$ $\mu$A. An analysis including grid current can readily be developed by the method given above, the grid-current fluctuation being analogous with that produced by the photo-current.

To summarize:—

High values of $R_1$, say greater than 1 megohm, should be used, the resulting loss of high frequencies being compensated in subsequent stages. The first valve should be chosen and operated such that $AIC_g^2/g^2$ is a minimum; this is vitally important when the photo-current is less than, say, $10^{-7}$ amp., for valve noise is then dominant. Grid current should be less than $10^{-8}$ amp.

Braude† has already discussed this problem and has arrived at the conclusion that $R_1$ should be kept high. He assumed the thermal hypothesis‡ of valve fluctuations, and accordingly believed that fluctuations generated in screen-grid valves were basically negligible, a belief which has been disproved experimentally.§

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* See Bibliography (6), Section (5).

* Braude's [see Bibliography, (12)] suggested "double amplifier," one dealing with low and one with high frequencies, is therefore unnecessary.

† See Bibliography, (12). ‡ *Ibid.*, (4), (6), and (11). § *Ibid.*, (6).

Further, Braude's analysis omitted the fluctuations generated in the photo-current itself. These two sources of fluctuations have been shown to determine the practical and ideal limits of background noise respectively. Hence Braude's calculation of these limits and of the improvement obtained by increasing $R_1$ are at fault, as also is his proposed method of eliminating background noise by using reactive impedances in the early stages. With values of $R_1$ greater than 1 megohm thermal noise is negligible; his proposed values of $R_1$, between $10^7\ \Omega$ and $10^9\ \Omega$, are therefore unnecessarily high.

## (9) CONCLUSIONS

The first series of experiments establishes the general formula (1) relating to thermal fluctuations in complex networks with non-uniform temperature distribution. The thermal effects can be represented satisfactorily by concentrated e.m.f.'s associated solely with the resistive elements of the network: the values of the e.m.f.'s depend only on the values and temperatures of the resistances—the temperatures of reactive elements do not influence thermal fluctuations.

The second series of experiments establishes the mutual independence of thermal and thermionic effects, and proves the suitability of the formula used to represent thermionic fluctuations. The slope resistance of the valve must there be taken as constant, and thermal fluctuations must not be ascribed to it.

In wide-band photocell amplifiers a " corrected " system employing a high resistance of the order of 1 megohm in the anode circuit of the photocell is vastly preferable to an " uncorrected " system with its associated low anode resistance. In typical circumstances thermionic noise in the first amplifying valve will be dominant. Accordingly, the first valve should be chosen to yield the lowest possible value of $AIC_1^2/g^2$. The operating conditions should be such that $AI/g^2$ is a minimum, provided $g$ is not thereby reduced to such an extent that the gain of the first stage at the highest wanted frequency falls below 2. In " corrected " amplifiers the noise output is concentrated towards the high-frequency end of the spectrum, and flicker effect can be neglected.

There exists a critical value of photo-current above which the thermionic amplifier yields a better signal/noise ratio than the electron multiplier. This critical current cannot be evaluated without further data relating to electron multipliers; it is probably so high as to lie in a region where background noise is comparatively unimportant.

## (10) ACKNOWLEDGMENTS

The research described in this paper was carried out in the Electro-Technics Department of Manchester University. The author is indebted to Prof. R. Beattie, D.Sc., for the facilities placed at his disposal, and to Mr. E. B. Moullin, M.A., for much helpful discussion.

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# RADIO DIRECTION-FINDING ON WAVELENGTHS BETWEEN 6 AND 10 METRES (FREQUENCIES 50 TO 30 MC./SEC.)*

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## SUMMARY

The following paper describes the development of simple experimental direction-finders for wavelengths between 6 and 10 metres, and their use in an investigation of the accuracy of direction-finding on these wavelengths. It is shown that the inherent accuracy of the instruments is well within the limits of ± 2°, while their sensitivity is sufficient for observation on an experimental 50-watt transmitter at ranges up to 22 miles over flat ground. Bearings have also been taken on signals emanating from London television transmitters, blind-landing beacons, and also from commercial and amateur transmitting stations some 3 000 miles away.

Detailed experiments have shown that the site on which the direction-finder is used must be clear of obstacles, particularly trees and vertical wires, for a radius of at least 50 to 100 yards; and there are indications that similar conditions are necessary for the site of the transmitter. When such conditions are satisfied, the bearings observed at distances up to 22 miles from the transmitter may be in error by as much as 8°, although in the majority of cases the error was less than 2°. Such errors tend to diminish in magnitude as the range increases, and they may or may not be affected by small changes in position of the direction-finder, by changes in frequency of the transmitter, or in the orientation of the transmitting aerial. For a given set of conditions the changes in bearings observed from day to day do not exceed about 2° for ranges of 20 miles, although in some long-distance observations made on signals from American stations at a range of 3 000 miles the variations in bearings were much larger.

A brief study has been made of the behaviour of the loop direction-finder when horizontally polarized waves are emitted at the transmitting station. It is shown that the errors experienced in this case can be almost entirely eliminated by the use of a rotating spaced vertical aerial arrangement in place of the loop. The evidence resulting from the use of this Adcock type of direction-finder indicates that the errors with the loop set are due to the reception of horizontally polarized waves.

## (1) INTRODUCTION

Radio direction-finding on ultra-short waves may really be said to have been initiated by Hertz, for in some of his classical experiments carried out some 50 years ago he used circular or rectangular loops of wire with small spark-gaps for detecting the directions of the fields in the waves radiated from his spark-coil-driven oscillators. The wavelength used in many of these experiments was approximately 10 metres or less. From that time very little work appears to have been carried out on the use of direction-finding receivers on wavelengths below 10 metres, although from time to time various systems for transmitting directional and rotatable beams have been employed. In recent years these have led to the development on a commercial scale of the course-indicating beacons and landing-beam systems now in use for the guidance of aircraft.

On wavelengths of about 1 metre S. Uda† has studied the transmitting and receiving properties of half-wave antennae and reflector systems. In addition he has described‡ a direction-finder for a wavelength of 0·50 metre, comprising a receiving antenna at the focal line of a cylindrical parabolic reflector. As this apparatus is rotated during the reception of signal the intensity passes through a minimum at about 35° on either side of its maximum value, and by observation of these minima the direction of the maximum can be located with fair accuracy. Various amateur experimenters have also used the ordinary loop direction-finder on wavelengths of about 5 metres.

The investigation of the performance of the single rotating-loop direction-finder was commenced at the National Physical Laboratory in 1930, and the results of some experiments made on a wavelength of 7·9 metres have been published previously.§ After being in abeyance for some years the study of this subject has recently been resumed, and the present paper describes the results of investigations carried out on wavelengths between 6 and 10 metres during the past two years.

## (2) DESCRIPTION OF THE DIRECTION-FINDERS EMPLOYED

The work described in this paper was largely carried out in the field, and the direction-finders were accordingly designed to be of a readily transportable, battery-operated type, with aural indication by means of head-telephones. While instruments of both the rotating-loop and the rotating spaced aerial or Adcock aerial pattern have been employed, attention was first devoted to the loop type on account of its compactness and simplicity.

### (a) Rotating-Loop Direction-Finder

The rotating-loop direction-finder depends for its operation upon the fact that the electromotive force induced in a vertical loop by vertically polarized electric waves is proportional to the cosine of the angle between the plane of the loop and the plane of propagation of the waves. Thus when these planes are perpendicular to one another the signal e.m.f. is of zero value, and the practical operation of the direction-finder consists in rotating the loop and determining the position at which the observed signal, detected aurally or visually, passes through a minimum or zero value. The satisfactory operation of a direction-finder in this manner depends upon the attainment of exact electrical symmetry of the loop and associated circuits, and this problem requires great care and attention in the design and construction of apparatus for use at very short wavelengths. In the two classes of loop direction-finder described in this section, this electrical symmetry has been attained by the use of a screened loop connected to a push-pull detector or amplifier circuit.

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* This paper will be included in a symposium of Direction-Finding Papers to be read before the Wireless Section during the 1938–39 Session.

† See Reference (1).  ‡ *Ibid.*, (2).  § *Ibid.*, (3).

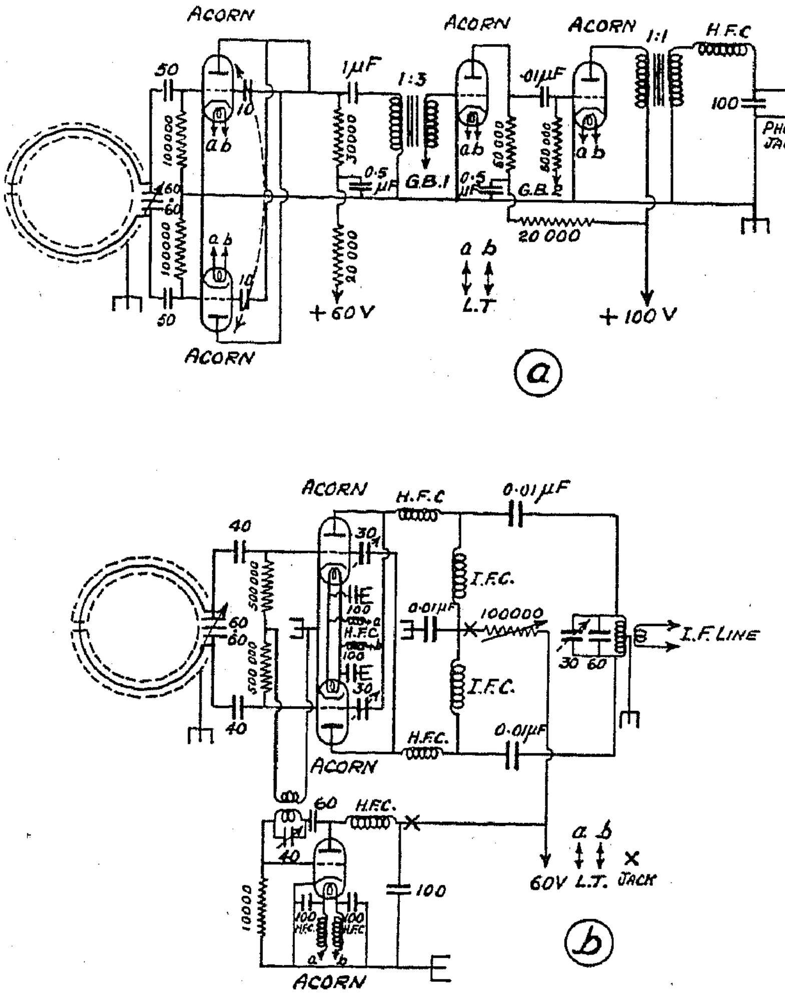

**Fig. 1.**—Loop direction-finders.

(*a*) Straight receiver.
(*b*) Frequency-changer of superheterodyne.

The capacitance and resistance values are in micro-microfarads and ohms, respectively, unless otherwise stated.

The first instrument utilizes a simple straight receiver and has been developed for use in relatively strong fields of the order of 0·1 mV per metre. The circuit diagram is given in Fig. 1(*a*), from which it is seen that the tuned receiving loop is connected to a push-pull detector, with ganged retroaction condensers and a two-stage audio-frequency amplifier. Either modulated or continuous-wave signals can be received. The whole receiver with its batteries is enclosed in a metal box which, together with the screened loop, rotates as a unit on a suitable turntable.

The more sensitive instrument, for use in weaker fields, employs a receiver of the supersonic heterodyne type. In this case the receiver is constructed in two units. The first or rotating unit comprises the frequency-changer in a screened box carrying the screened loop, while the second is a fixed unit containing the intermediate and audio-frequency amplifiers. The two units are coupled together by a screened flexible transmission line carrying the intermediate-frequency output from the frequency-changer.

The circuit diagram of the latter is shown in Fig. 1(*b*). The push-pull detector stage is screened from the local signal frequency oscillator, the output from which is applied in parallel, through a coupling coil, to each of the detectors. Retroaction is applied by means of small pre-set condensers, and is controlled by an adjustment of the anode voltage by means of a variable resistance. The intermediate-frequency output from the detectors is fed to the transmission line through a step-down transformer, which is isolated from the detector anodes by chokes designed to prevent the passage of signal-frequency current.

As already remarked, this transmission line connects the rotating unit of the receiver to the fixed unit containing the intermediate-frequency amplifier, second detector, and audio-frequency amplifier, giving output to the head-telephones. This unit is of a standard pattern and needs little description. An intermediate frequency of about 2 Mc./sec. is used, in order to permit of reasonably high amplification with a large band-width. The latter feature results in an improved performance in the reception of frequency-modulated transmissions, facilitates tuning, and minimizes the need for repeated adjustment due to drift of the frequency of the local oscillator. Provision for the reception of continuous waves is made by the inclusion of a beat-frequency oscillator, which can be switched on as required.

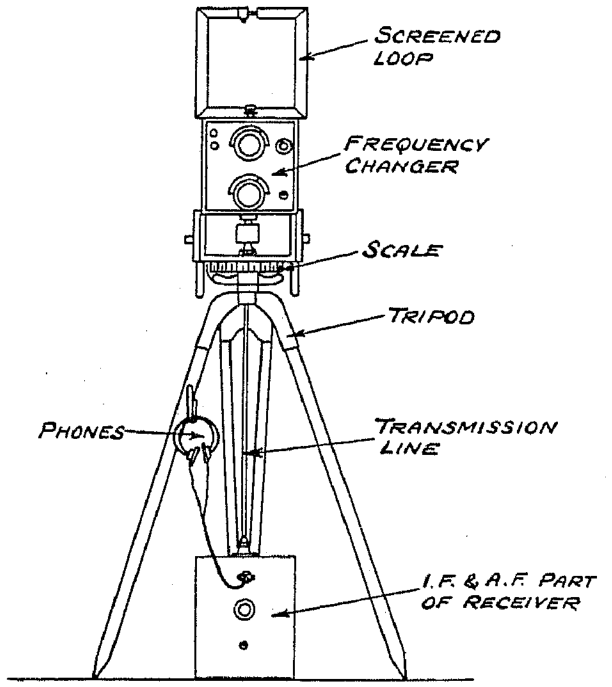

**Fig. 2.**—Loop direction-finder (super-heterodyne model).

Both the straight and supersonic heterodyne receivers cover a wavelength range of 5·5 to 10·7 metres (frequency 54 to 28 Mc./sec.) by means of two plug-in screened loops. These are square in shape and have sides of $11\frac{1}{2}$ in. and $7\frac{1}{8}$ in. respectively: they are made of $\frac{3}{16}$ in. diameter copper rod surrounded by a 1 in. diameter copper screen. This screen is split at the centre of the top side, and an adjustable collar sliding on the screen provides a convenient means of accurately balancing the input system.

The disposition of the more sensitive direction-finder is shown diagrammatically in Fig. 2. The rotating-loop unit is mounted on a tripod well above the observer, who sits on a stool as nearly as is practicable beneath the centre of the system. With this arrangement the actual position of the observer about the central axis of the direction-finder is not of great importance. The simpler direction-finder is carried in a similar way on a turntable mounted on a tripod, the telephone leads passing through the bearing shaft in place of the transmission line shown in Fig. 2. With each instrument a suitable hand-wheel is provided for rotating the turntable, and a graduated scale is set at a convenient level for the observer to read off bearings from the position of the pointer attached to the rotating portion. In setting up the instruments in the field a magnetic compass is employed to ascertain the direction of the meridian, and suitable sights are provided on the screened loops to facilitate this operation.

The sensitivity of the direction-finder with the straight receiver is such that bearing observations can be made with a swing through the minimum signal position of 5° when operated in a uniform vertically-polarized field, the intensity of which is of the order of 100 microvolts per metre. The more sensitive instrument enables the same observational accuracy to be attained in field intensities of about one-tenth this value. These figures apply to the reception of fully modulated signals. The effective sensitivity in continuous-wave reception is somewhat less than the above, owing to fluctuations of the audio-frequency note due to variations in the local oscillator frequency as the set is rotated. It is more difficult to obtain an accurate reading on this fluctuating note than on a steady modulation note.

### (b) Experimental Rotating Spaced Aerial Direction-Finder

It is now well known that one of the limitations of the closed-loop direction-finder is that it is subject to errors in observed bearing when employed in the presence of downcoming horizontally polarized waves. One of the most effective means of overcoming this difficulty is to replace the loop by a pair of spaced vertical aerials in accordance with the arrangement first described by F. Adcock. In Section (5) of this paper some experiments are described in which a source of horizontally polarized waves was employed, and for the purpose of investigating the phenomena observed a simple rotating Adcock direction-finder was developed. It is convenient to outline the main features of this instrument at the present stage.

The construction of this direction-finder involved the replacement of the loop attached to the supersonic heterodyne receiver by a suitable spaced-aerial system with the necessary transmission lines and transformer. The general layout of the instrument is shown in Fig. 3, from which it will be seen that the aerial pair is of the "H" type supported centrally at the vertical axis of rotation. The aerials are made of copper rod, $\frac{1}{4}$ in. diameter and of a total length of 4 ft., and are rigidly attached to wooden supports. Each aerial is supported at its centre by an aluminium tube, $\frac{3}{4}$ in. diameter, forming the screen for the transmission line which consists of two copper rods $\frac{1}{8}$ in. diameter and 3 ft. long. The inner end of each aluminium tube is rigidly attached to the top of the metal frequency-changer box, and the transmission lines are connected inside the box to the primary winding of an air-core screened transformer. The secondary of this transformer is outside the primary and coaxial with it, a screen of copper foil strips being placed between the windings. This secondary winding is tuned and connected to the push-pull detector in the same manner as was the closed loop in the sets previously described.

The satisfactory operation of this spaced-aerial direction-finder depends upon the attainment of symmetry of the two aerials and also of the two halves of each aerial. For this reason care must be taken to set up the direction-finder on a level surface, and also to place the observer close in to the central axis and on the line at right angles to the plane containing the two aerials. The latter precaution necessitates the observer changing his position for each working azimuth; and although in a fixed installation this could probably be avoided by elevating the aerial system, this would appear to be impracticable in a set designed for field use.

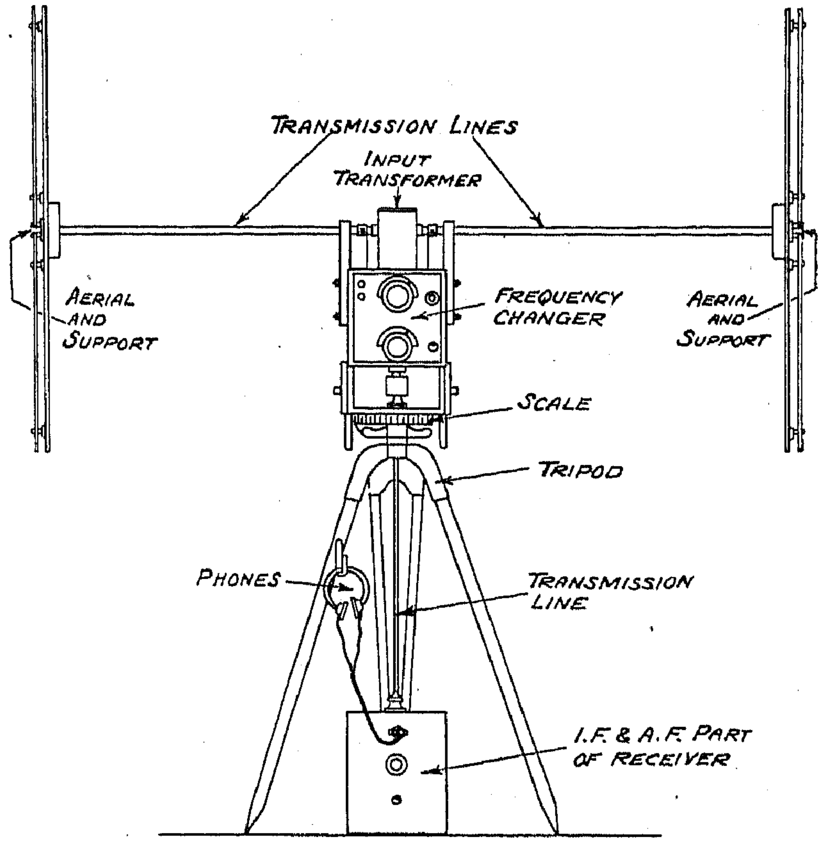

**Fig. 3.**—Spaced-aerial (Adcock) direction-finder.

The sensitivity of this Adcock direction-finder was found to vary over the working range of wavelengths, owing apparently to an approach to a resonance condition in the transmission-line system. When used for bearing observations on vertically-polarized waves propagated horizontally, the mean sensitivity was of the same order as that of the simple loop set described above, i.e. bearings with a swing of 5° could be taken in a field intensity of about 100 $\mu$V per metre.

## (3) ACCURACY OF DIRECTION-FINDERS

### (a) Instrumental Accuracy

In order to obtain some idea of the instrumental accuracy of these direction-finding sets and the conditions to be fulfilled for a site on which they can be operated, simple calibration tests were first made on a flat open field some 250 yards square. A portable transmitter was set up in the centre of this field, and the direction-finder was taken to various positions in circles of radii 30 to 60 yards, the radio bearing being directly compared with the visual bearing. Out of 64 observations made in this manner, 59 were less than 2° in error, two of the remaining five being 3° in error. These errors refer to the mean observation of a pair taken by rotation of the coil, the difference between direct and reciprocal bearings being usually less than 2°. This latter figure has been reduced to less than 1° in later models of these sets. It is thus considered to be fair to state that the instrumental accuracy of these direction-finding sets for observed bearings lies within ± 2°.

Experiments carried out on the same and other sites indicate that the above accuracy is maintained at distances up to 600 yards from the transmitter. The results obtained at greater distances are dealt with in a later Section.

### (b) Effect of Wires and Pipes on Direction-Finding Bearings

In the course of the calibration tests described above it was found that a long water pipe buried at a depth of 1 ft. 6 in. produced no effect on the bearings observed on a direction-finder placed over it, the height of the centre of the loop being about 6 ft. above the ground. Under similar conditions lengths of copper wire up to 50 yards long laid on the ground produced no effect; only when the wires were raised to the level of the base of the loop, and placed a few inches from it, were errors in bearing detectable. In later experiments it has been found that placing the direction-finding set immediately beneath a number of overhead telephone wires along a road produced a change in bearing of 3° on a distant station.

When short lengths of wire insulated from earth were placed vertically near the direction-finder, it was found that the effect was greatest when the wire was half a wavelength long. In this condition, the presence of the wire at a distance of 4 yards produced errors in bearing ranging up to 11°, the error being greatest when the direction of the wire was at right angles to the line joining transmitter and receiver, and practically negligible when situated on this line. The effect of such a vertical half-wave aerial became negligible when the aerial was removed to a distance of 30 yards from the direction-finder.

### (c) Effect of large Aerials

In one phase of the experimental work the opportunity occurred to examine the effect of an aerial system the dimensions of which were large compared with the wavelength on which the direction-finder was operated. The system comprised four vertical-cage aerials used as an Adcock direction-finder for medium and long waves. Four aerials were arranged at the corners of a square of 320 ft. side. Each aerial was 70 ft. high and was formed of an octagonal cage of wires with a side of about 4 ft. When the ultra-short-wave loop direction-finder was used at a distance of about 70 yards from the centre of this system, errors in bearing were experienced ranging up to 11° on a transmitter a few miles away. When all the aerials were removed this error was reduced to 3°, which was considered to be the error due to other obstacles—chiefly trees—surrounding the direction-finding site. A single cage aerial at a distance of about 100 yards from the direction-finder produced a maximum error of 5° on a transmitter about 20 miles away, while when the cage was replaced by a single vertical wire of the same height (70 ft.) the error was less than 1°.

These values of the errors are intended to indicate the order of the effects observed rather than to be taken as absolute values, for it was found that the actual values depended upon the frequency employed and the direction of the transmitting station. In the case of observations taken on the Teddington station, about 11 miles away, the bearing was found to vary with the frequency in a quasi-periodic manner, indicating the presence of an interference pattern in the received field arising from the superposition of re-radiation from the aerial system on the incoming field from the distant source. In a similar manner it was found that if the portable transmitter was moved in a straight line away from the direction-finder to distances up to 100 ft. the bearings varied in a periodic manner over an amplitude of 4° or 5°, owing to the same cause. Although an attempt was made to study this interference pattern systematically it was found not to be susceptible to a simple analysis, probably owing to the complicated conditions prevailing at the site and to the fact that the aerials were not the only obstacles affecting the observations.

### (d) Effect of a single Tree on Direction-Finding Bearings

Similar experiments to the above were carried out with the direction-finding set located near a single tree about 25 ft. high and with a spread of its branches of the order of 12 ft. A portable transmitter was used for these tests at a distance of 30 yards on a wavelength of 9·5 metres. With the direction-finder placed at a distance of 4 yards from the tree, an error in bearing up to 10° was experienced, the maximum effect being as in the case of the vertical wire, when the direction of the tree from the direction-finder was at right angles to that of the transmitter. When the set was moved to a distance of 20 yards from the tree, the maximum error was reduced to 1·5°. It is evident that a separation of 30 yards from such a tree would be sufficient to produce a negligible effect. On the other hand, later experience has shown that a large mass of trees may produce an appreciable error in observing the bearing of a distant transmitter with the direction-finder placed some 20 to 100 yards away.

### (e) Effect of Buildings

An experiment was next carried out in which a portable transmitter was set up on an open site and operated on a wavelength of 8·0 metres, while the direction-finder was moved in a straight line along a road with various laboratory buildings on either side. Up to a distance of

280 yards there was a clear air line between transmitter and direction-finding set, but errors varying from − 9° to + 29° were experienced, accompanied by the effects of flat signal minima. In other experiments in similar " built-up " areas, where the direction-finding set was not in direct visual range of the transmitter, errors of ± 24° were experienced at ranges of 360 yards. It was usually found that, in these situations, the error of the observed bearings altered appreciably when the direction-finder was moved a few yards. All these effects are to be expected as the result of the interference of two or more waves arriving at the receiver after reflection from the surrounding obstacles.

### (f) Conclusions as to Effect of Site Conditions and the Calibration of a Direction-Finder

The experiments described above provide considerable information as to the conditions to be satisfied for a site for an ultra-short-wave direction-finder in order that the observed bearings shall be free from the disturbing effects of obstacles in its locality. It appears that a flat, clear piece of ground some 100 yards in radius forms a satisfactory site, since the direction-finder is then practically unaffected by the presence of wires, trees, and similar obstacles. It has been shown, however, that a similar clear site is necessary for locating the transmitter if the " source error " which results from the re-radiation of waves from trees and other obstacles near the transmitter is to be avoided. Thus, in order to ascertain the actual instrumental accuracy of the direction-finder, it is necessary that a short-range calibration should be made on a site which is clear over a radius sufficiently large to accommodate both transmitter and receiver. The results obtained at three such sites will serve to illustrate the accuracy obtainable with the apparatus referred to in this paper.

(i) On a sports ground at Teddington, some 250 yards square, the errors observed in a complete calibration on a portable transmitter at a radius of about 60 yards were in general not more than 2°.

(ii) On a flat site at Dorney Common, Bucks, observations on a transmitter moved in a straight line away from the direction-finding set to distances up to 400 yards showed a maximum error of 2°. A similar run on another site has been carried out to the same order of accuracy at distances up to 650 yards.

(iii) In this case the direction-finding set was placed at the end of a concrete jetty, 1 120 yards long, in the middle of Queen Mary Reservoir, Staines, surrounded, except for the jetty, by an expanse of water of from 570 yards to 1 580 yards. When the transmitter was actually on the edge of the reservoir, the errors were very small (1° or less), except in positions near a wire fence, in which cases some source error was to be expected. Observations made over a distance of 360 yards at the projecting end of the jetty showed that the variations in the error were within ± 1½°, so that the concrete itself had only a small effect on the bearings.

The results of the experiments described in this Section thus confirm the previous conclusion that the instrumental accuracy of the direction-finders employed is of the order 1° or 2°, and that for short ranges up to 1 mile, when both the transmitter and direction-finding set are on good sites and there is clear, level ground in between, the accuracy of observed bearings is generally within 2°.

## (4) RESULTS OF DIRECTION-FINDING OBSERVATIONS AT GREATER RANGES (UP TO 25 MILES)

In order to ascertain the practicability of direction-finding on ultra-short wavelengths, a systematic series of tests was carried out in which the loop direction-finder was used at various ranges up to 22 miles from Teddington and in different directions.

### (a) Local Observations on Teddington Transmitter

The fact that conditions between transmitter and receiver can seriously affect the accuracy of the observed bearings was first illustrated by some experiments carried out in Bushy Park at ranges up to about ¾ mile. The bearings observed on the main transmitter, situated at from 300 to 1 250 yards, were in error by amounts ranging up to 25°. It is considered that these errors were due to the effects of the large number of trees situated in the path of transmission, but to track down this cause in a satisfactory manner presents a somewhat difficult problem.

### (b) Distant Observations on Teddington Transmitter

Table 1 gives the results of bearing observations made on the transmitter at the National Physical Laboratory, Teddington, operating on a wavelength of about 9 metres. Every endeavour was made to place the direction-finding set on a site which was considered to be good as judged by inspection, although no actual calibrations were made. In most cases the bearing was the mean of a number of consecutive observations, and in one or two cases repeat observations were made on a different day. In some instances these repeat observations were made with a different direction-finding set, and the results obtained were consistent. No difficulty was experienced in taking the bearings to an observational accuracy of 2° or less; for example, at the maximum distance of 22 miles the swing on which the observation was made was about 8° in some cases, the signal being inaudible at the minimum position. In some cases, at the shorter ranges, and particularly when the bearing was in error, larger swings were necessary and the minima were not always silent.

During the course of these tests it was noted that aeroplanes flying in the neighbourhood of the direction-finder produced considerable fluctuations of signal strength which sometimes rendered observations difficult.

An inspection of the results given in Table 1 shows that at 28 of the 43 sites tested the observed bearing was within the accuracy of ± 2° assigned to the instrument in Sections (2) and (3) above. In 15 cases the error was greater than 2°, the maximum recorded being 8°. It is to be noticed that there is a definite tendency for the larger errors to be of lesser magnitude as the distance of the direction-finder from the transmitter is increased, as was the case in the earlier work of 1931. While the conditions surrounding the transmitter, such as large masses of trees, and also buildings, may be a contributory influence in

### Table 1

RESULTS OF OBSERVATIONS ON TEDDINGTON WITH LOOP DIRECTION-FINDER

($\lambda$ = 9 metres approx.)

| Site | | From transmitter | | Error in observed bearing |
|---|---|---|---|---|
| Ref. No. | Place | Distance | True bearing | |
| | | miles | deg. | deg. |
| 43 | Hampton Court .. | 1·0 | 14 | −1 |
| 44 | Sunbury Court .. | 2·5 | 77 | +1 |
| 45 | Bushy Park .. | 0·75 | 114 | −4 |
| 46 | Bushy Park .. | 0·5 | 120 | +8 |
| 47 | Bushy Park .. | 0·75 | 285 | +8 |
| 39 | Claygate .. .. | 4·75 | 3 | −6 |
| 40 | Hersham .. .. | 4·5 | 36 | +3 |
| 41 | Lower Halliford .. | 4·75 | 77 | +4 |
| — | Queen Mary Reservoir .. .. | 5·25 | 84 | −1·5 |
| 31 | Bedfont .. .. | 4·25 | 112 | 0 |
| 21 | Hatton .. .. | 4·5 | 132 | 0 |
| 32 | Heston .. .. | 5·25 | 157 | +2 |
| 33 | Osterley .. .. | 5·75 | 173 | −2 |
| 35 | Richmond Park .. | 3·75 | 225 | −6 |
| 36 | Wimbledon Common | 4·5 | 250 | +1 |
| 38 | Ewell Court .. | 4·75 | 324 | −3 |
| 42 | Ruxley Farm .. | 4·75 | 329 | 0 |
| 17 | Effingham .. .. | 9·7 | 20 | −2 |
| 16 | Ockham .. .. | 9·7 | 32 | −6 |
| 15 | Pyrford .. .. | 10·2 | 46 | −1 |
| 14 | Addlestone .. | 8·0 | 67 | 0 |
| 13 | Virginia Water .. | 9·0 | 87 | +1 |
| 12 | Englefield Green .. | 10·2 | 95 | −4 |
| 11 | Runnymede .. | 10·0 | 101 | −4 |
| 19 | Sunnymeads .. | 10·2 | 112 | 0 |
| 10 | Dorney Common .. | 14·2 | 113 | −4 |
| 20 | Datchet golf course | 11·7 | 114 | −4 |
| 26 | Langley .. .. | 10·5 | 124 | 0 |
| 24 | Hillingdon .. .. | 9·7 | 151 | −1 |
| 23 | Northolt .. .. | 9·7 | 169 | −3 |
| 22 | Wembley .. .. | 10·0 | 186 | 0 |
| 30 | Guildford By-pass .. | 21·0 | 40 | 0 |
| 28 | Hog's Back .. | 20·0 | 47 | +1 |
| 27 | Frimley .. .. | 17·5 | 66 | −1 |
| 7 | Blackwater Flats .. | 22·0 | 73 | 0 |
| 8 | Caesar's Camp .. | 18·0 | 83 | −1 |
| 9 | Winkfield .. .. | 15·2 | 96 | +1 |
| 1 | Whitebrook Common .. .. | 18·0 | 120 | −3 |
| 2 | Wooburn Common | 18·2 | 127 | 0 |
| 3 | Tyler's Green .. | 21·5 | 135 | +2 |
| 4 | Coleshill .. .. | 20·2 | 144 | +2 |
| 5 | Bovingdon .. | 21·7 | 159 | +1 |
| 6 | Bedmond .. .. | 21·2 | 171 | −1 |

producing these errors, it is considered that at distances of more than 5 miles such direct influence is likely to be small. It would therefore appear that some portion of the errors, particularly at the greater distances, is due to deviations of the waves in travelling between transmitter and receiver. The arrival of two or more sets of waves at the receiver, and their consequent interference, is probably partly, if not wholly, the basis of production of this path error. In some cases where an appreciable error was experienced, it was found that moving the direction-finding set a few yards produced a change in the error; but the absence of such an interference pattern is not an infallible criterion for the absence of error. For example, Dorney Common (Site No. 10) is a wide expanse of open, flat common, and small changes in position (about 10 yards) of the direction-finder produced no effect on the observed bearing.

### (c) Observations on Heston

During the course of the field observation work from which Table 1 was constructed, bearings were taken on the homing beacon at Heston Airport on a wavelength of 9 metres at such times as this station was found to be in operation. In this case the transmitter operates on a directional aerial system (Lorenz type) used for the guiding of aircraft towards the Heston airport at times of bad visibility. It employs two broad beams, on one of which a series of dashes is emitted while on the other is a corresponding series of dots. In the position midway between the beams, which in this instance is in the direction 84°, the two signals merge into a continuous dash. The results of the direction-finding observations obtained on this transmitter are shown in Table 2.

In making these observations it was generally found that both the signal strength and the signal minima were of good quality, although in some cases it was observed that the dot-and-dash signals did not pass through a minimum value at the same position of the direction-finding coil. As a result a bearing differing by not more than 2° was obtainable on the two series of signals. This effect is attributed to the fact that the two beams are reflected or scattered by different reflecting objects in their paths, so that the resulting waves arriving at the direction-finder produce different interference effects for the two sets of signals. This observation, in fact, tends to confirm the deduction that some of the errors in bearing experienced are the result of a path error, introduced at a position intermediate between transmitter and receiver.

### (d) Observations on Alexandra Palace and other Transmitters over Extended Periods

With the regular transmissions available from the London television station at Alexandra Palace, the opportunity was taken to make regular observations at the working programme periods for several weeks. This programme was started in the winter of 1936 and continued in the summer of 1937. For this purpose two 1-hour periods were available, one each afternoon and evening in the winter, and an additional morning period in the summer. A continuous series of observations of bearings was made at the rate of about one bearing a minute, successive periods of 15 minutes being devoted to the sound and vision transmissions alternately. The direction-finder was set up in a hut at the site where the experiments with large aerials, described in Section 2(*c*) above, were carried out. It was known that this site was not perfect even with the aerials removed, and that a permanent error would probably be experienced. It transpired that this error was different for the two transmissions (sound and vision), due to the difference in wavelength. The observations carried out in the above manner showed that during any period of 15 minutes the bearings were in general constant to within 1°, but that occasionally a change of the bearing somewhat greater than this was observed from one period to a later one. A summary of all the results obtained is given in Table 3.

In comparing the results for summer and winter it should be borne in mind that the large Adcock aerial system already described was removed in the meantime. These results were obtained by three observers, but no consistent difference was noticed indicating any systematic personal error. It will be seen from the Table that out of several hundred observations the maximum departure from the mean bearing was 3½°, while 97·5 % of all the observed bearings differed from the mean by less than 2°.

It is to be concluded from these results that bearings taken on transmissions from the Alexandra Palace at a range of 20 miles show no diurnal variation exceeding about 2°. No estimate of seasonal effects can be made, owing to the change in receiver site conditions described above.

Similar observations were taken during the summer on transmissions from a television transmitter at Kensington, working on wavelengths of 6·39 and 6·12 metres respectively (frequencies 47 and 49 Mc./sec.). The results obtained from this source were of the same character as those described above.

During the course of this work, various observations were made on transmissions from American stations working on wavelengths between 8·5 and 10·7 metres (frequencies 25·6 to 28 Mc./sec.). In this case the

## Table 2

RESULTS OF DIRECTION-FINDING OBSERVATIONS ON HESTON, WITH LOOP DIRECTION-FINDER

($\lambda = 9$ metres)

| Site | | From transmitter | | Error in observed bearing | Error in bearing of Teddington (from Table 1) |
|---|---|---|---|---|---|
| Ref. No. | Place | Distance | True bearing | | |
| | | miles | deg. | deg. | deg. |
| 41 | Lower Halliford .. .. .. | 6·25 | 19 | −1 | +4 |
| 31 | Bedfont .. .. .. .. | 3·5 | 21 | +1 | 0 |
| 24 | Hillingdon .. .. .. .. | 4·5 | 152 | −6 | −1 |
| 22 | Wembley .. .. .. .. | 6·25 | 215 | 0 | 0 |
| 33 | Osterley .. .. .. .. | 2·25 | 244 | 0 | −2 |
| 36 | Wimbledon Common .. .. | 7·75 | 294 | −2 | +1 |
| — | Queen Mary Reservoir .. .. | 5·75 | 24 | +6 | −1·5 |
| 42 | Ruxley Farm .. .. .. | 10·0 | 330 | 0 | 0 |
| 39 | Claygate .. .. .. .. | 9·75 | 345 | +7 | −6 |
| 3 | Tyler's Green .. .. .. | 16·5 | 129 | 0 | +2 |
| 6 | Bedmond .. .. .. .. | 21·2 | 177 | −1 | −1 |

## Table 3

SUMMARY OF OBSERVATIONS TAKEN ON TRANSMISSIONS FROM ALEXANDRA PALACE (SOUND AND VISION TRANSMITTERS)

Distance 20 miles, true bearing 67·5°

| Date | Transmitter | Wavelength | Observed bearings | | Deviations from mean bearing | |
|---|---|---|---|---|---|---|
| | | | Number | Error of mean | Extreme | Within 2° |
| | | metres | | deg. | deg. | % |
| Nov.–Dec., 1936 .. .. | Sound | 7·25 | 314 | +0·8 | 3½ | 96 |
| | Vision | 6·7 | 432 | −2·9 | 1½ | 100 |
| July–Aug., 1937 .. .. | Sound | 7·25 | 122 | −1·5 | 2 | 97 |
| | Vision | 6·7 | 144 | +2·8 | 1½ | 100 |

observed bearings sometimes varied as much as 15° in the course of a few minutes' observations, although in most cases the variations were not greater than 10°. No opportunity has yet occurred to make a systematic investigation of this matter.

### (e) Effect of Frequency on Observed Bearings.

During the course of the investigations it was noticed that in some cases the observed bearing was considerably in error although the site of the direction-finding set appeared to be satisfactory, while in other cases the observed bearing altered appreciably when the direction-finding set was moved only a few yards. The observations indicate the possibility of two or more waves arriving at the receiver and producing interference effects. In order to ascertain the validity of this deduction, some experiments were carried out in which the frequency of the transmitted waves was varied over a limited range while observations were made of the corresponding direction-finder bearings. The results of such experiments are given in Table 4.

**Table 4**

Effect of Varying Frequency of Transmitter on Error in Observed Bearings

| Site No. | Transmitter | Site of direction-finder | From transmitter | | Error in observed bearing on wavelengths of:— | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Distance | True bearing | 10·5 m. | 9·7 m. | 9·4 m. | 9·1 m. | 8·8 m. |
| | | | | | deg. | deg. | deg. | deg. | deg. |
| 1 | Portable . . . . | Dorney Common . . | 100 yards | Various | — | +1·3 | +0·5 | — | +0·5 |
| 2 | Portable . . . . | Dorney Common . . | 300 yards | 249° | — | −1·5 | −1 | — | −1·5 |
| 3 | Teddington . . | Dorney Common (open site) | 14·5 miles | 113° | −5 | −6 | −6 | — | −4 |
| 4 | Teddington . . | Dorney Common (under telephone wires) | 14·5 miles | 113° | −3 | −3 | −3 | — | −2 |
| 5 | Teddington . . | Dorney Common (near trees) | 14·5 miles | 113° | −3 | −5 | −10 | — | −10 |
| 6 | Portable . . | Queen Mary Reservoir | 0·5 mile | Various | −1 | — | −0·5 | 0 | +0·5 |
| 7 | Teddington (Main aerials up) | Queen Mary Reservoir | 5·25 miles | — | 0 | −2 | −1·5 | — | −2 |
| 8 | Teddington (Main aerials down) | Queen Mary Reservoir | 5·25 miles | — | +2·5 | +2·5 | +2·5 | — | +1·5 |

The observations shown in this Table were taken at two sites which appeared to be good as the result of the calibration with a portable transmitter at a radius of 50 yards or more. The first of these, Dorney Common, was clear of trees and other obstructions for a radius of at least 200 yards, and in one direction the site was clear to a distance of over 400 yards. In this direction Test 2 shows that the error in bearing on the local transmitter was 1·5° or less, and that it did not change appreciably with frequency. With the direction-finder in the same position the error on the Teddington transmissions was between − 4° and − 6°. As a result of moving the direction-finding set very nearly under some telephone wires, the error decreased slightly (Test 4), while when the direction-finding set was moved to within 20 yards of a mass of trees the error ranged from − 3° to − 10°, according to the frequency.

At the much superior direction-finding site at Queen Mary Reservoir, Test 9 shows that on a local calibration at a radius of about ½ mile across the water, the maximum error was 1° and did not vary with frequency within the errors of observation. On the transmissions from Teddington there was only a small variation with frequency, but it is seen that the error changed by some 3° or 4° when the main transmitting aerial at Teddington was lowered to the ground. This main aerial is of " T " type, 450 ft. long and 100 ft. high, and normally the ultra-short-wave transmitting aerial is slung about 50 ft. beneath the centre of this. There are also two other small aerials which were lowered for the above experiments. The angle subtended by the large aerial at the direction-finder at the reservoir is much less than 1°, so that it is not easy to understand the manner in which the aerial is directly responsible for the above change of 3° or 4°. It is possible that the position of the main aerial affects the distribution of radiation, and so indirectly is responsible for a change in the direction of some portion of the radiation arriving at the receiver. The results obtained on observations on the Heston transmitter at different places also indicated that the observed bearings were in some way affected by the direction in which the maximum radiation is projected from the transmitter.

These experiences indicated that conditions at the transmitting aerial could materially affect direction-finding bearings even at ranges of several miles, and some further experiments were conducted on this point as described in the next Section.

## (5) STUDY OF EFFECT OF WAVE POLARIZATION ON DIRECTION-FINDER

### (a) Effect of Horizontally Polarized Waves on a Loop Direction-Finder

In all the experiments described above, the transmitting aerial was vertical and thus the results obtained refer in all cases to the emission of vertically polarized waves. With a view to studying the effect of horizontal polarization, the aerial at the Teddington transmitter was supported in a horizontal position at a height of about 50 ft. above the ground. This aerial was connected to the transmitter by a tuned parallel-wire feeder as used in the previous experiments. The loop direction-finder was set up at the Queen Mary Reservoir site, where it was known that the error in bearing on Teddington, when using a vertical aerial, was 2·5° or less (Table 4). With the horizontal aerial the errors in bearings were in general greater than this; also they were subject to marked variation with orientation of the aerial and with alteration of the wavelength employed. The detailed results of these observations are given in Table 5, from which it will be seen that the errors observed range up to nearly 30°.

### Table 5

Effect of Orientation of Horizontal Portion of Transmitting Aerial on Observed Bearings with Loop Direction-Finder

Transmitter at Teddington; direction-finder at Queen Mary Reservoir; distance 5·25 miles; true bearing 84°.

| Direction of top of transmitting aerial | Error in bearing for nominal wavelengths of | | |
|---|---|---|---|
| | 9·8 m. | 9·2 m. | 8·8 m. |
| deg. | deg. | deg. | deg. |
| 17 | +17 | +11·5 | +13 |
| 63 | + 5 | + 3 | + 6 |
| 82 | + 3·5 | − 1·5 | + 1 |
| 119 | 0 | +11 | − 5 |
| 143 | +17 | +20 | − 6 |
| 197 | +10 | +15·5 | −28·5 |
| 243 | + 7 | + 7 | +16 |
| 262 | + 3 | + 2 | +11 |
| 299 | −29 | −11·5 | − 7 |
| 323 | −16 | −13 | +14 |

In general the errors are reduced to a minimum value when the aerial is in the line from transmitter to receiver, although this is sometimes masked by the large change in bearing with alteration in azimuth and wavelength. When the horizontal aerial was used, the signals were of adequate strength for direction-finding purposes, although the quality of the signal minima varied with the actual orientation of the transmitting aerial. In certain positions of the latter the bearing changed rapidly with orientation, thus sometimes making an observation difficult when the transmitting aerial was swinging slightly in the wind.

Tests made with the aerial disconnected showed that the vertical feeder was radiating to some extent, so that throughout the experiments a certain amount of vertically polarized radiation was superimposed on the horizontally polarized waves. For this reason the actual errors observed were to some extent dependent upon the adjustments of the feeder, and somewhat larger errors, up to 50°, were experienced at the same receiving site when emission of vertically polarized radiation was minimized by correctly matching, instead of using the feeder in the tuned condition.

Similar large errors in bearing on the Teddington transmitter were obtained at the Radio Research Station, Slough. In order to see whether the bearing changed with time for constant conditions at the transmitting end, observations were carried out during day and evening periods on five separate days spread over two weeks. The transmitting aerial was maintained in a fixed position, with the horizontal portion at approximately 90° to the direction of the receiver. Under these conditions the bearings were very flat; the mean error in the observed bearing was 78° and this was subject to a variation from one period to the next of as much as 21° on the same day. Although the transmitter was operating on the nominal wavelength of 9·3 metres throughout, the change in bearing may have been partially due to change in frequency of the transmitter, as special tests showed that changing the frequency by about 80 kc./sec. altered the observed bearing by some 10°. Furthermore, although the horizontal aerial was held during the tests as rigidly as circumstances permitted, some rotation of the aerial may have occurred and this would cause a change in error. It seems, however, that these variable conditions at the transmitting end cannot entirely account for the variations observed.

### (b) Experiments with a Spaced-Aerial Direction-Finder

The effects described in the previous Section may have been due solely to the action of horizontally polarized waves on the loop direction-finder employed. It must be noted, however, that similar results would be obtained if the waves arriving at the receiver had a vertically polarized component, and if the direction of arrival showed a considerable lateral deviation from the line joining transmitter and receiver. It was in order to decide between these possibilities that the Adcock spaced-aerial direction-finder described in Section 2(*b*) was developed, since this instrument is insensitive to the action of horizontally polarized waves. When this instrument was completed, therefore, further experiments were carried out at the Queen Mary Reservoir site, in which a direct comparison was made of the bearings observed on both the loop and spaced-aerial direction-finder, while the horizontal aerial at the Teddington transmitter was rotated to various positions. The result of one of these tests made on a wavelength of 8·8 metres (frequency 34 Mc./sec.) is shown in Fig. 4. It is seen that while an error of nearly 60° was obtained with the loop when the transmitting aerial was broadside on to the receiver, the maximum error experienced on the Adcock set was only 6°. On a wavelength of 9·8 metres (frequency 30·5 Mc./sec.) the bearings with the Adcock set showed a total variation of about 4°, while the loop set gave errors up to about 30°.

It would appear from these experiments that although there is a vertically polarized component in the waves arriving at the receiver, this portion of the waves is not subject to any great lateral deviation. Such a vertically polarized component may have been present in the radiation leaving the transmitter, or it may have been produced indirectly by re-radiation from a secondary source. Although neither the above nor any other experiments so far conducted give proof that the horizontally polarized waves were confined to the vertical plane through transmitter and receiver, it seems reasonable to presume that these waves were subject to no greater lateral deviation than the vertically polarized waves. The large errors described in the previous Section must therefore apparently be explained in terms of horizontally polarized waves, the direction of travel of which is confined to the great-circle plane through transmitter and receiver.

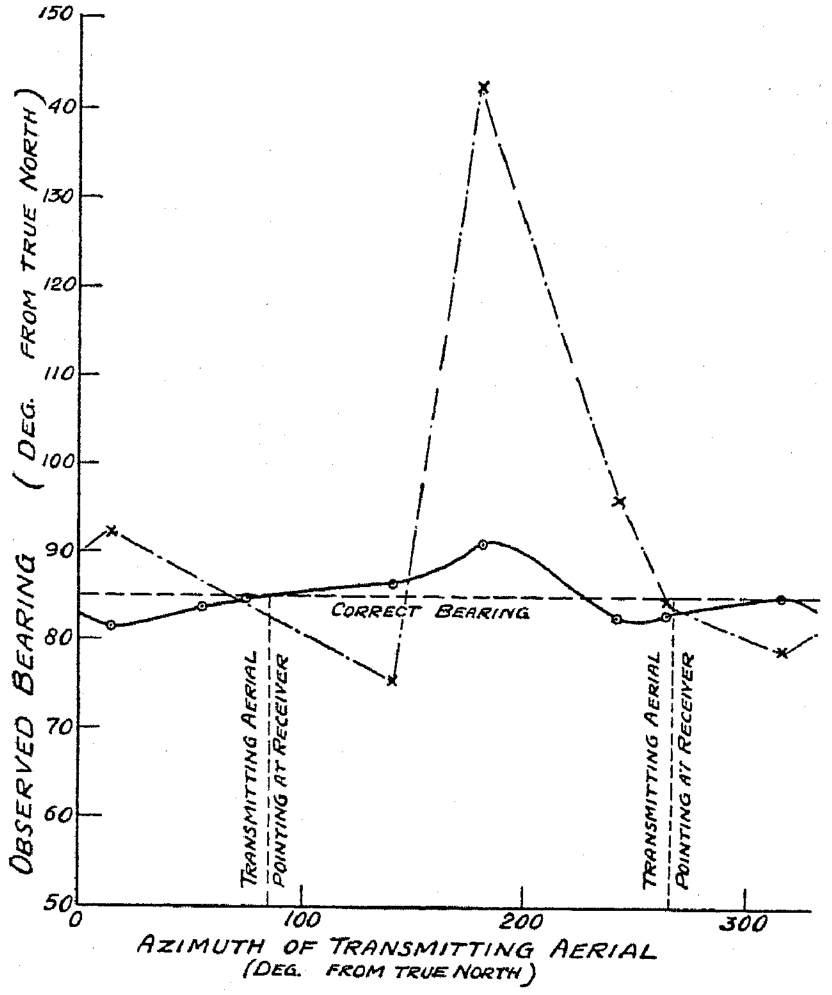

**Fig. 4.**—Comparison of the bearings obtained at Queen Mary Reservoir with the loop and Adcock receivers respectively, on the emissions from a matched horizontal dipole at Teddington. Working wavelength 8·8 metres (34 Mc./sec.) approx.

×- — — - — × Loop bearings.
⊙———— ⊙ Adcock bearings.

### (c) Further Investigation of the Action of the Loop Direction-Finder with Horizontally Polarized Waves

In seeking an explanation of the abnormally large errors experienced with the loop direction-finder, consideration was first given to the possibility of the loop receiver being susceptible to the action of horizontally polarized waves propagated horizontally. Such an effect might result, for example, from a serious lack of symmetry of the loop about a horizontal axis, so that the electromotive forces induced in the top and bottom sides of the loop produced a residual signal component in the receiver. In order to balance out such a component it would be necessary to rotate the loop away from the normal minimum position until an equal and opposite component was obtained from the e.m.f.'s in the vertical sides of the loop. The following tests appear to prove that the loop direction-finder employed was free from such a characteristic.

In the first place, when loops of different sizes and shapes were used on the direction-finder the error remained sensibly unaltered. Thus it appears that the error is not dependent upon a critical condition of lack of horizontal symmetry of the loops. Secondly, experiments carried out with a portable transmitter placed on the reservoir site at a distance of $\frac{1}{4}$ mile showed no large errors in bearing when a horizontal transmitting aerial was rotated in a horizontal plane. Furthermore, the observed bearing did not change appreciably as the transmitter was raised from ground-level to heights up to about 10 ft.

Thus we see that with a local transmitter presumably emitting horizontally polarized waves there is no appreciable error in observed bearing, while with a similar transmitter at a distance of over 5 miles large errors occur. The absence of error in the local case may have been due either to the small value of the horizontally polarized field arriving at the receiver owing to ground attenuation, or to the fact that the loop is not susceptible to horizontally polarized waves propagated horizontally. If the first explanation is true, it is to be expected that the horizontally polarized field from the Teddington transmitter would also be small. The presence of such large errors in this case seems, therefore, to indicate the presence of horizontally polarized waves arriving at an appreciable angle to the horizontal after being deflected in some part of the atmosphere between the transmitter and receiver.

Such downcoming radiation is of low intensity, since its presence only produces a serious effect when steps are taken to exclude the emission of vertically polarized radiation at the transmitter. At the shorter distance of $\frac{1}{4}$ mile at which the local test with the portable transmitter was made, the downcoming radiation was negligible compared with the residual vertically-polarized waves propagated along the ground. Furthermore, the results given in Table 4 showed that no large variations in bearing were observed on transmissions from stations, such as Alexandra Palace, employing vertical aerials. It thus appears that if any of such radiation is deflected from atmospheric regions in which a rotation of the plane of polarization might be produced, such horizontal downcoming radiation is weak compared with the vertically polarized direct radiation from transmitter to receiver.

Adequate opportunity has not yet occurred to study this problem in more detail, but a simple attempt has been made to measure the angle of incidence by comparing the signal intensity received on a loop and on an aerial in the manner employed some years ago on medium waves.* Experiments of this nature have been carried out at Queen Mary Reservoir, using vertically polarized radiation from Teddington. While the results obtained tended to indicate the presence of a downcoming component, certain difficulties were encountered which do not permit of a numerical interpretation of the results, e.g. fluctuations of signal strength due to the presence of aeroplanes in the neighbourhood.

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* See Reference (4).

## (6) CONCLUSIONS AND FURTHER WORK

The experimental investigations described in this paper show that it is a practical proposition to use simple radio direction-finders with a rotating aerial system for wavelengths between 6 and 10 metres. While care is needed in the design and construction of the instruments to secure electrical symmetry and stability of operation, no difficulties are encountered other than those normally associated with ultra-short-wave technique. The rotating-loop pattern forms a compact, easily-transportable instrument suitable for much field investigation work. An inherent instrumental accuracy of such a direction-finder of from 1° to 2° can be obtained, but some difficulty is experienced in finding a suitable site on the ground on which such an accuracy can be utilized. Obstacles such as buildings, trees, and aerials, within a distance of about 100 yards of the direction-finder produce appreciable errors.

When a loop direction-finder is used on the ground to observe the bearing of ground transmitting stations using vertical aerial systems at distances up to 22 miles, the errors may be as great as 8°, although in the majority of cases the errors are less than 2°. The changes in bearing from day to day do not exceed about 2°. The direction-finders developed have been shown to be capable, under certain conditions, of giving bearings on commercial transmitting stations some 3 000 miles away. In this case the observed bearings are subjected to much larger variations than those given above.

When a horizontal aerial is employed at the transmitting station, serious errors are experienced with the loop direction-finder except at very short distances from the transmitter. These errors are attributed to the reception of horizontally-polarized waves, and they may be avoided by using a spaced-aerial or Adcock direction-finder, an experimental model of which is described in the paper.

The future work on this subject will be devoted to the development of an improved and more sensitive form of this Adcock direction-finder and to an investigation of its performance under a wide variety of conditions.

## (7) ACKNOWLEDGMENTS

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# THE MEASUREMENT OF THE LATERAL DEVIATION OF RADIO WAVES BY MEANS OF A SPACED-LOOP DIRECTION-FINDER*

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## SUMMARY

The paper gives an account of some systematic measurements of the lateral deviations of wireless waves received at Slough from various short-wave transmitters over a period of several months. The measurements were made by means of a spaced-aerial direction-finder of the four fixed-loop type, and the first part of the paper is devoted to a description of the apparatus, together with an account of its performance in respect of instrumental accuracy, pick-up factor, and other characteristics.

The second part of the paper describes the measurements themselves, which were made photographically from a cathode-ray oscillographic goniometer. Some of the observations were made on pulse transmissions from Nauen ($\lambda = 30$ m.) and Dorchester ($\lambda = 37\cdot3$ m.), and the remainder on continuous-wave transmissions from Zeesen ($\lambda = 31\cdot4$ m.), Prague ($\lambda = 44\cdot6$ m.), and various American stations.

The results obtained show that the variations in bearings which arise from instrumental causes do not exceed about 2°, and are usually of the order of 1°. The records demonstrate that lateral deviation occurs for the above cases to an extent which depends on the range and type of the reflected waves observed. Deviations of 10° to 20° were recorded from the more distant stations, while in the case of Dorchester, distant 160 kilometres, the deviations were as much as 50° on occasions. From a study of these observations it is concluded that the effective points of reflection at the ionosphere may be as much as 50–100 km. out of the great-circle path.

## (1) INTRODUCTION

The measurement of the azimuthal departure of radio waves from the great-circle path has been one of the main objectives of the direction-finding research at the Radio Research Station, Slough, for many years past, for it is clear that the extent and accuracy of such knowledge can alone determine to within what limits of accuracy it is, or will be, possible to carry out practical direction-finding. The early stages of this investigation gave information on the subject which has been of much value. Owing, however, to the existence of residual instrumental error in the apparatus then used it was not possible, from these early experiments, to obtain any positive measurements of lateral deviation, although it was possible to assign a limiting value to its magnitude in many cases.

This work, which has been described in an earlier publication,† opened up the possibility of developing improved practical direction-finding apparatus, and, since without such apparatus further work on lateral deviation could not be carried out, it became desirable from two points of view to devote time to this development work.†

As this work has progressed, however, it has been becoming increasingly apparent that the degree of improvement achieved thereby in direction-finding design is not accompanied by a corresponding degree of immunity from error in practice.‡ This conclusion is supported by others who have developed the Adcock system. Eckersley,§ experimenting with short waves, demonstrated the existence of extensive scattering in the skip distance, coupled with the very large deviation of waves arriving from transmitters at distances comparable with the earth's semi-circumference or for which the great-circle path passes near the North Pole.

Further evidence of scattering has been obtained by Ratcliffe and Pawsey,|| employing a method based on the simultaneous measurement of fading at two adjacent sites and being thus independent of direction-finding apparatus.

It has, therefore, now become well established that a finite amount of lateral deviation exists which sets a definite and final limit to the accuracy to which a direction-finder can ever attain.

This paper describes the first stages of an investigation to determine these limits, dealing first with the experimental method adopted, and secondly with an account of the results of the first series of tests made with the apparatus.

## (2) THE SPACED-LOOP DIRECTION-FINDER

### (a) General Arrangements

In order to carry out this research an apparatus of the direction-finding type was required which would possess an instrumental accuracy—particularly as regards polarization and other variable errors—considerably higher than that of any which had hitherto been made. It was also necessary that this apparatus should be capable of giving almost instantaneous bearings, preferably with automatic recording thereof, and finally that it should be capable of working on pulse transmissions. The last two considerations could only be satisfied by a cathode-ray photographic recording technique. As regards the first, it had been found from a series of experiments that the complete elimination of polarization error with any form of spaced vertical-aerial system was very difficult, owing to the fundamental asymmetry introduced by the position of the earth's surface relative to the feeders or transmission lines connecting the aerials

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* This paper will be included in a Symposium of Direction-Finding Papers to be read before the Wireless Section during the 1938–39 Session.

† See Bibliography, (1).

† See Bibliography, (2) and (3).

‡ *Ibid.*, (4).

§ *Ibid.*, (5), (6), and (7).

|| *Ibid.*, (11) and (12).

to the central receiver. The substitution of loop aerials for vertical aerials, however, provides a simple solution to this problem of balancing, as can be appreciated by a consideration of Fig. 1, which contrasts the essential symmetry of a loop with respect to its feeder with the essential asymmetry of various forms of vertical aerials in this respect. The possibility of entirely screening by pipe enclosure a loop aerial from the effect of static coupling (or of accumulated charges), on the other hand, tends to render accurate balancing unnecessary, so that the combination of these two factors makes it easily possible to produce a spaced-loop arrangement with negligible polarization error.

There are certain disadvantages introduced by such a substitution which render unsuitable the application of the spaced-loop system to practical direction-finding. It is not relevant to the subject in hand to discuss these here, but their nature is such that they are not at all serious when the apparatus is used for research only.

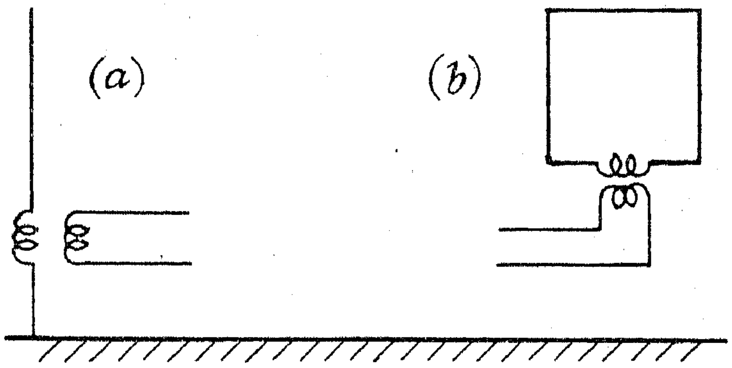

**Fig. 1.**—Asymmetrical and symmetrical coupling.

(*a*) Feeder coupled to aerial (asymmetry).
(*b*) Feeder coupled to loop (symmetry).

A spaced-loop direction-finder may be of the four-aerial fixed type or two-aerial rotating type. The latter type has been employed by Eckersley* for work of a similar nature, the results of which are referred to later in the paper, and it has also been developed at the Radio Research Station.† For the experiments here described, however, the four-loop system was employed and it is this which will now be described.

Four loops are arranged at the corners of a square of 12 metres diagonal, as shown in Fig. 2(*b*), and the diagonally opposite loops are coupled together through transformers and transmission lines so as to be in opposition. The system thus corresponds to an Adcock direction-finder in which loops take the place of vertical aerials. Each loop can be rotated about its own vertical axis, and all four are set to point in the same direction. Since the system is only used for downcoming waves of complex and variable polarization it will be clear that, as far as the "wanted" pick-up of the system is concerned, it does not matter which direction is chosen, provided each loop is accurately aligned thereon, for then the same field component will act on each loop and the output from each pair will depend solely on the phase differences. To lessen the risk of "unwanted" pick-up resulting from currents induced in the feeders, however, one of the two arrangements in which the loops all make angles of 45° with the feeders has been chosen for all observations made up to the present, as this arrangement is not only the most symmetrical but also the most suitable for the process of lining-up described below.

It is clear that a spaced-loop system, even when the loops are so oriented as to be acted upon wholly or partly by horizontally polarized waves, will be immune from polarization error, provided that the loops are accurately aligned in parallel planes and that the feeders connecting them do not pick up.

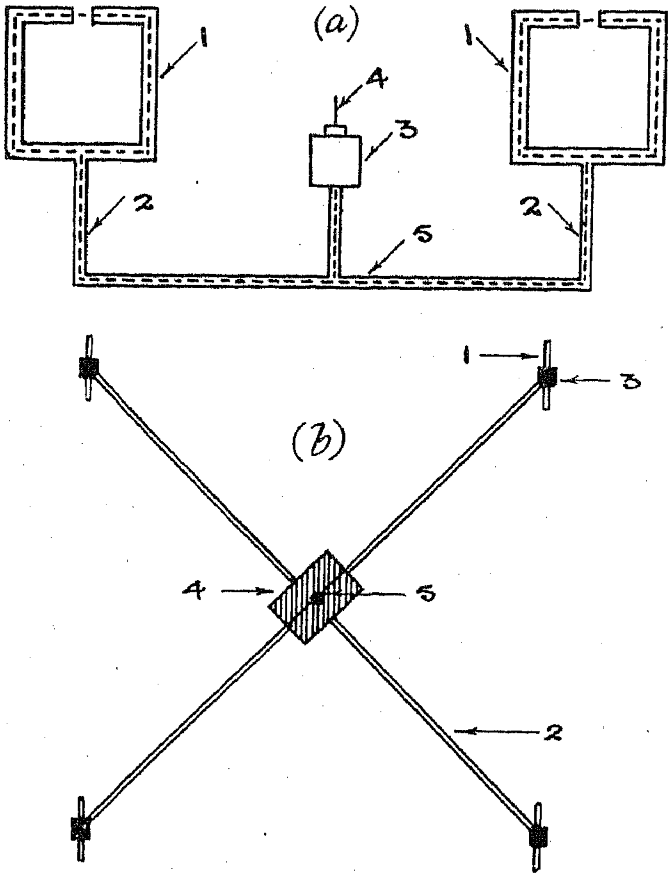

**Fig. 2.**—General arrangement.

(*a*) Elevation.

(1) Screened loop.
(2) Feeder.
(3) Central screening cage.
(4) Lining-up transmitter and vertical aerial.
(5) Feeder screen.

For simplicity the loops are here shown in line with feeders; actually they are at 45° as shown in (*b*).]

(*b*) Plan.

(1) Screened loop (1 m. square).
(2) Screened feeder.
(3) Screening box containing tuning condenser and matched transformers.
(4) Cage containing screened cathode-ray receiver.
(5) Central radiating aerial for lining-up.

## (b) Constructional Details

Each loop consists of four separate turns of insulated wire which, by series-parallel switching, can be made to form from one to four turns. These wires are enclosed in a rigid tubular screen formed of $1\frac{1}{2}$ in. diameter aluminium piping jointed together at the corners and having an insulated gap at the centre of the top member. The wire is so arranged on spacers that it occupies approximately the central axis of the tube. The method of tuning the loops and of coupling to the feeders is shown in Fig. 3. The tuning condenser and transformer are housed in a screening box fixed at the centre of the frame, the leads from the loop being brought into this box through a metal pipe fixed at the centre of the bottom limb of the frame. The construction is illustrated in Fig. 4. It was ascertained by means of special tests that the pick-up factor of the loop was not reduced by the screening pipe surrounding it.

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* See Bibliography, (13).

† *Ibid.*, (14).

The transformers are wound on ribbed formers and are mounted inside a copper pot. The primary is in parallel with the loop, and the windings are so designed that the circuits are matched at wavelengths of 40 and 100 metres (frequencies of 7·5 and 3 Mc./sec.), which gives approximate matching over the wavelength range 30–100 metres. There is an electrostatic screen between the windings of the transformer. The screened twin feeders between the loops and the central hut are of ordinary twin lead-covered electric lighting cable. They are buried to a depth of a few inches, partly to bring them in close proximity to the surface of the earth and partly to protect them from mechanical damage, and are brought up along the outside of the central axis of the loop to the screening box, to which the screen is bonded.

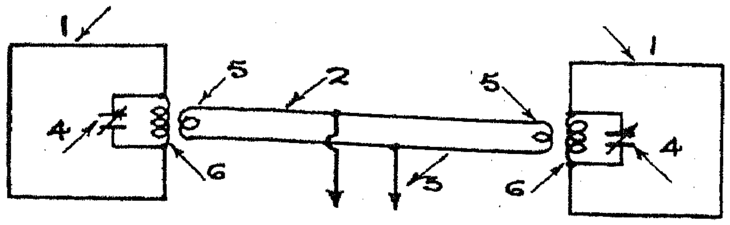

**Fig. 3.**—Circuit diagram.

(1) Loop.
(2) Feeder.
(3) Input to amplifier.
(4) Tuning condenser.
(5) Transformer secondary.
(6) Transformer primary.

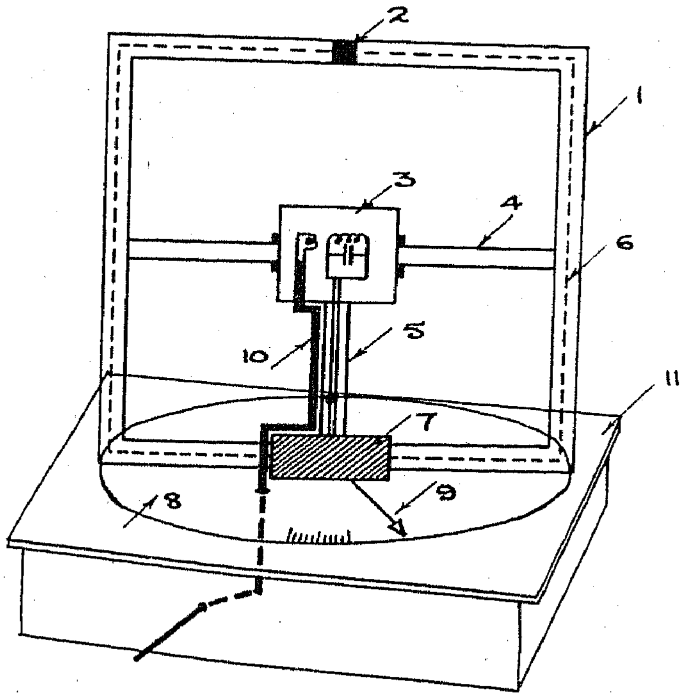

**Fig. 4.**—Loop details.

(1) Screening pipe containing loop and rotating about central vertical axis.
(2) Insulating gap in screen.
(3) Metal box containing tuning condenser and feeder transformer.
(4) Insulating supports.
(5) Metal tube containing leads from loop to (3).
(6) Four single loops.
(7) Series-parallel connection box.
(8) Scale.
(9) Pointer.
(10) Screened feeder to cathode-ray apparatus.
(11) Wooden supporting platform.

At the centre of the system inside the hut is fixed a screening cage approximately 6 ft. cube, consisting of a wooden framework covered with wire netting of 1 in. mesh to which the transmission lines are bonded at their point of entry at the centre point of its floor. The four feeders then pass into a small screening box (called the "mixing" box) where they are connected to a series of switches and variable resistances as shown in Fig. 5. By means of the switch the two incoming feeders constituting a pair are joined together (in conjunction or opposition as desired) and to the common input feeder leading to one of the amplifiers, while the resistance can be made to vary the relative output of the loops in order to obtain a perfect balance. This enables the process known as lining-up to be carried out in the manner described in another Section.

The two feeder-pairs from the "mixing" box pass directly to the input transformers of the pair of matched amplifiers whose outputs are connected to the plates of the cathode-ray tube. This combination of two amplifiers and cathode-ray tube constitutes a cathode-ray comparator. Essentially similar apparatus, the working thereof, and the methods of photographic recording adopted, have already been described in other publications.*

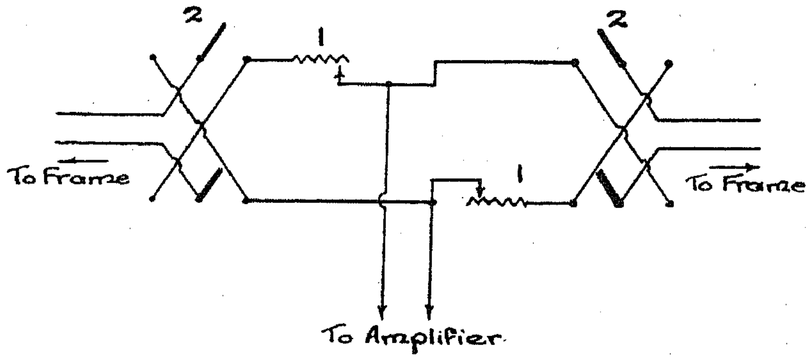

**Fig. 5.**—Circuit diagram of mixing box.

(1) Variable resistance.
(2) Reversing switch.

### (c) Lining-up the System

It will be clear from the principle of action of the direction-finder that it must be assumed that, provided each loop is acted upon by an equal and co-phasal field, it must with its associated feeder contribute exactly co-phasal and equal e.m.f.'s to the central points of the system, and that on passing through the two amplifiers the signal from each pair is subjected to the same gain and phase-change. To attain this end all corresponding parts of the system are made as nearly equal as possible, but in practice it is not possible to attain in this way the degree of equality necessary to ensure the desired overall accuracy of the instrument, and certain controllable adjustments are provided for the final matching. To make these adjustments it is necessary to provide a local source of energy which, acting equally on all four aerials, enables the adjustments to be made which give equal output. This is provided by installing a transmitter with a short vertical aerial on the top of the screening cage at its geometric centre, and tuning it to radiate at the frequency of the signal to be received [see Fig. 2(*a*)]. With this switched on, fine tuning adjustments can be made in each loop, together with further resistance adjustments and the usual lining-up process, stage by stage, of the amplifiers. In this way the two loops of a pair can be balanced against one another to give the required degree of equality, and the two pairs can be equalized by appropriate adjustments to the amplifiers. The degree of equality required for a given accuracy, and details of how it is attained, are given in another Section.

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* See Bibliography, (15).

### (d) Pick-up Factor

The " pick-up factor " of an aerial system is defined as the quotient of the potential difference across the first grid circuit of the amplifier and the field strength existing at the aerials. It is a measure of the sensitivity of the aerial system, and is determined by the size of the loops, the spacing of the aerials, the magnification factor of the tuned circuits, and the efficiency of the coupling transformers. It is found that the attenuation of the transmission lines themselves is negligible.

The pick-up factor of the system has been measured by experiment and found to have the values shown in Table 1, for a wave travelling along the ground.

**Table 1**

| $\lambda$ | Frequency | Pick-up factor |
|---|---|---|
| metres | Mc./sec. | metres |
| 100 | 3·0 | 1·0 |
| 75 | 4·0 | 1·5 |
| 45 | 6·7 | 3·0 |
| 33 | 9·0 | 6·0 |

These values were found to be in close agreement with those calculated from the known dimensions and characteristics of the various parts of the aerial system. For frequencies above 4·0 Mc./sec. the loops were used with only one turn on each. It might appear that the pick-up factor on any frequency could be increased considerably by increasing the number of turns used on each loop. Theoretical considerations, however, have shown that this is not so, and experiment confirms these conclusions. Briefly, the reason for this is that while, by increasing the number of turns on any loop, the e.m.f. picked up by the loop is increased proportionally, the inductance of the loop and its high-frequency resistance are both increased much more rapidly, more or less proportionally with the square of the number of turns. This rapid increase of the resistance of the loop has exactly the effect of cancelling the increase in pick-up factor brought about by the increased e.m.f. received by the loop itself.

Two turns were used on the loops at the lower frequencies, chiefly to achieve a high inductance in the loop circuits and thus to facilitate tuning of the loops.

It should be borne in mind that since the pick-up factor of the system depends on the angular spacing of the aerials, it is a function of the angle of incidence of the wave, being in fact proportional to the sine of the angle of incidence. Thus the pick-up factor becomes much smaller for steeply downcoming waves.

## (3) INSTRUMENTAL ERROR

Spaced-aerial direction-finders are liable to instrumental errors of several different types. It is proposed in this Section to deal with such errors as apply to the spaced-loop direction-finder, and to indicate the performance of the instrument in respect of such errors and to show to what extent the errors which remain in the instrument affect the accuracy of the results obtained.

### (a) Polarization Errors

It is important to consider to what extent the spaced-loop direction-finder is liable to polarization errors, i.e. errors whose magnitude and sign depend on the state of polarization of the received waves. With this direction-finder it is found the polarization errors may arise from two causes.

Firstly, energy may be picked up by the outer screen of the horizontal feeders running between the aerials and the receiver, and transferred to the aerials by radiation or by electromagnetic coupling between the aerials and the outer screens of the feeders. It is clear that, in the presence of downcoming waves, if the loop aerials are so oriented as to receive energy either from the vertically polarized component of the wave only or partly from the vertically polarized component and partly from the horizontally polarized component, such transfer of energy from the screening cover of the transmission lines to the aerials would give rise to polarization errors, since the energy received by the screening cover of the lines is derived from the horizontally polarized component of the wave.

Secondly, if the planes of all four loops are not exactly parallel, polarization errors will again arise, since the loops will not be all equally receptive to the vertically or horizontally polarized component of the received waves.

Returning to the case of the first type of polarization error mentioned above, it is evident that the magnitude of the error involved will depend on several factors, viz. the strength of the horizontal electric field at the transmission lines, the " wanted " pick-up of the aerials themselves, and some factor in the nature of a coupling coefficient representing the ratio between the e.m.f.'s induced in the loops by currents flowing along the outer screen of the transmission lines and the e.m.f. picked up by that screen on account of the horizontal electric field of the wave. It may be noted that, in the special case in which the loops are so oriented as to be receptive only to the horizontally polarized component of the received waves, the error introduced is not strictly a polarization error, since both the " wanted " and " unwanted " pick-up are due to the same component of the wave. The error, however, does vary with angle of incidence in this case and is therefore still undesirable.

For simplicity of treatment, however, we shall deal with the case in which the planes of the loops are parallel to the vertical plane of arrival of the wave, so that the loops are receptive only to the vertically polarized component of the wave, and the " unwanted " pick-up is due to the pick-up of the horizontally polarized component by the outer screen of the transmission lines. The polarization error resulting when the planes of the loops make any other angle with direction of arrival of the wave can be shown to be of the same order of magnitude as for the case here considered. We shall further suppose that the wave arrives along the direction of the line joining one pair of aerials and use the following notation:—

$A$ = area of one loop in square metres.
$n$ = number of turns on loop.
$d$ = distance between loops.
$\lambda$ = wavelength.
$E_V$ = intensity of vertically polarized component in wave.
$E_H$ = intensity of horizontally polarized component in wave.
$Y$ = intensity of horizontal electric field at the outer casing of the transmission lines.
$K$ = number representing the "coupling coefficient" between outer screen of transmission lines and loops referred to above.
$\rho_v$ = reflection coefficient of earth for vertically polarized waves.
$\rho_h$ = reflection coefficient of earth for horizontally polarized waves.
$e$ = error.
$\theta$ = angle of incidence of wave.
$\alpha, \beta$ = angular misalignment of one loop in horizontal and vertical planes.

We have the following:—

$$\text{“Wanted” pick-up} = E_V(1+\rho_v)\frac{2\pi An}{\lambda}\cdot\frac{2\pi d}{\lambda}\sin\theta$$

$$\text{“Unwanted” pick-up} = YdK.$$

Then the maximum value of $e$, obtained by assuming the "wanted" and the "unwanted" pick-up are in phase, is given by

$$\tan e = \frac{YK\lambda^2}{E_V(1+\rho_v)4\pi^2 An\sin\theta}$$

The coupling coefficient $K$ has been measured experimentally in the following manner. The transmission lines from one pair of aerials were removed from their trenches under the ground and suspended close to the ground but not touching it. These transmission lines were each encircled by a toroidal inductance, the two toroids being fed from a common oscillator and so wound as to induce currents in the same direction in the two halves of the transmission line. Knowing the current in the toroids (measured by a thermo-junction) and the coefficient of mutual induction between the toroids and the lines, the e.m.f. induced in these lines can be calculated. The e.m.f. induced in the loops by the currents flowing in the lines is observed by noting the signal produced in the receiver, the two loops being connected together in the same sense as when used as part of the direction-finder. By injecting a known e.m.f. directly into the loops and observing the value of e.m.f. required to produce the same signal in the receiver as that produced by the currents induced in the outer screen of the transmission lines, the factor $K$ can be found. The direct injection into the loops can be accomplished in several ways. One method is to insert a small resistance at the centre of the loop and to connect the output from a signal generator across the resistance. Another method, suitable in cases when it is not possible to reach the centre of the loops, is to radiate a signal from a small oscillator the field strength of which at the loop can be measured or calculated.

The value found for $K$, by experiment, was of the order of $10^{-4}$ for frequencies of 3 and 8 Mc./sec. This means that the error for a "standard wave" (angle of incidence 45° and equal vertically and horizontally polarized components) is not greater than 0·1° at these frequencies, and while errors of greater magnitude can occur they can only do so at the expense of signal strength. Thus errors of the order of 1° would occur only when the amplitude of the signal fell to about one-tenth of its normal value, that is to say when the "wanted" component in the wave became very much smaller than the "unwanted" component. As the state of polarization of ionospheric waves is constantly changing, provided bearings are only noted when the amplitude of signals is near the maximum the polarization error from this source is negligible.

As far as the second type of polarization error is concerned, namely that arising from the loops not being all strictly parallel, we may again consider a simple example to examine to what degree of accuracy the loops must be set up. The apparatus is supposed to be disposed in the same manner as described above, with the planes of the aerials parallel to the vertical plane of arrival of the wave and the line joining one pair of aerials pointing in the direction of arrival of the wave. We shall suppose that three of the loops are exactly parallel to the plane of arrival of the wave, while the fourth, which we shall assume to be one of the pair on the line at right angles to the direction of arrival of the wave, makes a small angle $\alpha$ with that direction.

The "wanted" pick-up for the pair of loops lying along the direction of arrival of the wave will be, as before,

$$E_V(1+\rho_v)\frac{2\pi An}{\lambda}\cdot\frac{2\pi d}{\lambda}\sin\theta$$

In the case of the other pair of loops, lying on a line perpendicular to the direction of arrival of the wave, the pick-up due to the vertically polarized component is zero, since it is equal (approximately) for the two loops which are in opposition, but since one loop makes a small angle $\alpha$ with the direction of arrival of the wave there is a residual "unwanted" pick-up due to the horizontally polarized component of amount

$$E_H(1+\rho_h)\cos\theta\frac{2\pi An\sin\alpha}{\lambda}$$

The maximum error, assuming the "wanted" and "unwanted" pick-up to be in phase, is given by

$$\tan e = \frac{E_H}{E_V}\cdot\frac{(1+\rho_h)}{(1+\rho_v)}\cdot\frac{\lambda}{2\pi d}\cot\theta\sin\alpha$$

For a standard wave $E_H = E_V$ and we may write $\rho_h = \rho_v = 1$ (approximately), from which it appears that $e$ is of the same order of magnitude as $\alpha$ for a standard wave. Again, the error may be many times as big, but only when the ratio $E_H/E_V$ becomes large or $\theta$ so small that the "wanted" pick-up becomes very small also. Thus, except for very small angles of incidence, if observations are confined to those made when the amplitude is large, the polarization errors due to this cause will be of the same order of magnitude as $\alpha$. It can be shown that in the more complex case, in which the planes of the loops do not coincide with the plane of arrival of the wave and thus each receives signals from both vertically and horizontally polarized components, the result is broadly the same. It therefore appears that misalignment of any loop from the plane parallel to the others gives rise to polarization errors of the same order of magnitude as the angular misalignment, except when the signal strength falls to small dimensions on account of changes in the state of polarization of the wave.

What has been said above refers to azimuthal misalignments, but it is also true that if all the loops do not make equal angles with the vertical, polarization errors will arise. Calculations show that if one loop is out of the vertical by an angle $\beta$ the standard-wave error is of the order of $\beta/|\sqrt{\kappa'}|$, where $\kappa'$ is the complex dielectric constant of the ground.

In practice, it has been found that by making use of sights attached to the frames of the loops and training the loops on distant posts spaced apart the same distance as the loops, the error in angular alignment of the loops has been kept to a value less than $0\cdot2°$.

As an indication that polarization errors in this instrument are as small as the foregoing would appear to indicate, it may be noted that in the course of making observations on the direction of arrival of pulses from Dorchester (distant 160 km. from Slough) using a wavelength of $37\cdot3$ metres, occasions have occurred when the two components of the first-order $F_2$ echo, separated in time on account of magneto-ionic splitting, have been observed to give exactly the same bearing. These two components are elliptically polarized, having opposite senses of rotation of the ellipse in the two cases. Polarization errors would thus have opposite signs in the two cases.

Occasions have also been observed when the two components did not give the same bearing, but these appear to be genuine cases of difference in the direction of arrival of the two. If such difference in apparent direction were due to polarization error it would always exist. This, however, is not in agreement with observed facts.

### (b) Other Forms of Instrumental Error

Of the other forms of instrumental error, the next in importance after polarization errors are those which arise from lack of balance between the aerials. Such errors will arise, for instance, if, when equal and co-phasal fields exist at the aerials, the signals produced at the receiver by all four aerials are not equal.

The theory of this type of error has been discussed in another paper,* and all that requires to be said here is that the magnitude of the error depends on the ratio of the signal obtained from a pair of aerials connected in opposition to the signal obtained from the pair connected in conjunction when equal and co-phasal fields exist at the two aerials of the pair. Now, as mentioned elsewhere in the paper, in order to achieve a sufficiently high value of pick-up factor the aerials used in this system were tuned to resonance. As stated in an earlier Section, it is found, in practice, that if the four aerials are separately tuned to resonance the phases and amplitudes of the currents in them are not in general sufficiently equal, and some methods of fine balancing are required. The method finally adopted makes use of a small oscillator with a short vertical aerial located exactly at the centre of the system. The four aerials are set up in such a manner that their planes are parallel and at 45° to the line joining their centres to the centre of the system. It is assumed that the oscillator will produce equal and co-phasal fields at all four aerials. After tuning all four aerials to resonance (i.e. until the amplitude of signal received from the central oscillator is a maximum) the opposite aerials in each pair are opposed. Ideally, the signal obtained from a pair of aerials should then be zero. In general it is found not to be so. By making slight adjustments to a trimming condenser connected in parallel with the main tuning condenser of one aerial of each pair and to a small variable resistance connected in series with the feeder from that aerial, the residual signal from a pair of loops connected in opposition can be controlled until its amplitude in comparison with the signal obtained either from one loop alone or from the pair connected so as to be in conjunction, is sufficiently small for the degree of accuracy required. Having " balanced " each pair of aerials in this way the aerials in each pair are connected so as to be in conjunction with respect to the signal from the central oscillator. They are then in opposition with respect to a distant oscillator, and are thus in the operating condition. With the members of each pair connected in conjunction, the signals induced in each pair ought to be equal and co-phasal, and these signals, after passing through a pair of identical amplifiers to the orthogonal pairs of deflecting plates of a cathode-ray oscillograph, ought to produce on the fluorescent screen of the oscillograph a line at 45° to the axes of deflection of the tube. The amplifiers themselves are balanced stage by stage as described in other publications,* so that when equal and co-phasal signals are applied to the input of each the trace on the oscillograph screen is a line at 45° to the axis.

It is found that when the two pairs of aerials are connected, one pair to each amplifier, with the aerials in conjunction with respect to the central oscillator, the figure on the oscillograph screen when signals are received from the oscillator is usually a thin ellipse whose major axis is nearly but not quite at 45° to the axis of the tube. This indicates that, while the members of each pair have been " balanced " with respect to each other, the one pair is not " balanced " with respect to the other pair. The necessary balance can, however, be easily obtained. All that it is necessary to do is to make a small adjustment to the tuning of one stage of one of the amplifiers or to the attenuator of the amplifier, or to both, until the figure on the tube takes the form of a line at 45° to the axis of deflection of the tube. The apparatus is then ready for use on the frequency at which the adjustments have been made.

It is necessary to deal with several questions of major importance in connection with instrumental errors arising from lack of balance or adjustment of the aerial systems. In the first place, the absolute accuracy of the apparatus after adjustment depends essentially on the accuracy with which the central oscillator produces equal and co-phasal fields at the four aerials. If it does not do this exactly, an error will be introduced by carrying out the aerial adjustments and balancing on the assumption that it does. Such an error will, however, always remain the same for a given direction unless changes are made in the disposition of the oscillator and aerials, since it is found in practice that the aerial system can be balanced to an accuracy better than ± 1°. There are some indications that the central oscillator does not produce exactly equal and co-phasal fields at the four aerials, since the mean of a large number of bearings taken on a distant fixed transmitting station usually shows an error of 2° or 3°. The means of small groups of observations on the same station taken at different times, however, usually lie within ± 1° of the mean of all observations.

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* See Bibliography, (16).

* See Bibliography, (15).

Once the aerial system has been balanced, it is important that balance should be maintained at least during the period that the apparatus is used. It has been found that in favourable circumstances (i.e. fine, dry, calm weather) the aerial balance remains sufficiently good for periods of as much as one or two hours continuously, and even in adverse circumstances the balance is nearly always maintained for at least 15 to 30 minutes continuously. In very damp weather, however, there is usually a steady drift in the balance, necessitating readjustment after such a period. In operation, the balance and general line-up of the apparatus is frequently tested, and readjustments are always made whenever a drift great enough to change the instrumental error by more than 1° has occurred. In addition to such slow changes in balance, there is usually also a more or less rapid " flutter " of the balance, particularly on very windy days. The " flutter " is never greater than would cause an oscillation of the bearing of any station by more than $\pm \frac{1}{2}°$, except for waves arriving at very small angles of incidence, say less than 30°. For smaller angles of incidence the uncertainty can be taken as inversely proportional to the angle of incidence.

The process of adjusting the apparatus for operation on any frequency has been described. It has been found by experiment that the adjustment of the apparatus only holds over a limited band of frequencies in the neighbourhood of the frequency at which adjustment was made. This appears to be essentially due to the fact that the aerials in operation are tuned to the incoming signal. Theoretically one might suppose that four identical aerials accurately matched to each other at one frequency would remain so matched. In fact, however, this state of affairs is unattainable, chiefly because the process of balancing does not adjust every part of one aerial to be identical with the corresponding part of the others, but simply ensures that the output of one aerial as a unit shall be identical with that of the others at the frequency of adjustment. It has been found that the frequency band over which the adjustments of the aerials hold is about 10 to 20 kc./sec. on each side of the frequency of adjustment. Outside these limits the error due to aerial unbalance becomes steadily greater than the instrumental accuracy of ± 1° aimed at. A further limit to the frequency range of the instrument for any particular adjustment is imposed by the limitations of the amplifiers available for use. It was found necessary to perform the " lining-up " process for these within about 3 to 5 kc./sec. of the frequency on which it was desired to operate. In practice, adjustment and lining-up of the apparatus is always carried out on as nearly as possible the exact frequency on which it is desired to operate, the central oscillator being tuned to produce a low audio note with the received signals if the latter take the form of continuous waves.

Finally, the apparatus is liable to other minor errors, such as errors arising in calibration of the protractor drawn on the cathode-ray tube face. This calibration is done at the intermediate frequency by the aid of a specially constructed potentiometer giving 10° steps round the protractor. Errors arising in calibration have a maximum of 2° and are generally considerably less. They are of course unimportant when studying changes in bearing and only affect the absolute accuracy of the instrument, since they are obviously fixed errors of the instrument.

It is convenient here to summarize very briefly the state of development of the apparatus so far as instrumental accuracy is concerned. The absolute accuracy of the apparatus is such that fixed errors in bearing of the order of 2° or 3° may be expected. Such errors will be constant for a given azimuth, though they may vary to some extent with angle of incidence since the effective spacing of the aerials depends on the angle of incidence. For a given ray, the angle of incidence of which remains within a few degrees of a fixed value, such errors in the absolute accuracy of the apparatus will not give rise to any variations in the apparent bearing. This is an important point since, in its application, the apparatus is used to investigate changes in bearing, the absolute accuracy of the bearings being of lesser importance.

In addition to such more or less fixed errors, there remain residual variable instrumental errors arising on account of fluctuations of the aerial balance, or very small fluctuations due to polarization error. These, however, would not give rise to fluctuations in bearing greater than ± 1° for all rays except such as are incident at small angles (less than, say, 30°) to the vertical. For such small angles of incidence the probable fluctuation in bearing is approximately inversely proportional to the angle of incidence.

Spurious fluctuations in bearing can arise in yet another way when receiving a continuous-wave signal which may be composed of several rays at different angles of incidence. If the instrumental error is different for the different rays each ray would have a different apparent bearing, and it can be shown that in these circumstances the apparent bearing of the composite continuous-wave signal will fluctuate as the relative phases of the separate rays change. The form of the trace on the cathode-ray tube will also pass through various forms of ellipse. At the same time the amplitude of the signal will vary and, at times when the amplitude is least, the apparent error of the bearing of the continuous-wave signal may be several times as great as the difference in the instrumental errors for the separate rays. Thus in the course of such fading the apparent bearing will fluctuate. Provided, however, note is only taken of the bearing at large amplitudes, say when the amplitude is at least as great as $0 \cdot 5$ times the maximum, such spurious fluctuations will not exceed ± 2° at the utmost.

*Plate 1*

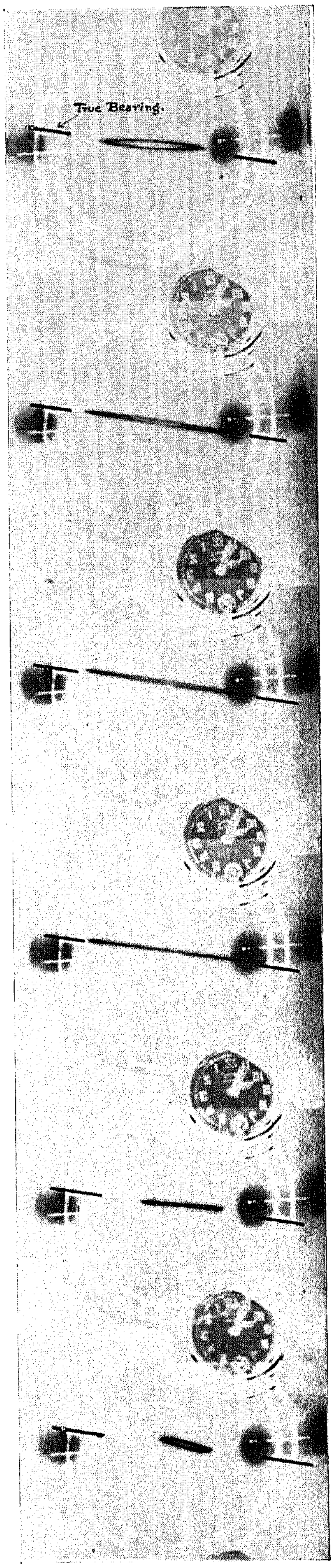

**Fig. 7(a).**—Zeesen (31·4 m.); 0955 G.M.T.; 2.10.36. Exposure = $\frac{1}{25}$ sec.; interval between photographs = 5 sec. Receiver beyond skip distance.

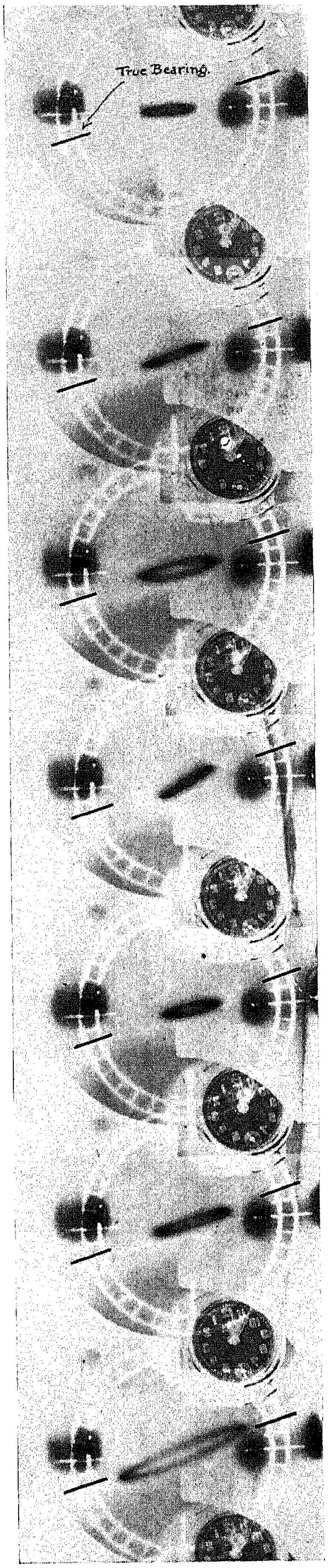

**Fig. 7(b).**—Sayville (36 m.); 2353 G.M.T.; 27.10.36. Exposure = $\frac{1}{25}$ sec.; interval between photographs = 2 sec. Typical bearings.

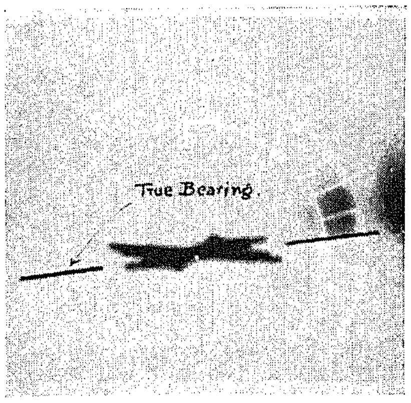

**Fig. 7(c).**—Prague (44·6 m.); 1530 G.M.T.; 8.3.37. Exposure = $\frac{1}{25}$ sec. High-speed morse transmission giving complex pattern.

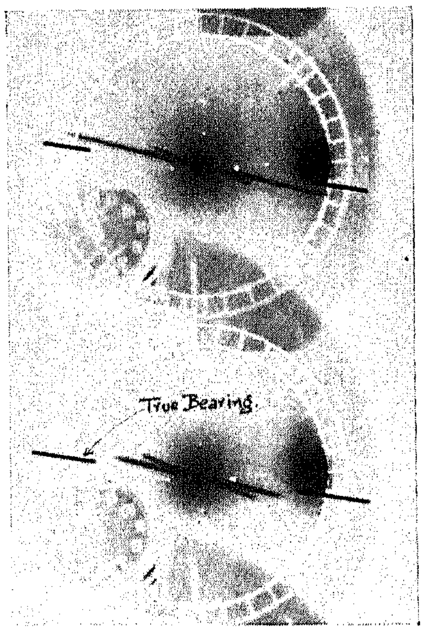

**Fig. 7(d).**—Nauen (30·6 m.); 1300 G.M.T.; 15.12.36. Exposure = $\frac{1}{10}$ sec. Pulse transmission, 1st- and 2nd-order $F_2$ rays giving different bearings.

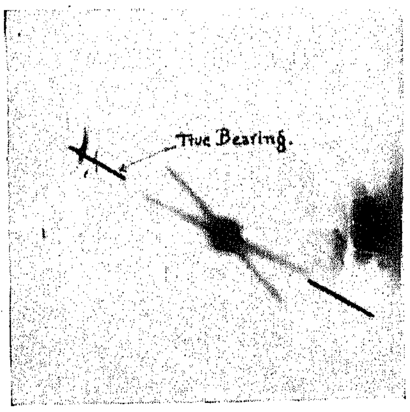

**Fig. 7(e).**—Dorchester (37·3 m.); 1200 G.M.T.; 7.5.37. Exposure = $\frac{1}{8}$ sec. Pulse transmission. Ordinary and extraordinary components of ray giving different bearings.

*Plate 2*

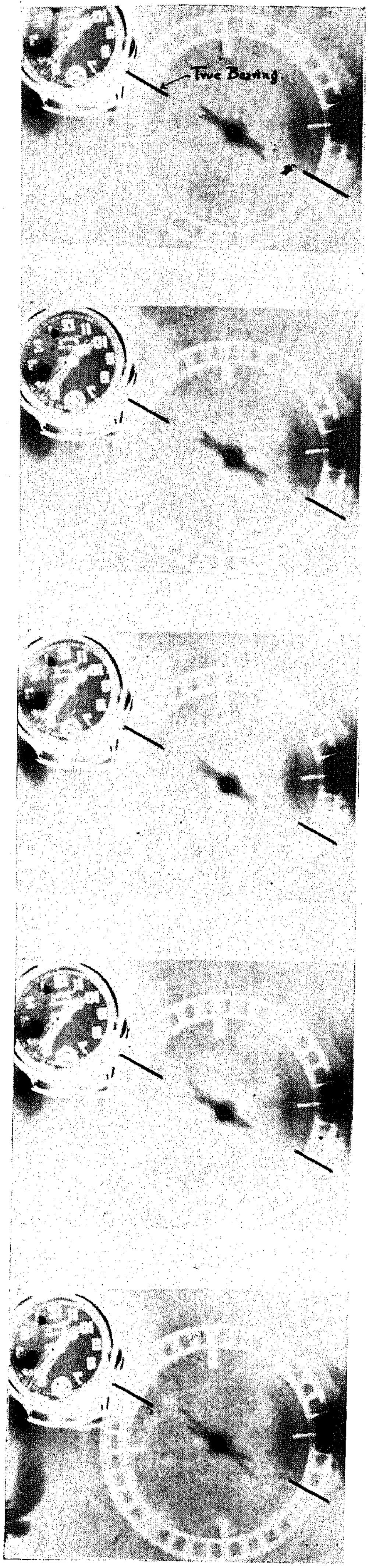

**Fig. 7(f).**—Dorchester (37·3 m.); 1020 G.M.T.; 23.4.37. Exposure = $\frac{1}{10}$ sec. Interval between photographs = 10 sec. Pulse transmission. Two rays (unidentified) giving different bearings.

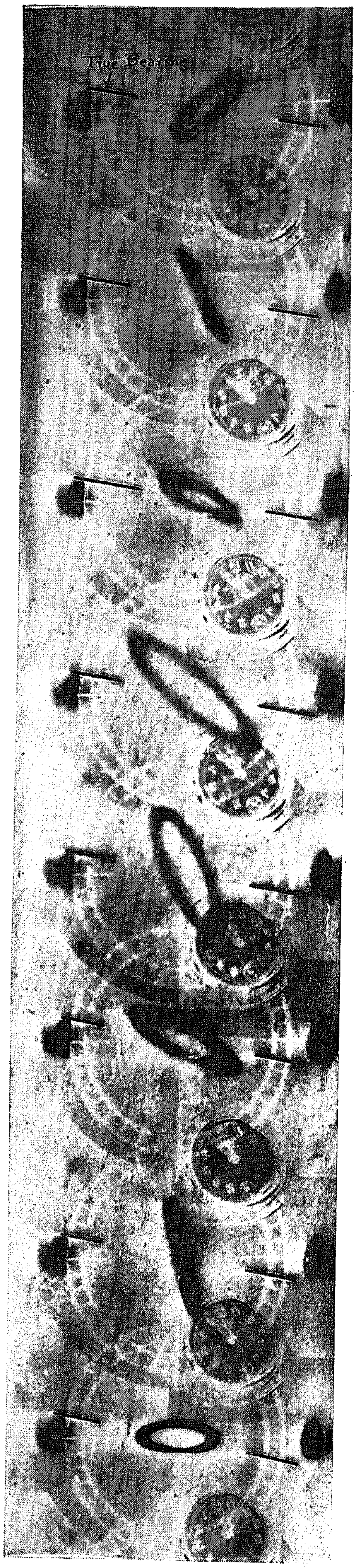

**Fig. 7(g).**—Zeesen (31·4 m.); 0004 G.M.T.; 18.9.36. Exposure = $\frac{1}{25}$ sec. Interval between photographs = 2 sec. Receiver within skip distance.

Thus, provided note is taken of bearings only when the amplitude of the signal is reasonably large, the instrument itself cannot introduce fluctuations in bearing greater than a maximum of ± 2°, and the probable fluctuations arising from instrumental causes is of the order of ± 1°. In the course of many observations fluctuations outside these limits have frequently been observed. This renders it probable that many of the smaller fluctuations observed do not arise instrumentally, since these larger fluctuations cannot do so.

The rejection of bearings observed when the amplitude is small, further ensures that actual deviations occurring in propagation are not magnified. Such magnification in the apparent deviation of a wave can occur in the case of continuous-wave signals composed of several distinct rays. If each, or any, of the rays is subject to some small amount of lateral deviation, the apparent bearing of the continuous-wave signal will vary as the relative phases of the various rays change, and may suffer apparent deviations several times as great as those of the individual rays. Such spurious effects, however, can again be minimized by observing bearings taken only when the amplitude is large, as mentioned above.

In all results mentioned later in the paper, these conditions have been observed and it is felt that the deviations noted arise predominantly in the process of propagation of the wave.

## (4) OBSERVATIONS AND TESTS

The apparatus reached the stage of development recorded in this paper in the summer of 1936, about which time it was felt that it might profitably be taken into use for observations on the direction of arrival of waves.

Bearings were recorded by taking snapshot photographs (usually several per minute) of the fluorescent screen of the cathode-ray tube (see Fig. 7, Plates 1 and 2, facing page 104). By this means it is possible to record bearings practically instantaneously and at regular intervals of time. The exposures used were $\frac{1}{25}$ sec. for continuous-wave signals and about $\frac{1}{5}$ sec. for pulses. Bearings were taken in groups usually extending over about 5 to 15 minutes, the adjustment of the apparatus being checked at the beginning and end of each short period. The results of several different series of observations are recorded below.

### (a) Zeesen.

The first series of tests with the apparatus was made by observing the bearings given by the German short-wave broadcasting station at Zeesen. This station is situated 920 km. from the Radio Research Station, and uses a wavelength of 31·4 metres. Observations were made whenever transmissions were available during the months of September and October, 1936, at which period the station was transmitting between the hours 0500 and 1000 G.M.T. and between 2200 and 0300 G.M.T.

It was found that results fell into two distinct classes. At times when the receiver was not within the " skip-distance " from Zeesen, a reasonably definite and steady bearing was obtained [Fig. 7(*a*)]. The bearing was usually subject to continuous and variable oscillation about some mean position quite close to the true bearing, the oscillation being usually within an arc of about 5°; deviations up to about 20°, however, were observed very occasionally when the amplitude was small. Disregarding the signals of smaller amplitude, however, the maximum deviation from the mean which was observed was about 10°. Bearings were usually recorded at the rate of about 12 per minute, those taken when the amplitude was greatest being finally selected from the recorded bearings. In the observations here referred to, all those for which the amplitude fell below 0·7 times the maximum were rejected. The probability that the deviations observed in those results which remain occur in propagation is thus very high.

Table 2 gives the distribution of bearings relative to the mean and is based on about 300 results.

### Table 2

ZEESEN ($\lambda = 31 \cdot 4$ METRES, DISTANCE 920 KM.)

Percentage of observations having deviations from the mean not exceeding:—

| 1° | 2° | 3° | 5° | 10° |
|---|---|---|---|---|
| 54 | 85·5 | 96 | 99 | 100 |

The observations in each group were averaged and it was found that the group means lay very close to the mean of all the observations. The mean of all the observations had an error of + 2°, which represents the more or less fixed instrumental error. Of the 15 groups of bearings on which the analysis is based, 7 had a mean bearing within 1° of the mean of all the observations, and all were within 2° of that mean. Variations in the state of adjustment of the apparatus could account for variations of the absolute instrumental error of not more than 1°. The results obtained, therefore, show that during these tests there were no occasions on which a steady lateral deviation of more than between 1° and 2° persisted throughout the duration of a group of observations. Table 2 shows, however, that the waves received are subject to continuous changes of bearing of not more than 10° about a mean position which appears to be close to the true bearing. The probable deviation of a single observation is seen to be of the order of 1°.

At times when the receiver was within the skip-distance from Zeesen (from a few hours after sunset until about sunrise) the results obtained were different. Signal strength fell some 30 to 40 decibels. The form of the trace on the oscillograph screen passed rapidly and continuously through all forms of ellipses from a straight line to a circle, the major axis at the same time swinging through all possible orientations [Fig. 7(*g*)]. There was a slight tendency for a predominance of bearings in a direction some 20° from the true bearing, but the bearings recorded were very widely scattered about this direction—about 50 % showing bearings more than 20° different from the means of all the recorded results. The mean bearing indicated agrees very approximately with the bearing to be expected if the scattered signals received at this time came mainly from points along the beam of the transmitter. It is, however, clear that no reliable bearings could be obtained at such times.

### (b) Nauen.

To throw further light on the results obtained in the tests mentioned above, a short series of special pulse transmissions from the German station at Nauen (920 km. distant, wavelength 30·6 metres) were arranged with the German Post Office through the kind co-operation of Dr. T. Walmsley of the British Post Office. The transmissions took place in December, 1936, between the hours 1300 and 1400 G.M.T. It was found that with the pulses used ($\frac{1}{3}$ millisec. long) in general two rays were received, corresponding to the first- and second-order reflections from $F_2$ region. The third-order ray was also received at times, and very occasionally others which may have been reflections from E or $F_1$ regions [Fig. 7(*d*)].

It was found that all the rays were subject to continuous and variable deviations. In the case of the ray corresponding to the first-order $F_2$ reflection, the trace on the tube was nearly always linear but occasionally became slightly elliptical, while fading, though not so rapid as in the case of continuous waves, was usually present. This indicates that the ray cannot always be regarded as a single ray but is composite. In view of this, in producing Tables 3 and 4, observations made at small amplitudes have been neglected. The effect on the distribution of errors was found to be slight and much less than in the case of continuous waves. The greater deviations (between 10° and 20°) were, however, eliminated by this process of selection, such deviations being probably occasioned by the interference effects referred to above. The distribution of bearings relative to the mean is given in Table 3, which is based on about 550 results.

### Table 3

NAUEN ($\lambda$ = 30·6 METRES, DISTANCE 920 KM.)

1st-order $F_2$ reflection

| Percentage of observations having deviations from the mean not exceeding:— | | | | |
|---|---|---|---|---|
| 1° | 2° | 3° | 5° | 10° |
| 49·5 | 73 | 85 | 98 | 100 |

The mean of all the observations had an error of − 3°. The observations were, as before, taken in groups at the rate of several per minute over intervals of about 10 minutes. Sixteen such groups of bearings were taken and, of these, 13 groups had a mean error with respect to the mean of all observations of 1° or less, the remaining 3 having relative errors of − 2°, + 2·5°, −3·4°. This indicates that steady lateral deviation of more than 1° persisting for several minutes on end is uncommon, but does occasionally occur and may have an amplitude of 3° to 4°. Individual bearings, however, are subject to a continuously variable amount of lateral deviation up to about 10° maximum, the probable deviation being about 1° as shown in Table 3. The angle of incidence for this ray was measured and found to be about 50°.

Results for the second-order ray were very different from those for the first-order ray. Of 13 groups of bearings obtained on this ray only two groups had mean bearings differing by 1° or less, 5 had mean bearings differing by 2° or less, and 8 had mean bearings differing by 5° or less from the mean of all the observations. The mean bearings of the remaining 5 groups had deviations from the mean of all the observations of − 5·8°, + 6·2°, + 7·2°, − 7·7°, + 8·2°. These groups extended over 5 to 10 minutes and it is seen that, in the case of this ray, steady deviation of one sign might persist for several minutes continuously and might have a maximum value of about 8°. The individual bearings in each group, of course, fluctuate rapidly about the mean bearing for the group. Table 4 shows the distribution of the individual observed bearings relative to the mean of all the observations and has been compiled from 250 results, signals of small amplitude being neglected.

### Table 4

NAUEN ($\lambda$ = 30·6 METRES, DISTANCE 920 KM.)

2nd-order $F_2$ reflection

| Percentage of observations having deviations from the mean not exceeding:— | | | | | |
|---|---|---|---|---|---|
| 1° | 2° | 3° | 5° | 10° | 20° |
| 18 | 30 | 42 | 58 | 85 | 99 |

The probable deviation is seen to be between 3° and 5°. Only a few observations were made on the third-order ray, but these showed that the deviations suffered by this ray were still greater, the probable deviation being between 5° and 10°. The number of results is too small to make the construction of a table of much value. It may be added that the first-order ray was usually much the strongest, being usually at least twice as strong as the second-order ray. Continuous-wave signals from such a station, and in similar circumstances, will therefore consist mainly of the first-order ray, and might be expected to show results similar to those obtained with the first-order ray. The results obtained in the Zeesen tests are in confirmation of this supposition, as a comparison of the results will show.

The results of the Nauen measurements agree on the whole satisfactorily with those obtained by Eckersley* from measurements made with a single-pair spaced-loop arrangement on pulses from the same transmitter.

### (c) Prague.

A number of observations have been made on signals from Prague on a wavelength of 44·6 metres at a distance of about 1 040 km. during March and April, 1937, and at all hours of the day between 1000 G.M.T. and 0100 G.M.T. In this case the emissions are morse signals, the carrier wave being modulated. The figure on the cathode-ray-tube screen was frequently extremely complex, several thin ellipses being seen at once in addition to other fainter ellipses apparently corresponding to key-clicks [Fig. 7(*c*)]. The bearings indicated by the different ellipses were generally quite different. On some occasions only one ellipse was visible. When several ellipses were present, the bearing of the one of greatest amplitude only has been taken into account, all signals whose amplitude fell to below 0·5 times the maximum being rejected. The results obtained are similar to those obtained on Zeesen, although the deviations noted are somewhat greater.

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* See Bibliography, (13).

The mean of all the observations had an error of — 4°. Of 27 groups of bearings recorded, 15 had mean bearings within 1° and 23 had mean bearings within 2° of the mean of all. The remaining 4 groups had deviations from the mean of 2·5°. The nature of the results was practically the same at all times when transmissions were observed, and at no time did the receiver pass into the skip region from Prague. The distribution of bearings for all the individual observations noted is shown in Table 5, which is based on 700 observations. There were occasions when the distribution was better than that shown in Table 5, such times usually occurring during the middle of the day. There are insufficient data, however, to correlate any particular type of results with a particular time of day, except to note that all four occasions on which the mean relative error of a group exceeded 2° occurred in the late evening.

### Table 5

PRAGUE ($\lambda$ = 44·6 METRES, DISTANCE 1 040 KM.)

Percentage of observations having errors relative to the mean not exceeding:—

| 1° | 2° | 3° | 5° | 10° | 20° |
|---|---|---|---|---|---|
| 39 | 58 | 72 | 85 | 98 | 99·7 |

The results appear to show that the waves received were quite frequently subject to steady deviations of about 1° to 2°, since 12 out of 27 groups had deviations of more than 1° from the mean of all the results. Such steady deviation, however, was not found to exceed 2·5°. The probable deviation of a single observation is between 1° and 2°.

### (d) Various American Fixed Stations.

Observations were made during October and November, 1936, on stations situated at Sayville (wavelength 36 metres), Chatham, Mass. (36·5 metres), Millis, Mass. (31·4 metres), Schenectady (31·5 metres), and New Brunswick, N.J. (31·7 metres), all some 5 000 km. from the Radio Research Station.

Observations were made when transmissions were available and could be picked up at sufficient strength, namely between the hours of 2000 G.M.T. and 0400 G.M.T. It was found that deviations were usually somewhat larger than those observed on the stations already mentioned, and the tendency for a steady deviation to persist for several minutes was somewhat greater [Fig. 7(*b*).] Thus of the 9 groups of observations made, only 5 had mean bearings differing by 2° or less from the mean of all, and the mean of one group had a deviation of 4·6° relative to the mean of all. It has been assumed that, since all the above stations lay in approximately the same direction and used wavelengths quite close to each other, the fixed error would be approximately the same for all of them. This fixed error appeared to be about — 3°. The distribution of bearings for individual observations is given in Table 6, which is based on about 250 observations. About 500 of the observations actually made were rejected as having too small an amplitude. The nature of the results was inclined to be very variable, bearings being occasionally much steadier than Table 6 implies, and at times even less steady. The probable deviation of a single observation appears to be between 2° and 3°.

### Table 6

VARIOUS U.S.A. TRANSMITTING STATIONS (WAVELENGTH 31·4 TO 36 METRES, DISTANCE 5 000 KM. APPROX.)

Percentage of observations having errors relative to the mean not exceeding:—

| 1° | 2° | 3° | 5° | 10° | 20° |
|---|---|---|---|---|---|
| 24 | 39 | 56 | 77 | 95 | 100 |

These results again may be compared with those obtained by Eckersley* from a few observations made on Montreal ($\lambda$ = 32·15 m.) at Chelmsford with his single-pair spaced-loop system, when deviations of the same order were recorded.

### (e) Dorchester.

Observations have been made on pulse transmissions from Dorchester on a wavelength of 37·3 metres during the months of April, May, and June, 1937. This station is about 160 km. from the receiver. The signals which are received are, therefore, incident at quite a small angle to the vertical. For the first-order reflection from the $F_2$ region, the angle of incidence of the ray was probably between 12° and 18° during the period of the tests. At such angles of incidence the instrumental accuracy is, of course, not of such a high order as for rays at much greater angles of incidence, since the effective spacing between the aerials is less, and small misadjustments of the aerials may produce quite appreciable errors. This affects most particularly the absolute accuracy of the direction-finder. The minor fluctuations in aerial balance which occur more or less continuously during observations also produce correspondingly larger fluctuations in the apparent bearing. Any residual polarization errors also become increasingly great as the angle of incidence decreases. For equal horizontally and vertically polarized components in the wave, however, the polarization error for an angle of incidence of about 12° is not greater than about 1°. It will increase, of course, as the state of polarization of the wave changes in such a manner as to reduce the wanted pick-up of the aerials, but provided bearings are only noted when the amplitude of the signals is near the maximum the error cannot exceed, say, 2° at most. Tests have shown that for signals from this station the average strength of the horizontally and vertically polarized components are equal. At the most, therefore, instrumental errors cannot give rise to variable errors in the bearing of more than 4° or 5°, and probably the instrumental variations do not cause any deviations greater than about 2° or 3°.

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* See Bibliography, (13).

The results obtained in the course of tests on Dorchester are, in the main, strikingly different from those obtained on the more distant stations and are most conveniently illustrated by graphs. This has been done in Fig. 6. For any particular ray the bearing fluctuates about some mean position, but this mean position is frequently many degrees from the true bearing. It has been observed to be as much as 50° to 60° in error, and moreover is frequently observed to drift steadily in one direction or the other. Thus a continuous change in the mean bearing of as much as 25° in 10 minutes has been observed (Fig. 6, 30.4.37, 0940 G.M.T.). Occasionally, too, the drift may change direction after several minutes, so that the mean bearing returns to its original value (Fig. 6, 30.4.37, 0940 G.M.T.). The mean bearing has also been observed to oscillate slowly about the true value (Fig. 6, 23.4.37, 1020 G.M.T.). The fluctuations of the bearing about its instantaneous mean position are usually of small amplitude, i.e. ± 3° or 4° at most. Moreover, when more than one ray is present the two rays frequently show very different bearings, as much as 10° to 20° difference in bearing being observed for the first- and second-order reflections from $F_2$ (Fig. 6, 21.5.37, 1030 G.M.T.). At times also the first-order reflection has been observed to be split into its two components by magneto-ionic action. When this occurs the two components have been observed sometimes to arrive in exactly the same direction and to continue to do so for several minutes together, although that direction of arrival may not correspond to the true bearing (Fig. 6, 4.6.37, 0930 G.M.T.—first 7 minutes), while at other times they may have bearings differing by up to 30° (Fig. 6, 7.5.37, 1050 G.M.T., and 4.6.37, 0930 G.M.T.—last 10 minutes). It should be noted that the effect of any substantial polarization error in the direction-finder would be to produce a difference in apparent bearing for these two components of the first-order $F_2$ reflection of more or less constant amplitude and of constant sign and such that the mean bearing of the two was approximately true. This is deduced from the known fact that the two components will be nearly circularly polarized, having opposite senses of rotation. The results show, however, that the difference in bearing for the two components can be anything from zero to 30° and is not of constant sign, sometimes the one and sometimes the other being most northerly. The mean bearing may not be even approximately true.

On a few occasions, when a single ray only has been received, results have been very erratic, the bearing changing by 20° or 30° from one instant to the next and not showing any tendency to indicate a mean direction (Fig. 6, 4.6.37, 1100 G.M.T.). This might be expected to occur at times when the ray consisted of several components (such as the two magneto-ionic components, not actually separated in time) each of which was subject to a large amount of deviation. As the phase relationship

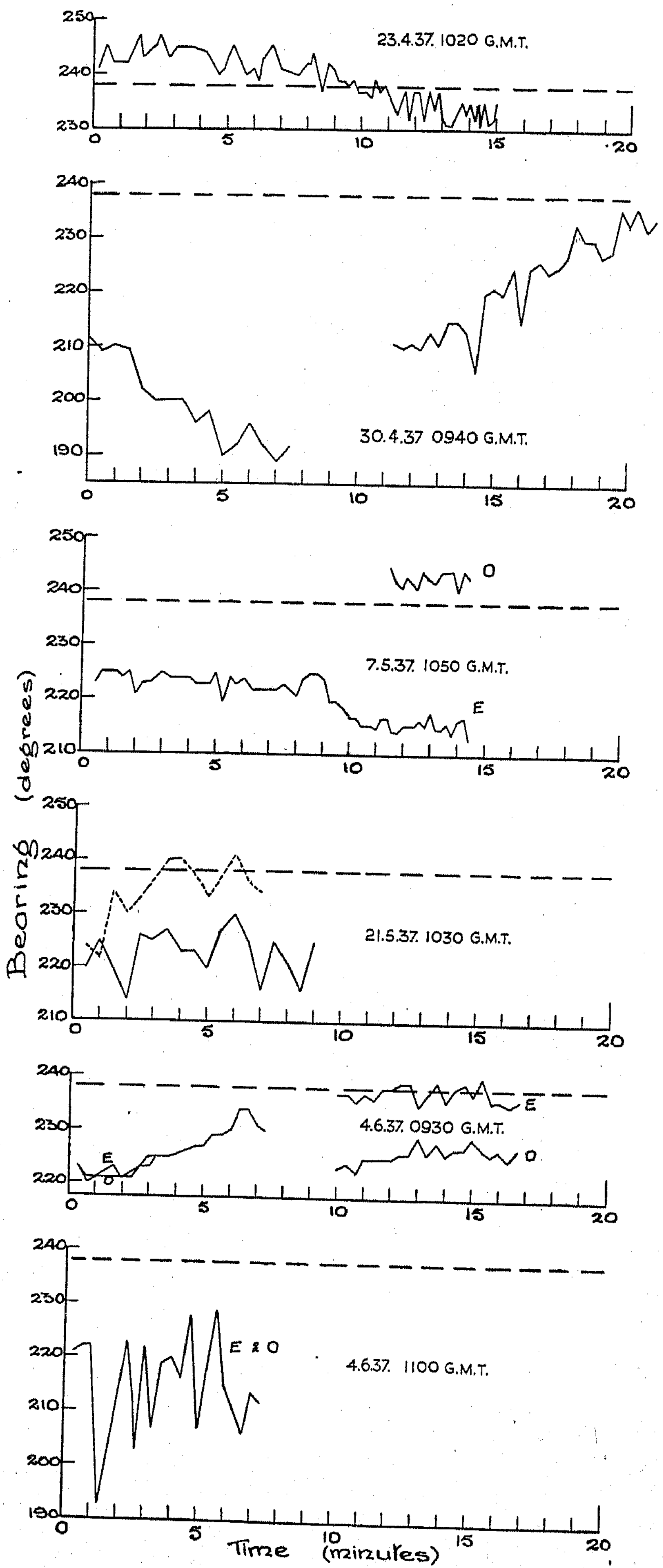

**Fig. 6.**—Observations on pulse transmissions from Dorchester ($\lambda$ = 37·3 m.).

— — — True bearing.
———— 1st-order $F_2$ reflection.
- - - - - - 2nd-order $F_2$ reflection.

O = Ordinary component.
E = Extraordinary component.

of the components changed, the apparent bearing would fluctuate to a very great extent. At such times also, the trace, which is usually practically linear, may become very much opened out into an ellipse.

These results can only be interpreted as implying that considerable lateral deviation of waves which have travelled by way of the ionosphere can exist. Instrumental errors might possibly account for the small fluctuations of the bearing from instant to instant in the case of waves incident at such small angles to the vertical as those received from Dorchester, but cannot account for the large deviations of up to 50° which have been observed. Such deviations imply that the region of the ionosphere effective in reflecting the waves received at the direction-finder may be displaced laterally by as much as 50 to 100 km. from the great-circle path joining the transmitter and receiver.

## (5) CONCLUSIONS

It has been shown that the spaced-loop direction-finder is to a very large extent free from instrumental errors of a variable nature, and that, provided care is taken in rejecting bearings obtained when the amplitude is small, the variations in apparent bearing which can arise from instrumental causes have a maximum of about 2° and are probably usually of the order of 1°. The results of various tests made with the apparatus show that deviations in apparent bearing much in excess of these figures are found, deviations up to 10° or 20° being noted on distant stations and up to 50° on a station at close range. It seems practically certain that these deviations arise in propagation and that, in fact, the wave does not travel along the great-circle path. It appears that the equivalent point of reflection at the ionosphere may be as much as 50–100 km. out of the great-circle path. Usually the deviations suffered by waves change rapidly from instant to instant in such a way that the mean bearing is very close to the true one, but occasions are not rare on which the mean direction of arrival of the waves remains deviated in one direction for several minutes on end.

Waves which have had several reflections at the ionosphere appear to be liable to greater deviation, as might be expected. This is borne out by direct observations on waves known to have suffered two reflections, and by observations on certain American stations for which all the waves must have suffered multiple reflections at the ionosphere.

There is some indication that so-called " individual" rays are perhaps something in the nature of cones of rays. This deduction is based on the fact that, when receiving pulses, the traces for the " individual " rays are quite often slightly elliptical. This can only occur if more than one ray is present in each so-called " ray " and the constituent waves do not arrive along precisely the same path. The direction indicated will be some mean direction, usually within the cone of rays except when the rays interfere so as to produce a signal of very small amplitude, when the apparent direction may lie quite outside the cone. The variations in apparent bearing from instant to instant may therefore be due, not so much to instantaneous changes in the direction of arrival of the rays, but to phase changes amongst the components of the ray, giving rise to changes in the apparent bearing and degree of ellipticity of the trace. It is clear, too, that if this interpretation of the results is true, the whole cone of radiation constituting a ray may, and apparently does, suffer deviation from the great-circle path.

The experience gained with the apparatus described in this paper has indicated certain directions in which the direction-finder might be further improved so as to attain a higher degree of precision and certainty in the measurement of small amounts of lateral deviation. An improved model of the direction-finder is therefore at present being designed, with which it is believed that the problem of lateral deviation of the ionospheric ray may be adequately attacked.

## (6) ACKNOWLEDGMENTS

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# DISCUSSION ON "ELECTRICITY DEMAND AND PRICE" *

**Mr. E. V. Clark** (Australia) (*communicated*): That the author is seriously astray in some of his assumptions or deductions seems evident from a paragraph on page 191 (vol. 82), which I quote in full, the italics being my own.

"It follows that the monopoly of supply in a necessary service forms a very powerful weapon for ill or good. In private hands it tends inevitably to abuse unless checked by strict external control. In disinterested hands and wisely directed it can be an effective instrument of public service. *For when both necessity and luxury consumption are comprised in a single supply, it will often be possible to overcharge on the necessity consumption and use the proceeds so as to undercharge on the luxury consumption*; thus developing the latter without materially damaging the former."

For let us apply this principle to something other than electricity. Suppose, for example, that a Municipal Council should secure an Act of Parliament authorizing the establishment of a municipal bakery and prohibiting the local sale of any bread or cake not made therein. Bread is typically a necessity. Iced almond cake is typically a luxury. If the author's principle as italicized above is to be followed, the municipal bakery will overcharge for bread in order that it may undercharge for iced almond cake; or, in other words, it will overcharge the poor in order that it may undercharge the wealthy!

If we assume that whilst the demand-price (d.p.) elasticity of bread is low, that of iced almond cake is exceptionally high, then it is certainly true that by overcharging for bread and undercharging for the cake the gross revenue of the undertaking will be increased. But surely it is palpable that to overcharge for a necessity such as bread in order that those who want rich cake may buy it at a relatively low price is typical of those grave abuses of monopolies to which the paragraph cited refers.

This matter hinges upon the word "overcharge," which is the author's and is not introduced by me. If the municipal bakery considers that by confining its activities to bread only it could sell bread at a price of *N* pence per loaf, but at nothing less, then it is not overcharging for bread if it continues to sell at *N* pence when combining the activities of bread-making and cake-making. This is possibly what the author means; but in that event the use of the word "overcharge" is misleading. Bread, in these circumstances, is the primary object of the undertaking, and cake-making but a side-line, so that cake is a quasi by-product. No complaint can rightly be levelled at an undertaking, however low the price at which it sells by-products, provided that the price of its main products is not thereby increased, or, in other words, as long as it does not overcharge for its main products in order that it may sell its by-products more cheaply.

The fundamental fallacy in the author's argument appears to lie in his assumption that, whereas a necessity has a d.p. elasticity of a low order, that of a luxury may be expected to be more than unity. For surely it is evident that no commodity—luxury or necessity—can have a d.p. elasticity greater than unity unless it so happens that the price is in a region of active competition; and this may occur whether the commodity belongs to the necessity or luxury category.†

Electricity for heating and cooking is certainly not a commodity with an inherent d.p. elasticity greater than unity. It is quite certain that no reduction in price of electricity for these purposes would increase the revenue from houses which are already all-electric and have no other means of heating and cooking. The high d.p. elasticity shown by the author's curves and figures for electricity used for these purposes is, as he himself points out, due to the fact that, with a tariff of about 0·5d. per unit, electricity is highly competitive for these purposes.

Again, the author seems to be entirely astray in suggesting (at the top of page 194) that electricity for lighting, cleaners, and wireless, is a necessity, and for heating and cooking is a luxury, if the economic meaning of these words is intended. Indeed, the exact converse is more nearly the case. A wireless set is unquestionably a luxury; and though electricity is a necessity for its use, this does not make electricity so used a necessity in the economic sense. Cleaning, illumination, heating, and cooking, are necessities in the English climate, the last two more so than the first two. A vacuum cleaner is certainly a luxury, since the necessary minimum of cleaning may be more economically done without it. Electric lighting must also be regarded as a luxury in a private house, since nobody installs it for the sake of economy, but always for the sake of the convenience, which is such that we do not trouble much about the price.

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* Paper by Mr. D. J. Bolton (see vol. 82, page 185).

† It must be borne in mind that no purchasable commodity is an absolute necessity. Though food is a necessity, bread is only so in those places where its price is highly competitive with possible alternatives such as potatoes, rice, and biscuits.

Thus I should summarize the position by saying that electricity for wireless, cleaners, and illumination, comes into the category of luxuries; and a high price may well be asked, since the luxury value of these is so high that people will willingly pay a considerable sum for their enjoyment. Electricity for heating and cooking, on the other hand, comes rather into the category of necessities, since heating and cooking are necessities and other ways than electrical are reasonably satisfactory. Sales of electricity are therefore almost negligible for these purposes unless the price is cut low. With a price low enough to make electricity competitive, sales may increase enormously, with a d.p. elasticity much higher than unity, just because heating and cooking are necessities. But for necessities, as for many luxuries, there is only a limited market, however low the price; and when electric cooking and heating have become as nearly universal as electric lighting now is, the d.p. elasticity of electricity for these purposes may be expected to be lower than that for illumination.

It is unfortunate that the author has gone so astray in this matter of necessities and luxuries; for the paper is well worthy of detailed study by all interested in the economics of electricity supply. I fully agree with him that some complexity of tariffs is essential for any progressive supply undertaking, and that the day is far distant when a flat rate per unit for all purposes will become practical politics. However, I do not consider that electricity occupies such a unique place in the field of economics as is implied in the third paragraph of the paper, since the economics of railway transport is closely akin; and many useful analogies with electric supply rates may be drawn from such varied matters as the periodic sales of drapery establishments and the different prices of theatre seats.

**Mr. R. B. Rowson** (*communicated*): In view of the author's remarks in the Summary, one rather hesitates to make any comments on Part 1 of the paper. I feel, however, that it should have been made clearer that the equation for equilibrium under perfect competition represents an almost impossible position. The various assumptions are set out quite clearly in Mrs. Robinson's book and are more than sufficient to show that they are not all likely to happen together for any length of time. Of late the growth of rings, in which the electrical industry abounds, has also ensured that free competition (usually called " price cutting " or " wasteful competition ") shall not occur. As a consequence, I would suggest that for an undertaking to sell at price *mq* in Fig. 3 is not such a serious crime, as it is unlikely that price *pn*—or, more correctly, *rt* in Fig. A—would obtain in practical affairs.

Fig. 3 is also rather unfair, as the average cost curve has not been shown. As the marginal cost curve is shown falling, the average cost curve must lie above it. As a consequence the average cost curve must cut curve D to the left of *p*, as shown at *r* in Fig. A. This, of course, reduces the disparity between the price under monopoly and that under perfect competition.

Throughout the paper I think that the author has been a little unfair in his criticism of supply engineers. The fact that few supply engineers have published papers advocating an equating of tariffs with demand schedules does not necessarily mean that the subject is overlooked. That engineers have long been interested in the subject is indicated by the fact that a paper was read by Mr. Cowan before the British Association in 1910 (quoted in Diblee's " The Laws of Supply and Demand ") advocating different prices for different uses owing to the differing elasticities which from *a priori* grounds were thought to exist for electricity sold for light, power, and heat. I would also mention that the average cost curve over a wide range of outputs is not so easy to calculate as is inferred by the author, and it thus justifies the number of articles which have been written on its calculation.

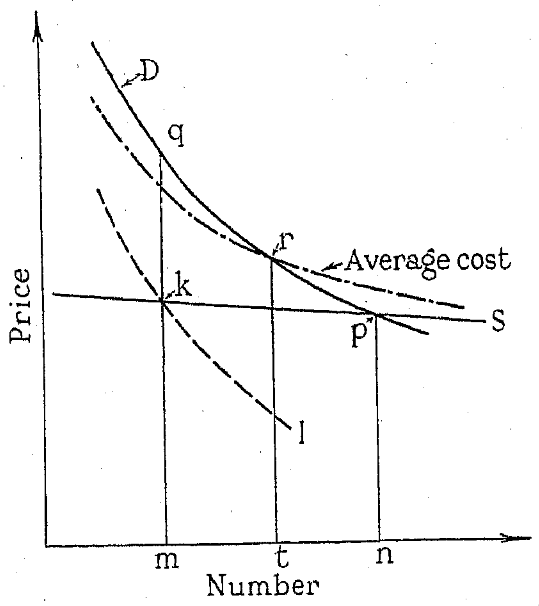

**Fig. A**

In this section of the paper it would have been helpful had the author pointed out that the average revenue curve for an individual has considerably more slope than that for a community and is likely to reach saturation at finite prices. This basic fact would help to explain the poor response in units per consumer for decrease in price, as indicated by the Hampstead figures.

In mentioning that monopolies of essential services are without the usual economic checks, it is pertinent to remember that they are more subject to the force of public opinion than, say, the makers of fountain-pens or chairs.

The difficulties referred to by the author on page 193, with regard to obtaining statistics of the various uses of electricity, disclose an unfortunate state of affairs. This is particularly the case as supply undertakings have, since 1932, been submitting very detailed figures of their sales; to obtain which they have, in many cases, had to rearrange their record systems completely. In not publishing these figures the Electricity Commissioners are withholding information which might be of considerable use in tariff-fixing.

I think that on page 194 the author means that the solid curve in Fig. 6 is generally less steep (i.e. $dP/dN$ is less) than the unit-elasticity hyperbola.

The curve of total revenue in Fig. 7 is misleading, as it gives the impression that it will continue to rise as prices are still further reduced. That this is not so was shown by Lardner in 1850, who applied this type of analysis to railways. Three important conclusions may be argued *a priori*:—

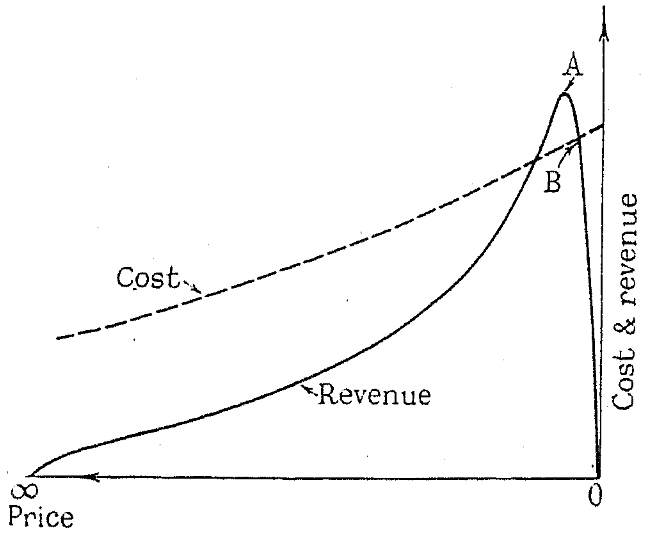

**Fig. B**

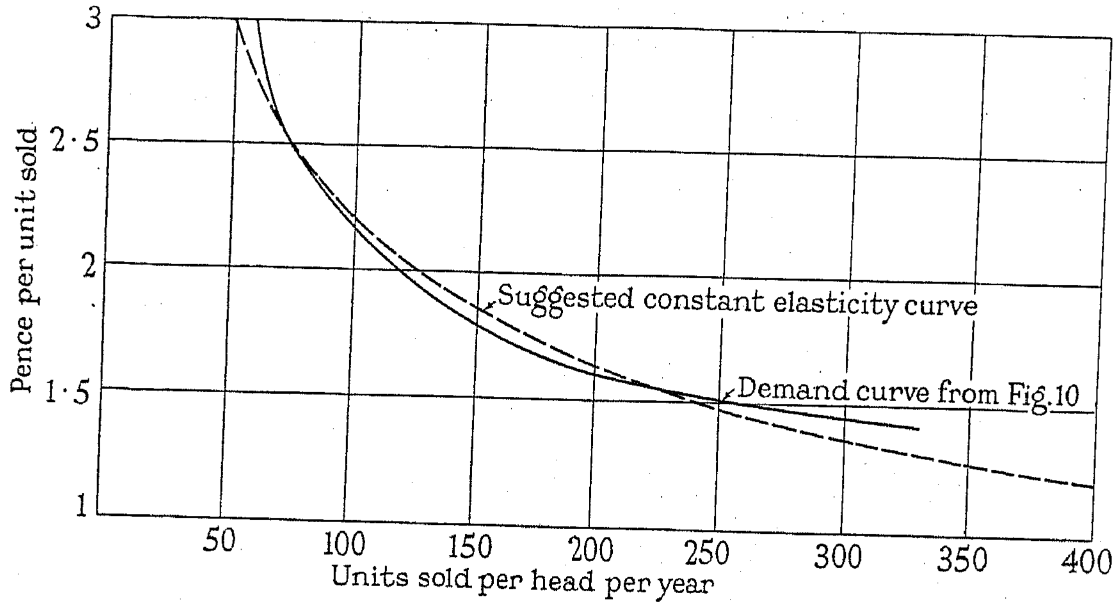

**Fig. C**

(1) If price is infinitely great, revenue is zero;
(2) if price is zero, revenue is zero; and
(3) costs are usually greatest under condition (2).

The theoretical curves have, as a consequence, shapes on the lines of those shown in Fig. B. The importance of research to ascertain the price per unit corresponding to points A and B will be realized.

On page 200 the author suggests that a follow-on rate of 0·5d. is indicated if the average price $P$ is 1·5d. Now the consumption per consumer depends on the marginal price $I$, and for equilibrium $I$ would equal 0·5d. It has recently been shown by Lerner ("Review of Economic Studies," vol. 1, p. 39) that

$$E = \frac{P}{P - I}$$

and if $P = 1 \cdot 5$d. and $I = 0 \cdot 5$d. then $E = 1 \cdot 5$, which may be compared with approximately 2·4 in Fig. 11.

I would suggest that the curves in Fig. 11 are based on insufficient evidence, and I would refer the author to a contribution by Mr. Melling (*Journal I.E.E.*, 1933, vol. 73, p. 135) who put forward reasons for thinking that elasticities less than unity prevail. Figures supporting this view are also given in the *Electrical World* (1934, vol. 104, p. 778, and 1937, vol. 107, p. 1390). As will be shown later, the Hampstead results also tend to support this view.

In two recent contributions to the derivation of demand curves (*Economic Journal*, 1937, vol. 47, p. 451, and *Journal of the Royal Statistical Society*, vol. 100, p. 625) it was suggested that the most likely possibility was that $E$ is constant. A curve of this type with an elasticity of 1·5 was mentioned in *World Power* in September, 1932, and another with an elasticity of 2·4 was given in the *Electrical World* in 1930 (vol. 96, p. 404). A similar curve is shown in Fig. C and is compared with Mr. Bolton's Fig. 10. The differences between the curves are so small and the scatter of the results in Fig. 10 so great that I would suggest that it is impossible to draw any definite conclusions on the shape of the elasticity curve.

From a practical point of view Table 2 is most interesting and, at the same time, alarming; interesting because it is an attempt to split up the aggregate demand curve into its components, and alarming because of the fall in the revenue per consumer. The revenue figures calculated from the Table are:—

| | Price per unit | Revenue per consumer | Revenue per assessment |
|---|---|---|---|
| | d. | d. | d. |
| Lighting .. .. | 8 | 2 944 | 1 944 |
| | 3 | 1 680 | 1 929 |
| Heating .. .. | 2 | 1 832 | 254 |
| | 0·6 | 1 278 | 528 |

This Table shows an unmistakable fall in revenue in three out of the four cases calculated. From this it appears that, if price reductions continue, the industry is likely to be faced with a serious difficulty in maintaining its trading balance as soon as a reasonable consumer saturation is obtained.

The investigation in the paper on the Hampstead results is unsatisfactory, inasmuch as no attempt has been made to separate the increased consumption due to price change and that due to other external influences. As regards heating, for instance, the increase from 1928 to 1931 (during which price was constant) in units per assessment is about 150, and the average compound interest rate of growth is about 7 %. If it could be assumed that this growth would have persisted throughout the period irrespective of fall in price (and no reasons have been given to assume otherwise) then the growth due to fall in price would have only been approximately 7 % and the elasticity less than unity.

Replotting the lower curve in Fig. 13 on ratio paper shows two distinct rates of growth, one from 1920 to 1927 and the other from 1928 to 1935. During the first period the rate of increase was 19·4 % per annum, compared with a fall in price of 9·43 % per annum, thus indicating an elasticity of about 2·06. In the latter period the rate of growth in units per assessment was only 7·5 % whilst the fall in price was 5·64 % per annum, indicating an elasticity of about 1·3 if growth due to other influences is neglected. This is interesting as indicating a lower overall elasticity for lower prices and may be compared with Fig. 11, which shows diametrically opposite results.

The author's statements with regard to distributing a surplus merit further analysis. In tabular form the conditions are as follows:—

| For $E$ equal to | 1 | 3 |
|---|---|---|
| Present sales | $N_1$ | $N_1$ |
| Present average price | $P_1$ | $P_1$ |
| Present revenue | $R_1$ | $R_1$ |
| Present costs | $C_1 = 0\cdot9R_1$ | $C_1 = 0\cdot9R_1$ |
| Proposed new price | $0\cdot95P_1$ | $0\cdot8P_1$ |
| Expected new sales | $1\cdot05N_1$ | $1\cdot96N_1$ |
| Expected new revenue | $R_2 = R_1$ | $R_2 = 1\cdot57R_1$ |
| New cost for equilibrium | $C_2 = R_2$<br>$= 1\cdot1C_1$ | $C_2 = R_2$<br>$= 1\cdot74C_1$ |

Thus it will be seen that, if $E = 1$, to distribute the 10 % surplus by a 5 % reduction in average price is quite practical, as costs can increase by 10 % for a 5 % increase in units sold. If $E$ is 3, however, and the price is reduced 20 %, the undertaking must be able to sell 96 % more units for an increase in total costs of only 74 %. If this fact is realized it is felt that there would be some hesitation before taking the plunge.

Another possibility is that the elasticity of 3 would not be realized, but, instead, the elasticity might be unity, or thereabouts. If this happened, 25 % extra units would be sold but costs must not increase by more than 10 %. As a consequence it is very likely that a loss would be sustained.

**Mr. J. A. Sumner** (*communicated*): I should like to commend the author for providing a paper of this nature, which is in line with the modern trend of thought as regards electricity distribution. Although I do not agree that it is practicable to charge for electricity on the basis of its value to a consumer, except in a very arbitrary and anomalous manner, I am firmly of the opinion that the present unregulated rule-of-thumb methods of charging are repressing consumption, and any attempt to consider tariffs in a co-ordinated manner is to be welcomed. It is quite true, as the author states, that the term "elasticity of demand" has been employed in tariff discussions before, and also that no precise definition of this term or appraisal of its value has been attempted. In the light of the reactions produced from a personal attempt to introduce the fringe of this subject as an oral addition to a recent paper read before The Institution, I would suggest that this is due to the unsatisfactory reception which a highly abstract subject of this nature would receive in a discussion on a paper read before The Institution.

On the other hand, the value of Mr. Bolton's paper is increased because it is presented at a time when so many thoughtful engineers are beginning to inquire how far the somewhat abstract science of economics can be used to help them to solve the difficult problems associated with the modern distribution of electricity.

The author appreciates fully the great difficulty which arises in applying purely economic reasoning, which is essentially abstract, unless there is a fairly complete appreciation by the reader of the concept of "elasticity." This paper is probably the first attempt to apply the abstractions of analytical economics to the concrete, if insufficient, facts of electricity supply and may thus prove to be of historic value. It will undoubtedly be criticized, and I feel that many of the conclusions may be reached more easily without reference to marginal theory, but I think that it should receive some praise for being in the van of other papers of a similar nature which will undoubtedly give a new impetus to scientific management in distribution.

The first part of the paper sets out the general principles governing the science of economics. Whilst I have a very real sympathy with, and appreciation of, the purpose of the paper, I am at once led to question whether it is possible to apply these principles, as has been done in the second part of the paper, so as to produce working results in terms of costs and charges for electricity. If I may quote from Mrs. Joan Robinson's book "The Economics of Imperfect Competition," where the various marginal and other types of curves used in Mr. Bolton's paper are referred to as "a box of tools for the analytical economist"—she states that "the area within which these tools can work is very narrowly bounded . . . the reader who is interested in results immediately applicable to the real world has every right to complain that these tools are of little use to him." The danger which I apprehend, therefore, is that a process of deduction may be used in order to obtain results which are beyond the limit of our premises, and I am apprehensive that we may be led into adopting a still greater number of tariffs and charges (on the basis of a separate cost per consumer) instead of attempting to limit the tariffs and charges to the very minimum.

There are, therefore, several points on which I must disagree with the author and regarding which I should be pleased to have his views.

The first point of difference is the general implication contained in the three paragraphs under the heading "Dual Aspects" (page 185). To a great extent my criticism of this section is fundamental to the whole paper. The implication is that electricity is a "special case" of a commodity, and the statement naturally follows from this implication that "it seems probable that there must always be a variety of electricity tariffs, differing in kind as well as in size." Although it may be considered heretical on my part, I fail to see that electricity is a special type of commodity. As the author says later, "the price of electricity, like the price of everything else, is governed by the twin considerations of supply and demand," and since the price of "everything else" is simple, why should the price of electricity be so complicated? The price of water is probably the simplest of all and that of electricity the most complicated, yet there is a remarkable similarity in the conditions governing the production and distribution of water and the production and distribution of electricity. Admittedly, water can be stored in bulk to meet peak requirements, whereas electricity requires coal, boilers, and generating plant, instead of a reservoir, to be kept available for this purpose, but the capital charges required to provide this requirement for water are probably higher than for electricity. (On page 206 of his book "Electrical Engineering Economics" Mr. Bolton shows that for five large towns the capital charges for water supply absorbed 57 % of the total receipts, and for electricity only 47 %). Is it not true that electricity can only be used for two primary purposes—to produce either heat or motion? If this is so, why do we find it necessary to produce a complicated and anomalous system of tariffs and charges such as is used for the sale of no other commodity?

I am led from this to question the possibility of attempting to attach a real value to the *marginal utility* of electricity. It does not seem possible to draw a curve of satisfaction to any numerical base, and I do not therefore consider that the author is justified in using a utility curve in order to arrive at a demand curve. My basic objection is that even if one were able in practice to measure "the marginal utility of the last increment that is just worth purchasing" (page 187), the attempt to apply this measurement so as to ascertain the charge per kWh which should be made to a consumer would result in each kWh being charged at a different price. This would surely lead away from unification in tariffs and charges, instead of assisting unification, which I maintain should be the main object of the supply industry. This is where I am bold enough to quarrel with the entire theory of utility, as used in an economic sense. The author, as an economist, is bound to consider the utility of electricity in the light of its *importance* to a consumer —I prefer to consider electricity in the light of its *usefulness* to the consumer, and a big gulf is fixed between the two views. Logically, and in pursuance of his argument in the paper, the author is bound to support the theory that we should charge a consumer for electricity on the basis of what electricity is worth to him, and, therefore, that we must have knowledge of the consumer's ability to pay. The first unit would be charged at a certain price (which would be different for each consumer), the next unit at a lower price, and so on throughout the whole range of *each consumer's* consumption. The supertax payer would, presumably pay in pounds sterling for his first kWh, and the labourer in shillings, whilst the value of each incremental kWh for each of the 8 million consumers would be determined by Mathematical Commissioners who could satisfactorily explain the theory of marginal utility to, chiefly, the supertax payers.

All our difficulties in relation to tariffs are due to the fact that we have attempted in a somewhat blind and confused manner to apply the theory of marginal utility to our methods of charging. When Dr. Hopkinson promulgated the theory of the two-part tariff, he presumably had in mind a system of charging based entirely upon the cost of production and related to the consumer only as regards his maximum demand. To-day we have retained the two-part tariff, divorced almost entirely from the scientific basis, and have divided it into two parts which we call the "primary charge" and the "promotional element." In progressive undertakings the latter charge is quite properly made so low ($\frac{1}{2}$d. or less per kWh) that the humblest consumer can afford to use electricity. Then comes the unconscious attempt to apply the marginal-utility theory by imposing a primary charge which, directly or indirectly, has some relation to the consumer's income or social standing, or to his "physical possessions," i.e. to the "importance" of electricity to the consumer. Surely the better method would be to ensure that there should be a price for electricity, common to rich and poor alike yet low enough to permit the poorest consumer to consume sufficient electricity for all purposes; at this common price the richer consumers will obviously consume more than the poorer consumers, if only because their houses are larger and their ability to buy consuming apparatus is greater. If the total income at this standard price of, say, $\frac{1}{2}$d. per kWh were insufficient to meet the cost of providing supply, then a primary charge might be imposed which was of a marginal nature and had some relation to the consumer's ability to pay more for his electricity. The best method of making this a true "marginal" charge would be to relate it to his actual payment for electricity during a current year. As I have pointed out in Fig. 9 and Table 14 of my recent paper,* the *average* primary charge would not be more than 15s. per quarter in 1939–40, and would grow less as time proceeded, until a virtual flat rate of $\frac{1}{2}$d. per kWh was reached.

I have one further point in relation to the author's definition of "elasticity of demand." I think that his explanation of economic principles may lead readers to consider that elasticity is necessarily high when prices are low, and I note that in my own paper which refers to this subject, a similar implication is made. I think that it should be stressed that elasticity can be low at low prices as well as at high prices. This is clearly explained by Marshall, who states: "in general, the elasticity of demand is small when the price of a thing is high in relation to the means of a class of consumer; and again when it is very low; while the elasticity is much greater for prices intermediate between the high and low level." The significance of this is only truly seen when we inquire what happens to the elasticity curves in the paper at the point where they terminate. It requires great caution not to read into the unknown future trend of a curve the continuation of its past history. The elasticity of the national domestic supply (figures for which are quoted later) also illustrates this point.

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* "Modern Factors affecting Electricity Costs and Charges," *Journal I.E.E.*, 1937, vol. 81, p. 429.

Turning now to Part II of the paper, several interesting points arise.

*Fig.* 7.—It is unfortunate that the data available to the author do not permit the value of elasticity to be determined beyond the terminating point of 1d. per kWh. Bearing in mind the remarks of Marshall on elasticity (quoted above), the tendency of the elasticity curve to turn at its termination of 1·2d. per kWh may be important.

*Fig.* 8.—I think there is an error in the values shown on the ordinate—the point marked 1·8d. should, apparently, be marked 1·6d.

*Figs.* 8 *to* 11.—Is it a correct assumption which the author makes (on page 200) that "the variations in elasticity are almost entirely due to the consumption response of existing consumers rather than to any growth in their numbers"? My reading of this curve gives an opposite conclusion. Fig. 11 shows that the elasticity for "consumers per head" increased by 120 % between 1·8d. and 1·5d. per kWh, whilst over the same range the elasticity for "units per consumer" increased by less than 40 %. This would mean that the rate of increase in consumers is greater than the rate of increase in consumption and that the average consumption per consumer was declining—a contention which I maintained in my recent paper.

### Table A

VALUES OF ELASTICITY AT VARYING PRICES PER kWh FOR COMBINED SUPPLIES FOR ALL UNDERTAKINGS

(From curves in *I.E.E. Journal*, vol. 81, page 488, Fig. E)

| Combined lighting, heating, and cooking | | Power supplies | |
|---|---|---|---|
| Average price per kWh | Elasticity | Average price per kWh | Elasticity |
| d. | | d. | |
| 0·6 | 2·12 | 1·7 | 0·3 |
| 5·5 | 1·78 | 1·6 | 0·56 |
| 5·0 | 2·18 | 1·5 | 0·79 |
| 4·5 | 2·21 | 1·4 | 1·16 |
| 4·0 | 2·32 | 1·3 | 1·48 |
| 3·5 | 2·05 | 1·2 | 1·98 |
| 3·0 | 2·38 | 1·1 | 1·93 |
| 2·5 | 2·41 | 1·0 | 1·77 |
| 2·25 | 1·96 | 0·9 | 1·76 |
| | | 0·8 | 1·53 |
| | | 0·7 | 2·3 |

*Fig.* 12.—I am rather puzzled about Fig. 12, and should be interested to have the author's views on the following points:—

(1) Is the marginal curve used to represent the marginal costs for an individual supplier, or for the supply industry as a whole?

(2) The marginal curve shows a condition of multiple equilibrium which is not referred to in the paper, and is not the greater profit obtained at the other point of equilibrium than that referred to by the author, i.e. at approximately 0·9d. × 110 kWh?

The difficulty in making deductions from Fig. 12 which I foresee, if the curve relates to the industry as a whole, is that there is no guarantee that the marginal supply curve is similar for any two undertakings.

Finally I note that the author has analysed the results of a single undertaking (Hampstead) because of the difficulty of obtaining marginal price and other data from the averaged figures of the results for combined undertakings. But the comparison data which are used (page 204) are based upon the combined results for all authorized undertakings. I am of the opinion that conclusions of a more useful nature can be obtained by a non-marginal study of the averaged national statistics for all undertakings. In proof of this I give in Table A the values of elasticity which I have calculated from the national curves shown in Fig. E on page 488 of vol. 81 of the *Journal*, from which it will be seen that they confirm to a remarkable degree the values of elasticity arrived at by Mr. Bolton.

**Mr. D. J. Bolton** (*in reply*): The policy which Mr. Clark criticizes has undoubted dangers, but there is one great difference between electricity and the commodities he mentions. In the former case the two types of consumption occur in the same household, so that there is no question of undercharging one section of the community at the expense of another. The convenience of electric heating, etc., is a potentiality for every single consumer, and it is to his advantage that the tariffs be so adjusted that he can realize these potentialities. Moreover, the terms under-charging and over-charging refer only to the immediate costs. If the policy criticized is not pursued, and lighting energy is priced at its bare cost, the effect will be a large increase in purely lighting connections, and the *ultimate* result may be a higher price all round.

Regarding Mr. Clark's further comments, there seems to be a confusion between the service and the energy which operates it. A wireless receiver is a luxury, but, once the wireless receiver is decided upon, electricity becomes a necessity, since the receiver cannot be operated by anything else. Heating is a necessity, but electricity for this purpose is a comparative luxury since there are several other good ways of satisfying the heating requirements. There seems also to be some confusion between the height of the curve and its slope. Mr. Clark mentions the high price of luxury articles, but I am concerned not with its actual magnitude but with what happens when you change it, and this concerns the shape of the curve rather than its height.

Substantially the difference between us concerns the terms rather than the facts. The names "luxury" and "necessity" are admittedly unfortunate, only I do not know of any better ones. Substitute "flat" and "steep" or "elastic" and "inelastic" and the facts will remain, and are hardly in dispute. If the price (whether high or low is not in question) of electricity for wireless reception is greatly reduced the wireless receiver will not be used much more than it now is. But a big reduction in the heating rate will result in a large increase in the sales.

In reply to Mr. Rowson regarding Fig. 3, I do not agree that average cost rather than marginal cost is the determining factor, and the authorities Mr. Rowson mentions (e.g. Mrs. Robinson) support me in this. In any case, when the marginal curve is as nearly level as it is in this figure the difference between the two curves is barely noticeable. Fig. A is misleading in this respect.

I am glad Mr. Rowson has emphasized certain factors (such as public opinion) hardly mentioned by me, and he is correct in saying that there is a misprint on page 194.

I agree as to the truth of his comment on Fig. 7, but not as to its relevance. Revenue is zero at an infinite price and at zero price, but this is purely academic and gives no clue to the shape of the curve, say, from ½d. to ¼d. A better base scale for Fig. 7 would be a logarithmic one with the zero at an infinite distance on the right. The revenue curve on such a base would probably continue to rise—one cannot say without further data. The fact that it becomes zero at zero price is a mere *reductio ad absurdum* and tells us nothing of its behaviour over the practicable portion of the curve.

Referring to his criticism of page 200, we are talking about two different things. The equation quoted from Lerner is identical with the relationship given at the end of my Appendix for the " profits " basis. But on page 200 I am not talking about the difference between the incremental and the mean price due to a changing unit rate. I am talking about the difference between the unit rate and the overall price on a two-part tariff. It is the former that is governed by the laws referred to, but it is the latter that is given in the Commissioners' Return. The laws operate according to the formula but at a different price from that which appears in the Return, and it is therefore necessary to view Figs. 8–11 with a shift in the base scale.

In reply to the quotations from economic journals (that the most likely possibility is that elasticity is constant) one can only ask "Why?" Electrical utilization is so varied that a change in the price level may well introduce it to a comparatively untouched field and entirely change its elasticity. In any case this *a priori* reasoning is very unsatisfactory, and the only sound plan is to analyse the actual data. My objection to many of the curves Mr. Rowson quotes is that they contain too many variables—industrial and domestic, units per consumer, and consumers per head of population, etc.

As to Fig. 13, it is true that there is a slight falling-off in elasticity in the later years and lower price-levels, but I think that Fig. 11 is more typical. The Hampstead curves are complicated by two elements, namely the time factor and the fact that in any one area a growth in connections means an extension of supplies to smaller consumers and hence a tendency to reduce the average consumption per connection. When using a target diagram of separate undertakings the former element does not come in, and the latter element is probably less marked.

My figures as to distributing a surplus were purely random ones, selected so as to illustrate the relative effect of lighting and power reductions (the lighting price reduction might equally well have been 10 % or even 12 %). Mr. Rowson has done well to apply exact figures and so illustrate the case more precisely. Undoubtedly there is a risk in making a big price reduction (especially if it is not backed by adequate propaganda), but it is a risk which has often proved to be worth taking. The converse is, I think, definitely proved by my figures, namely that a small reduction (in a high price) is seldom worth making.

Regarding Mr. Sumner's criticisms of Part I, he refers to my theory " that we should charge a consumer for electricity on the basis of what electricity is worth to him, and *therefore* to have knowledge of the consumer's ability to pay." My theory stops short at the word I have italicized, as I think the second half does not follow from the first. What the electricity is worth depends only upon what it can do and what the alternatives cost: it has nothing to do with the consumer's income. There would therefore be no need for the "Mathematical Commissioners," nor would the supertax payers be charged a higher price. My idea is not to make electricity so cheap that even a pauper can afford it, but to make it cheaper than its competitors for the same service. If the pauper cannot buy it we should raise his income, not depress the price of electricity.

Mr. Sumner, in my opinion, entirely misinterprets the fixed charge of the all-in tariff, which is *not* related to the user's income or social standing. A large house has a bigger fixed charge not because the owner can afford it but because the large house is likely to need more lighting, and no other sort of lighting is so good at the price. Basing tariffs on utility does not involve assessing incomes, nor do I agree that it leads away from unification, even though it requires each kWh to be charged for at a different price. We already do this under a plain all-in tariff, since each successive unit lowers the average price.

I am glad Mr. Sumner has stressed the fact that there is no necessary connection between elasticity and price, or between slope and height of the demand curve. My paper was evidently not clear on this point (see my reply to Mr. Clark). I am also obliged to Mr. Sumner for pointing out the missprint in the ordinate marked 1.8 in Fig. 8.

Regarding Part II, I disagree about Fig. 11. The elasticity of consumers per head is the *difference* between the two curves, and it only rises from 0·5 to 1·1 whilst the other rises from 0·3 to 2·4. But my conclusions and Mr. Sumner's are quite compatible, since they relate to different bases. I plotted the elasticity variations for different energy prices at a single time point, whereas he plotted on a time base and showed that the consumption per consumer was declining as the years went on.

Fig. 12 is for the whole industry and it does, as he says, show two possible equilibrium points. Starting with, say, 100 kWh sold at 2·2d., one can increase profits either by lowering or by increasing prices, and in each case a maximum profit point can be reached. I only explored the former, as being the practical case; no undertaking is likely at this stage to shrink their sales back to a scarcity value.

My reasons for plotting a single undertaking over a period of years are given in the paper. It illuminates certain points that the other method misses, but I still maintain that the other is generally preferable, and that values such as those Mr. Sumner quotes from his curves are partially vitiated by the change of date.

# THE DESIGN OF A POLARIZED TELEGRAPH RELAY*

By K. L. JENSEN, Ph.D., B.Sc.(Eng.), Associate Member.

(*Paper first received* 5*th November*, 1937, *and in final form* 15*th January*, 1938.)

## SUMMARY

Polarized telegraph relays have been used since the early days of the telegraph, but information regarding the design has hitherto been limited practically to a discussion of static conditions only.

The paper discusses static and dynamic conditions of polarized telegraph relays. From the point of view of performance relays are divided in two classes, viz. those with stiff and those with springy tongues. It is shown that a satisfactory performance as regards short transit time and freedom from chatter can only be obtained with relays of the second class. The necessity for damping the motion is pointed out.

In the latter part of the paper a practical design based on the theoretical part of the paper is described, and various test methods are discussed with regard to this design.

## LIST OF SYMBOLS

$\Phi$ = total field produced by permanent magnet.
$\Phi_1$, $\Phi_2$ = field produced by permanent magnet in pole-shoes.
$\Phi_L$ = field produced by line current.
$B$ = flux density in armature.
$A_1$ = cross-sectional area of armature.
$l$ = length of armature.
$l_1$ = length of armature plus tongue.
$V$ = magnetic potential across air-gaps.
$A$ = cross-sectional area of pole-shoes.
$b$ = breadth of pole-shoes.
$h$ = depth of pole-shoes and armature.
$w$ = thickness of armature.
$a$ = air-gap between armature and pole-shoes with armature in neutral position.
$x_1$ = distance of tip of armature from neutral position when armature deflected angle $\theta_0$.
$\theta$ = angular deflection of armature (radians).
$\pm\theta_0$ = maximum angular deflection (radians) of armature with stiff tongue.
$S_0$, $S$, $S_1$, $S_2$ = reluctances of air-gaps.
$G$, $g_1$, $g_2$ = conductances of air-gaps.
$F_m$ = force on armature produced by magnetic fields.
$\tau$ = torque on armature produced by magnetic fields.

## LIST OF SYMBOLS—*continued.*

$F$ = magnetomotive force produced by current in line coils.
$y$ = variable distance from pivot of armature (Fig. 25).
$x$ = variable distance from face of armature (Fig. 25).
$t$ = time.
$t_0$ = transit time of primary relay in line circuit of relay considered.
$t_t$ = transit time of relay considered.
$T$ = time-constant of relay circuit (unless where otherwise stated).
$L$ = self-inductance of relay circuit.
$R$ = resistance of relay circuit.
$\mathbf{T}$ = torque per unit radian produced by flux from permanent magnet.
$\mathbf{T}_f$ = torque produced by line field.
$\tau_1$ = impulsive torque due to impact.
$\mathbf{I}$ = moment of inertia of armature.
$\lambda = \sqrt{\frac{\mathbf{T}}{\mathbf{I}}}$
$\delta t$ = duration of impact.
$\alpha$ = acceleration of armature (unless where otherwise stated).
$\omega_a$, $\omega_b$, $\omega_1$ = velocities of armature.
$q$ = coefficient of restitution for tongue and contact butt.
$\theta_n$ = angular deflection of tongue away from contact butt after $n$th impact.
$N$ = number of turns of line coil(s).
$\sigma$ = density of armature.
$i$ = current in line coil(s).
$s$ = sensitivity of relay.
$K$ = torque exerted on armature by springy tongue per unit radian of deflection of armature from deflections $\pm\theta_0$.
$\pm\theta_a$ = deflection of armature with springy tongue due to action of torque produced by flux from permanent magnet alone.
$\pm\theta_b$ = deflection of armature with springy tongue due to action of torque produced by flux from permanent magnet, and field produced by line current.
$\gamma = \sqrt{\left(\frac{K - \mathbf{T}}{\mathbf{I}}\right)}$

## INTRODUCTION

Polarized telegraph relays are used both for the direct reception of line signals (line relays) and as secondary relays feeding automatic instruments or retransmitting signals to another line. It is generally not considered advisable, and in many cases it is not possible, to let the primary (line) relay perform these functions.

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* Abstract of a thesis approved for the Degree of Doctor of Philosophy in the University of London.

The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of The Institution not later than one month after publication of the paper to which they relate.

The principles, on which the magnetic construction of polarized relays is based are not new. One of the earliest forms is the Siemens relay, a modification of which is the Post Office polarized standard relay shown in Figs. 1 and 2. (In Figs. 1–6 the permanent-magnet portions of the relays are shown in bold lines to differentiate from the electromagnet portions, and the permanent fields are similarly drawn in a heavy dotted line whereas the line fields are indicated by dotted lines of normal thickness.) In these designs the *differential principle* is employed, the difference between two almost equally strong fields $\Phi_1$ and $\Phi_2$ supplying the required force of attraction between armature and pole-shoe. The only difference between these two designs is that the Siemens relay is provided with a single armature, while in the Post Office relay two armatures are carried on a common spindle.

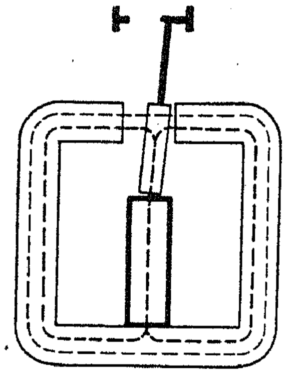

Fig. 1

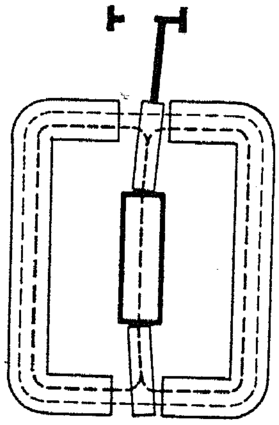

Fig. 2

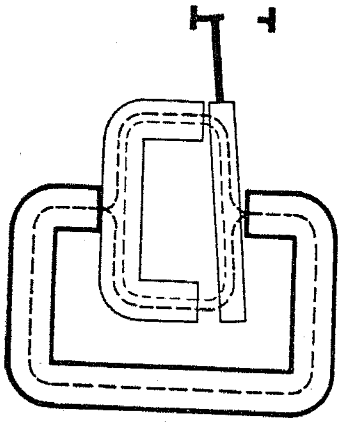

Fig. 3

The differential principle has been employed in a modified form in such old instruments as the polarized sounder (original pattern)* and in the German " Flügel Anker " (" wing armature ") relay,† both of which were of the form shown in Fig. 3.

Another principle, the *bridge principle*, Fig. 4 seems to have been introduced by Allan and Brown‡ and has also been used in relays of more recent date,§ among which occurs the modification shown in Fig. 5.|| The line coil is sometimes placed round the armature so that the whole line field is carried by the cross-sectional area of the armature, but in the old Allan and Brown relay the coil moves with the armature, whereas in more recent designs this is not so. In one case the line coil is placed on the pole-shoes of the electromagnet. All these relays except the modification shown in Fig. 5, make use of the fields across all the four air-gaps. The configuration of these relays is similar to that of a Wheatstone bridge, the armature taking the place of the galvanometer, and the permanent magnet the battery branch.

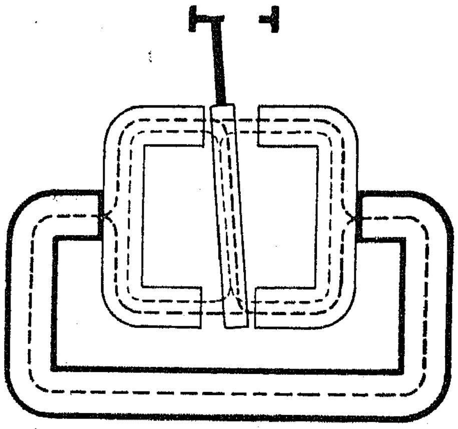

Fig. 4

In relays based on the differential principle the armature carries the whole of the magnetic field of the permanent magnet, while the armatures of bridge relays carry only the " out of balance " field which produces attraction.

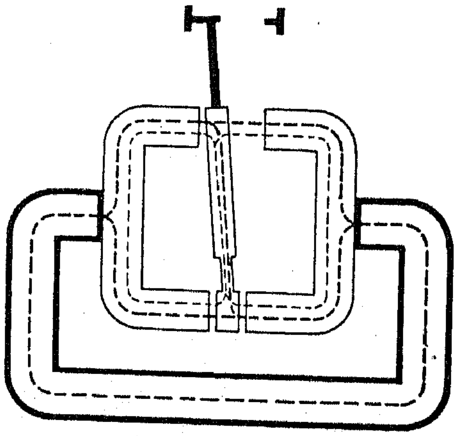

Fig. 5

Although polarized relays are used extensively in

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[*] See p. 179 of Bibliography (1).

[†] See p. 50 of Bibliography (2).

[‡] See Bibliography, (3).

[§] *Ibid.*, (4) and (5).

[||] *Ibid.*, (6).

telegraphy, many factors affecting the design do not seem to have been discussed.

In the following the design of a relay built on the differential principle, and intended especially as a secondary relay, will be discussed. For simplicity's sake an ideal relay is considered, i.e. a relay with no magnetic leakage.

In general, relays may be grouped in two classes, viz.:—

(1) Relays with ideally stiff tongues.
(2) Relays with springy tongues.

Clearly, all tongues are springy as they must deflect a certain amount when resting on either contact while at the same time under the influence of attraction of the field acting on the armature, but in practice many relay tongues are so short and the forces produced by the fields so small that it becomes natural to include such relays in the first group, while relays with very long tongues and/or strong permanent fields may be grouped in the second class.

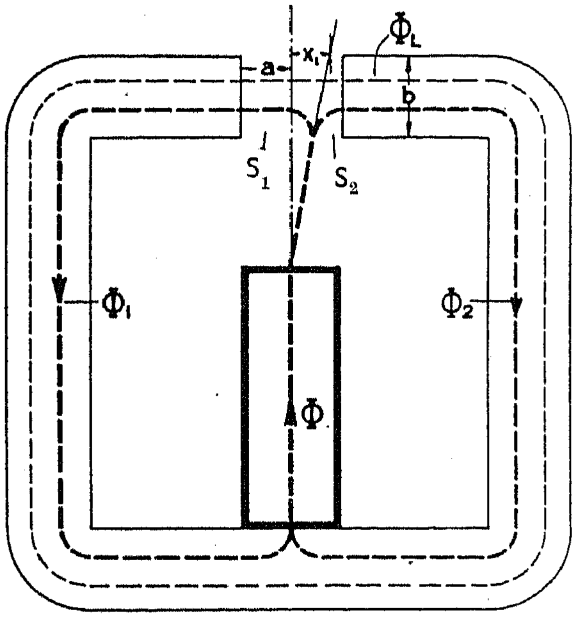

Fig. 6

## PART I. RELAYS WITH IDEALLY STIFF TONGUES*

### (1) General Static Equations

With reference to Fig. 6 we have:

Reluctances of air-gaps:

$$S_1 = \frac{a + \dfrac{l - \frac{1}{2}b}{l}x_1}{A}$$

$$S_2 = \frac{a - \dfrac{l - \frac{1}{2}b}{l}x_1}{A}$$

Joint reluctance $S = \dfrac{S_1 S_2}{S_1 + S_2} = \dfrac{a^2 - \left(1 - \dfrac{b}{2l}\right)^2 x_1^2}{2aA}$

Now, $\Phi = \Phi_1 + \Phi_2$

Hence $\Phi_1 = \Phi \dfrac{S_2}{S_1 + S_2} = \Phi \dfrac{a - \left(1 - \dfrac{b}{2l}\right)x_1}{2a}$

$$\Phi_2 = \Phi \frac{S_1}{S_1 + S_2} = \Phi \frac{a + \left(1 - \dfrac{b}{2l}\right)x_1}{2a}$$

The force on the armature with no current in the line coil(s), i.e. $\Phi_L = 0$, is given by

$$F_m = \frac{1}{8\pi A}(\Phi_2^2 - \Phi_1^2)$$

$$= \frac{1}{8\pi A}\cdot\frac{\Phi^2}{4a^2}\left[\left\{a + \left(1 - \frac{b}{2l}\right)x_1\right\}^2 - \left\{a - \left(1 - \frac{b}{2l}\right)x_1\right\}^2\right]$$

$$= \frac{1}{8\pi A}\Phi^2\left(1 - \frac{b}{2l}\right)\frac{x_1}{a}$$

i.e. the force is proportional to the deflection ($x_1$).

When current flows in the line coil(s), the line field $\Phi_L$ is superposed upon the fields $\Phi_1$ and $\Phi_2$, and the equation for the force becomes

$$F_m = \frac{1}{8\pi A}\left\{(\Phi_2 - \Phi_L)^2 - (\Phi_1 + \Phi_L)^2\right\}$$

$$= \frac{1}{8\pi A}\left\{(\Phi_2^2 - \Phi_1^2) - 2\Phi_L(\Phi_1 + \Phi_2)\right\}$$

It should be noted here that it is not strictly correct to assume a single value for the line flux across either air-gap, owing to the asymmetrical position of the armature away from the neutral position. In practice, however, the assumption may be considered valid. (For the practical design relay to be described later the difference between the line field across either air-gap is only 0·2 %, even when the deflection of the armature is the largest possible.)

The armature can begin to move* when $F_m$ is reduced to zero, i.e. when

$$\Phi_L = \frac{\Phi_2^2 - \Phi_1^2}{2(\Phi_2 + \Phi_1)} = \frac{\Phi_2 - \Phi_1}{2}$$

Expressed in terms of $\Phi$ this becomes

$$\Phi_L = \frac{\Phi}{4a}\cdot 2\left(1 - \frac{b}{2l}\right)x_1 = \Phi\left(1 - \frac{b}{2l}\right)\frac{x_1}{2a}$$

The sensitivity of the relay depends upon $\Phi_L$, but this question will be discussed more fully in a later Section.

The above simple theory is based on two assumptions, first that the magnetic field between two inclined surfaces is the same as that which would exist if the surfaces were parallel and separated by an amount equal to the mean distance of the former, and, second, that the total field, $\Phi$, is constant whether the armature be deflected or not. These questions are discussed in the Appendix.

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* After presentation of this thesis the author has become acquainted with a recent work on stiff-tongue relays [see Bibliography (16)] which to some extent covers the same ground as Part I of the present thesis; the conclusions with regard to design criteria differ, however.

* See p. 134 of Bibliography (7).

### (2) Dynamic Equation

It was shown above that when the line field is superposed on the permanent field the force on the armature is

$$F_m = \frac{1}{8\pi A}\left\{(\Phi_2^2 - \Phi_1^2) - 2\Phi_L\Phi\right\}$$

i.e. the force is equal to the difference between the force that would act on the armature in the absence of the line field and another force which is proportional to $\Phi_L$.

We shall denote the torque produced by the former by $\mathbf{T}\theta$—it has already been shown that the torque is proportional to the deflection—and that produced by the latter force by $\mathbf{T}_f f(t)$, where $f(t)$ indicates the way in which the current, and therefore the torque, varies with time.

For primary relays it is natural to assume a sinusoidal wave-shape of the current* as the signals after propagation along a line of great length are, generally, sinusoidal in shape, but for a secondary relay—the type under discussion—it is natural to assume that

$$f(t) = (1 - e^{-t/T})$$

where $T = \dfrac{\text{self-inductance of secondary relay circuit}}{\text{resistance of secondary relay circuit}}$

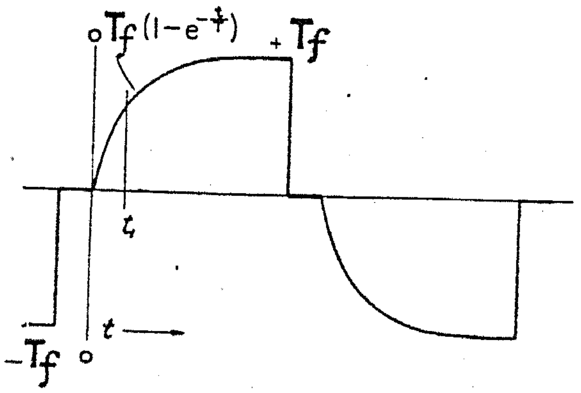

Fig. 7

is the time-constant of the circuit which is fed from the contacts of the primary relay.

The variation of the torque with time is shown in Fig. 7. During a certain interval the torque is zero, due to the transit (including rebound) of the tongue of the primary relay. This time interval, however, will have no effect on the calculations as the variation from, for instance, $-\mathbf{T}_f$ to 0 does not alter the position of armature and tongue, it being remembered that the tongue is considered ideally stiff.

It will be convenient to count the time from $t = t_1$, where $t_1$ is given by

$$\mathbf{T}\,\theta_0 = \mathbf{T}_f(1 - e^{-\frac{t_1}{T}})$$

i.e. the time at which the two opposing torques equal each other and at which, therefore, the movement of the armature may begin.

Consequently, the equation of motion becomes

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta = \mathbf{T}_f(1 - e^{-\frac{t_1+t}{T}})$$

where damping as well as any possible interaction between $\theta$ and $\mathbf{T}_f$ (e.m.f. induced in the circuit of the line coil due to the motion of the armature) is neglected.

The case considered is that of the movement of the tongue from the negative to the positive contact, $\mathbf{T}_f f(t)$ being positive. The product $\mathbf{T}\theta$, therefore, is respectively negative or positive, according to whether $\theta$ is negative or positive.

The solution of the equation of motion is

$$\theta = -\frac{\mathbf{T}_f}{\mathbf{T}} + \frac{T^2(\mathbf{T}_f - \mathbf{T}\theta_0)}{\mathbf{I}(\lambda^2T^2 - 1)}e^{-\frac{t}{T}} + C_1e^{\lambda t} + C_2e^{-\lambda t}$$

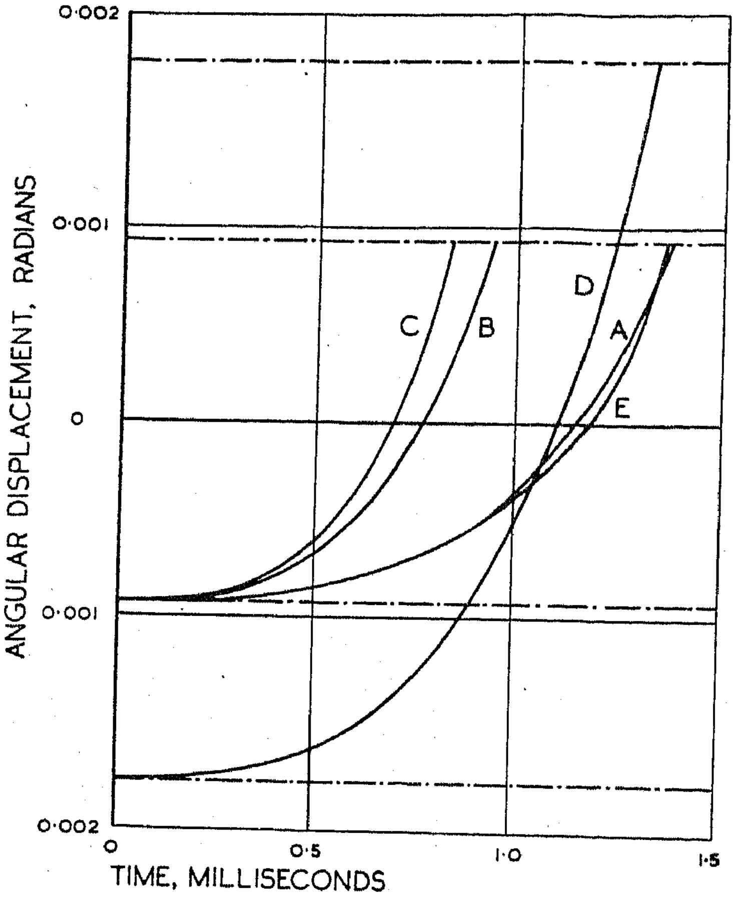

Fig. 8

where

$-\theta_0$ = the deflection of the armature (in radians) when the tongue rests on the negative contact.

$$\lambda = \sqrt{\frac{\mathbf{T}}{\mathbf{I}}}$$

$$C_1 = -\frac{\theta_0}{2} + \frac{\mathbf{T}_f}{2\mathbf{T}} - \frac{T(\mathbf{T}_f - \mathbf{T}\theta_0)}{2\mathbf{I}(\lambda^2T^2 - 1)}\left(T - \frac{1}{\lambda}\right)$$

$$C_2 = -\frac{\theta_0}{2} + \frac{\mathbf{T}_f}{2\mathbf{T}} - \frac{T(\mathbf{T}_f - \mathbf{T}\theta_0)}{2\mathbf{I}(\lambda^2T^2 - 1)}\left(T + \frac{1}{\lambda}\right)$$

The most important terms are $C_1e^{\lambda t}$ and $C_2e^{-\lambda t}$. In the practical examples worked out below $C_2$ becomes negative, and it is seen, therefore, that in order that $\theta$ shall quickly attain positive values it is necessary that the value of $\lambda$ should be as high as possible.

In Fig. 8 curve A shows the motion of the tongue from the negative to the positive contact.

The different constants used in calculating the curve are taken from a practical design of a relay the contact pressure of which is 40 g. for a tongue-travel of 0·1 mm.

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* See p. 137 of Bibliography (7).

with no line current flowing, $\lambda = 2\,570$, $T = 1 \times 10^{-3}$ sec., $\mathbf{T}_f = 0{\cdot}317 \times 10^6$ dyne-cm. for a final value of the current in the line coil of 7·5 mA, and the self-inductance $L = 0{\cdot}7$ henrys.

The relative importance of the different constants is best borne out by calculating the curve of motion for various values of the more important factors. Curve B, therefore, shows the motion of the tongue for the current in the line coil doubled; curve C for both current and contact pressure doubled; and curve D for current and length of travel doubled but contact pressure per unit radian (with no line current flowing) the same as for curves A and B.

Comparison of curves A and B shows that doubling the ratio $\mathbf{T}_f/\mathbf{T}$ improves the transit time from 1·4 to 0·9 millisec. Doubling the contact pressure, i.e. multiplying $\lambda$ by $\sqrt{2}$ and keeping the current the same as in curve B, further improves the transit time to about 0·8 millisec. (curve C). Finally, curve D shows that doubling the length of travel, while at the same time the current is doubled, has but little effect on the transit time; the velocity, however, with which the tongue strikes the positive contact is somewhat higher than for curve A.

For curve E, $\mathbf{T}$ has been halved, which increases the ratio $\mathbf{T}_f/\mathbf{T}$ but decreases the value of $\lambda$. It is seen that the transit time practically coincides with that for curve A, and no advantage, therefore, seems to be obtained from reducing the contact pressure.

In order not to complicate the calculations the dimensions of the armature have been assumed constant. It will be shown later that, actually, in order to obtain the utmost efficiency from the relay, the cross-sectional area of the armature for a relay of the differential flux type should be varied according to the value of the permanent flux required to obtain the desired contact pressure.

### (3) Rebound

At the instant the tongue strikes the positive contact screw it will receive an impulsive torque due to the impact which will, in general, repel the tongue from the contact.

The impulsive torque, $\tau_1\delta t$, produced by the impact may be found from the following simple equation

$$\tau_1\delta t = \mathbf{I}(\omega_a - \omega_b)$$

where $\omega_a$ is the angular velocity of armature and tongue when the impact commences, $\omega_b$ is the velocity after the impact, and $\delta t$ is the duration of the blow.

In order that the tongue shall not leave the contact $\omega_b = 0$, i.e.

$$\tau_1\delta t = \mathbf{I}\omega_a$$

Unfortunately both $\tau_1$ and $\delta t$ are unknown, otherwise the problem to be investigated would be whether the torques produced by the magnetic fields are at least equal to $\tau_1$, which is the condition for the above equation to be true.

We are, therefore, not justified in assuming that the tongue will remain on the contact but must assume that it is repelled.

We shall make use of the following law: When a moving body strikes a fixed body with velocity $v$, its velocity after the impact will be $-qv$, where $q$ is the coefficient of restitution. The value of $q$ must be determined experimentally.

Provided the deflections away from the contact screw are small it may be safe to assume that the torques acting on armature and tongue are constant while the rebound takes place. This is not strictly true as $\mathbf{T}\theta$ varies with $\theta$ and $\mathbf{T}_f f(t)$ varies with time. On the assumption stated the acceleration of armature and tongue is constant.

The question may now be investigated in the following way:

The angular velocity with which the tongue strikes the contact screw at the first impact is $\omega_1$; after the impact it is reversed and reduced to $q\omega_1$.

The tongue will now move an angle $\theta_1$ away from the contact screw, given by

$$\theta_1 = q\omega_1 t - \tfrac{1}{2}\alpha t^2$$

At the displacement $\theta_1$ the velocity is zero, which gives

$$q\omega_1 - \alpha t = 0$$

Hence $$t = \frac{q\omega_1}{\alpha}$$

The formula for the displacement thus becomes

$$\theta_1 = \frac{q^2\omega_1^2}{\alpha} - \frac{q^2\omega_1^2}{2\alpha} = \frac{q^2\omega_1^2}{2\alpha}$$

At each impact the velocity is reduced $q$ times and we get, therefore,

$$\text{displacement after } n\text{th impact, } \theta_n = q^{2n}\frac{\omega_1^2}{2\alpha}$$

Clearly, as $n$ increases $\theta_n$ becomes so small as to be neglected and the tongue may be considered at rest. In order that the tongue shall not be repelled at all $\theta_n$ must be vanishingly small even for $n = 1$.

It is seen, therefore, that

(a) $\alpha$ should be as high as possible, provided that increasing $\alpha$ $m$ times does not increase $\omega_1$ more than $\sqrt{m}$ times.

(b) $q$ should be as small as possible.

Very high values of $\alpha$ seem to have been employed by Franzke* but, as mentioned in the Introduction, tongues of relays having very strong permanent fields are actually to be regarded as springy, and, as will be shown later, the question of rebound can be most usefully studied for springy-tongue relays.

It has been proposed† to reduce $q$ by setting the contacts (generally of platinum) in lead.

If the current increases very rapidly with time, one will not be justified in assuming that the torque acting on the armature is constant during the rebound period, and in this case the tongue will come to rest sooner than it would in the case treated theoretically above. A steep wave-front of the current wave may be obtained by shunting the series resistance by a condenser; care should be taken so to determine the capacitance of the condenser that the possibility of an oscillatory current is avoided.

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* See Bibliography (10).

† *Ibid.*, (11).

Practice shows that most relays with stiff tongues exhibit the phenomenon of rebound, and since it is usual to add the time of rebound to the transit time, both being regarded as lost so far as the useful time of contact is concerned, it will be understood that the time of transit plus the time of rebound may easily amount to several milliseconds.

In the design of a relay tongue shown in Fig. 9 two flat springs carrying contact plates press against one another, and it is asserted that by this means the kinetic energy of tongue and armature is converted into heat by the rubbing action of the springs at the impact.

This design, which seems to have been introduced in America,* is also used in Germany† and by the British Post Office‡ but, actually, it is only an example of a springy tongue.

### (4) Factors Affecting Design

The performance of a polarized telegraph relay is determined by

(*a*) Transit-time,
(*b*) Operating current,
(*c*) Self-inductance.

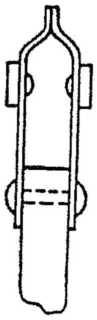

Fig. 9

The transit time should be short. High ratios of **T/I** and of $\mathbf{T}_f/\mathbf{T}$ both yield this result, but it is not possible to formulate an equation from which the value of the transit time may be found even approximately.§ If the value of **T/I** be low the relay may function for a smaller minimum current, but for the same transit time it may require a current at least equal to that of the relay having a high ratio **T/I**. As far as a practical design is concerned, therefore, it should be attempted *a priori* to make the ratio **T/I** as high as possible.

However, the question of sensitivity instead of that of operating current will be discussed in the present Section because it is possible to demonstrate how the sensitivity varies with the other properties and with the dimensions of the relay. Since, however, it is impossible to show how the sensitivity varies with transit time, the only alternative is to show how it varies with the ratio $\lambda = \sqrt{\mathbf{T}/\mathbf{I}}$.

Rebound should be absent, but this is practically unattainable with relays having a stiff tongue.

A high value of **T/I** means a high value of the permanent field and a small value of the moment of inertia of the armature, but these are two conflicting requirements as a high value of the permanent field calls for a high value of the cross-sectional area of the armature, i.e. a high value of **I**.

It is a fortunate property of relays built on the differential principle, however, that the flux density in the major portion of the armature is unaltered by the deflections of the armature and by the superposing of the line field across the pole-faces of area *A* as demonstrated in the Appendix. And, further, that at the pole-faces the permanent field divides into two almost equal fields each of approximate magnitude $\frac{1}{2}\Phi$ (Fig. 10), so that for the general case where $A > A_1$ the flux density in the pole-shoes is less than half that in the armature. Also, the superposed line field is, in general, a fraction of $\frac{1}{2}\Phi$, so that if the relay be designed for a certain flux density in the major portion of the armature the flux densities in the part of the armature which faces the pole-shoes, and in the pole-shoes, will be much smaller.

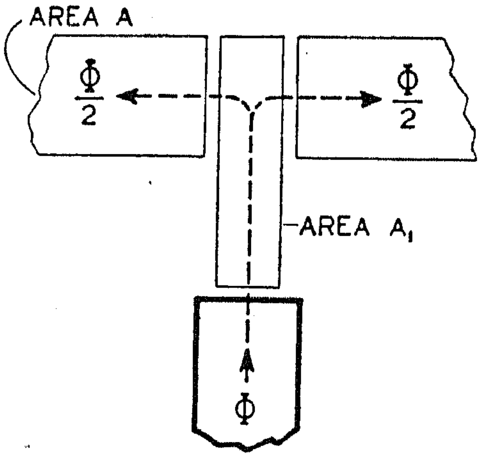

Fig. 10

Due to the fact that the field through the major portion of the armature is practically constant under working conditions, a fairly high value for the flux density may be assessed.

For a single-armature relay (Fig. 6) the torque produced by the permanent field $\Phi$ is shown (in the Appendix) to be

$$\tau = \frac{1}{8\pi A} \cdot \frac{x_1}{a} \Phi^2 \left(1 - \frac{b}{2l}\right)\left(l - \frac{b}{3}\right)$$

hence $$\mathbf{T} = \frac{\tau}{\theta_0} = \frac{\tau}{x_1/l} = \frac{1}{8\pi A}\Phi^2 \frac{l^2}{a}\left(1 - \frac{b}{2l}\right)\left(1 - \frac{b}{3l}\right)$$

The moment of inertia of the armature is

$$\mathbf{I} = \tfrac{1}{12} l A_1 \sigma (4l^2 + w^2)$$

where $\sigma$ = density of armature.

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* See Bibliography (6). † *Ibid.*, (2), p. 45. ‡ *Ibid.*, (9), p. 156.

§ By assuming $T = 0$, i.e. that the line current attains its maximum value instantaneously, Werner [see Bibliography (16)] deduces for the transit time a formula which with the notation employed in the present paper is

$$t_t = \frac{1}{\lambda} \text{ arc cosh } \frac{\dfrac{\mathbf{T}_f}{\mathbf{T}\theta_0} + 1}{\dfrac{\mathbf{T}_f}{\mathbf{T}\theta_0} - 1}$$

As $\mathbf{T}_f$ is proportional to $\Phi$ and, therefore, may be put equal to $ik\sqrt{\mathbf{T}}$, this formula may be transformed into:

$$t_t' = \sqrt{\frac{\mathbf{I}}{\mathbf{T}}} \text{ arc cosh } \frac{\dfrac{ik}{\theta_0\sqrt{\mathbf{T}}} + 1}{\dfrac{ik}{\theta_0\sqrt{\mathbf{T}}} - 1}$$

from which is seen that $t_t$ varies inversely as $\lambda (= \sqrt{\mathbf{T}/\mathbf{I}})$ only if **I** be the only variable.

The tongue is assumed to be so light that it does not add to the moment of inertia of the armature. This assumption, of course, is not a general one; for relays with small armatures the assumption is, in most cases, not valid.

The sensitivity depends on the field produced by the current in the line coil(s), and in Section (1) it has been shown that the minimum value of the line field for the armature to begin to move is given by

$$\Phi_L = \Phi\left(1 - \frac{b}{2l}\right)\frac{x_1}{2a}$$

But $$\Phi_L = \frac{F}{2a/A}$$

where $F$ = magnetomotive force applied to the magnetic circuit when line current flows.

$2a/A$ = reluctance of the two air-gaps in series.

The reluctance of the iron is neglected; we shall design the relay so that it becomes negligible compared with $2a/A$.

Now $$F = \frac{4\pi}{10}iN$$

$$4\pi\frac{iN}{10} = \Phi\left(1 - \frac{b}{2l}\right)\frac{x_1}{A}$$

where $N$ = number of turns on coil(s).

$i$ = least current to operate the relay (in amperes).

The sensitivity, $s$, is inversely proportional to $i$.

The self-inductance of the relay under the same conditions as those assumed for the sensitivity is

$$L = \frac{4\pi N^2A}{10^9 2a}$$

### (5) Interdependence of Salient Properties

The Siemens and the Post Office relays are both representative examples of relays designed on the differential principle, and as the former is provided with only one armature while the latter has two it will be interesting to know the relative merits of the two designs. An investigation of this problem has shown that the single-armature relay permits of a greater value of $\mathbf{T}/\mathbf{I}$, all other factors being the same. Partly for this reason and partly also because this form has been preferred for certain practical reasons, the present paper is confined to the discussion of a single-armature relay.

$\lambda$, $s$, and $L$, are all functions of the dimensions of the relay, so that by expressing one as a function of the two others we shall expect to obtain an equation which will be a useful guide when settling the dimensions of the relay.

We have

$$\Phi_L = \frac{FA}{2a} = \Phi\left(1 - \frac{b}{2l}\right)\frac{x_1}{2a} \quad \text{(Section 4)}$$

$$\therefore\ F = \Phi\left(1 - \frac{b}{2l}\right)\frac{x_1}{A} = BA_1\left(1 - \frac{b}{2l}\right)\frac{x_1}{A}$$

where $BA_1 = \Phi$

$$\mathbf{T} = \frac{1}{8\pi A}\cdot\frac{l^2}{a}B^2A_1^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)$$

hence

$$B = \sqrt{\left[\mathbf{T}\frac{8\pi Aa}{A_1^2l^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)}\right]}$$

$$\therefore\ F = A_1\left(1 - \frac{b}{2l}\right)\frac{x_1}{A}\sqrt{\left[\mathbf{T}\frac{8\pi Aa}{A_1^2l^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)}\right]}$$

But, also

$$F = \frac{4\pi iN}{10}$$

$$L = \frac{4\pi N^2A}{10^9 2a};\quad N = \sqrt{\left[\frac{10^9 2aL}{4\pi A}\right]}$$

$$\therefore\ F = \frac{4\pi i}{10}\sqrt{\left[\frac{10^9 2aL}{4\pi A}\right]} = \frac{i}{10}\sqrt{\left[\frac{4\pi 10^9 2aL}{A}\right]}$$

Equating the two expressions for $F$ we get

$$\frac{i}{10}\sqrt{\left[\frac{4\pi 10^9 2aL}{A}\right]} = F$$
$$= A_1\left(1 - \frac{b}{2l}\right)\frac{x_1}{A}\sqrt{\left[\mathbf{T}\frac{8\pi Aa}{A_1^2l^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)}\right]}$$

Re-arranging, we obtain

$$i = 10\left(1 - \frac{b}{2l}\right)x_1\sqrt{\left[\frac{\mathbf{T}}{L}\cdot\frac{1}{l^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)10^9}\right]}$$

and the sensitivity becomes

$$s = \frac{1}{i} = 10^3\frac{\sqrt{\left[\frac{L}{\mathbf{T}}\cdot l^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)10\right]}}{x_1\left(1 - \frac{b}{2l}\right)}$$

The expression so far only includes $\mathbf{T}$ but not $\lambda = \sqrt{(\mathbf{T}/\mathbf{I})}$.

The formula for the moment of inertia (Section 4) can be simplified. Remembering that $w << l$ we get, approximately,

$$\mathbf{I} \simeq \frac{1}{12}4l^3A_1\sigma = \frac{l^3}{3}A_1\sigma$$

Inserting this value in the expression for $s$ we get

$$s = 10^3\frac{\sqrt{\left[\frac{\frac{\mathbf{I}}{\mathbf{T}}\cdot Ll^2\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)10}{\frac{1}{3}l^3A_1\sigma}\right]}}{x_1\left(1 - \frac{b}{2l}\right)}$$

which becomes

$$s = \frac{10^3}{\lambda x_1}\sqrt{\left[\frac{30L\left(1 - \frac{b}{3l}\right)}{lA_1\sigma\left(1 - \frac{b}{2l}\right)}\right]}$$

It is worth remembering that when evolving this expression it has been tacitly assumed that no limitations were imposed on the magnitudes of $B$, $A$, or $a$.

The formula shows that no matter what be the values of $A$ or $a$, the sensitivity is constant for given values of $L$ and $\lambda$ (the factors $x_1$, $l$, $A_1$, $\sigma$, and $b$ being constant). The designer, therefore, may vary the main dimensions to suit practical requirements without affecting the principal properties of the relay, provided the limitations regarding $B$, as mentioned below, are not exceeded. Franzke* states that, by increasing $A$, $L$ is increased, but he apparently has overlooked the fact that for a given sensitivity $N$ may be reduced, so that the original value of $L$ is retained.

In practice $B$ may not be varied indiscriminately as saturation governs its maximum possible value.

Reverting to the expression for $B$, which may be put into the form

$$B = \lambda\sqrt{\left[\frac{8\pi A a l \sigma}{3A_1\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)}\right]}$$

it is seen that, for $B$ fixed, $\lambda$ increases with decreasing $a$ and $A$.

$B$ should, therefore, be fixed at as high a value as possible, and $A$ and $a$ should be as small as possible in order to obtain a high value of $\lambda$.

As the ratios

$$\frac{1 - \frac{b}{3l}}{1 - \frac{b}{2l}} \quad \text{and} \quad \frac{l}{\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)}$$

appearing in the expressions for $s$ and $B$ respectively do not vary very much for different ratios $b/l$ it is further seen from the expression for $s$ that $l$ should be small but, in order not to increase the angle of travel, $x_1$ should be small also.

Of course, the minimum possible value for $l$ is $b$, but in practical design ratios of $b/l$ about $0 \cdot 5$ or less are necessary. Generally, when working out a practical design, it is found that space does not permit increasing the value of $b/l$, the only alternative then being to decrease $x_1$, which can be done only by increasing the length of the tongue at the same time in order to keep the contact travel constant. On the other hand, the length of the tongue may then be increased to such an extent that its addition to the moment of inertia is no longer negligible and the formulae above cease to be true.

It is seen from the above discussion that the principal factors which may improve the design are $a$ and $A$. Experience will tell the designer how far these dimensions may be reduced.

It has been suggested† for polarized telegraph relays to introduce a term "specific minimum operating current," equal to the minimum operating current divided by the contact pressure.

From the formula here given for the sensitivity, it is seen that the minimum operating current varies directly as the square root of the contact pressure and inversely as the square root of the self-inductance. Having obtained the minimum operating current for a certain relay, it would appear to be more strictly correct, therefore, to convert it to a "specific minimum operating current" by dividing the test result by the square root of the contact pressure and multiplying it by the square root of the self-inductance. It should be clearly understood that the value so obtained refers only to a definite contact distance, which in practice might be $0 \cdot 10$ mm.

It will be noted that the question of the dimensions of the permanent magnet has not been touched on at all in connection with the influence of dimensions on the design. The reason for this is that with the magnetic materials now at the disposal of the manufacturer of permanent magnets no difficulty exists in meeting the requirements of strong magnetic fields, saturation of the armature being the only limit in the design.

## PART II. RELAYS WITH SPRINGY TONGUES

### (6) General Dynamic Equations

Whereas for a relay provided with a stiff tongue the deflection of the armature is constant for all values of $\mathbf{T}_f f(t)$ less than $\mathbf{T}\theta_0$, the problem of the motion of an armature provided with a springy tongue is complicated by the fact that the armature will assume a deflection greater than $\theta_0$ even when no current flows through the line coil(s) and another, still greater, deflection when line current flows, and the velocity of the armature will no longer be zero for the deflection equal to $\theta_0$, i.e. at the instant when the tongue leaves either contact screw. Indeed, the short transit time which may be obtained with a relay with a springy tongue is due to this property. Furthermore, a suitable choice of the springiness of the tongue makes it possible to avoid rebound almost entirely.

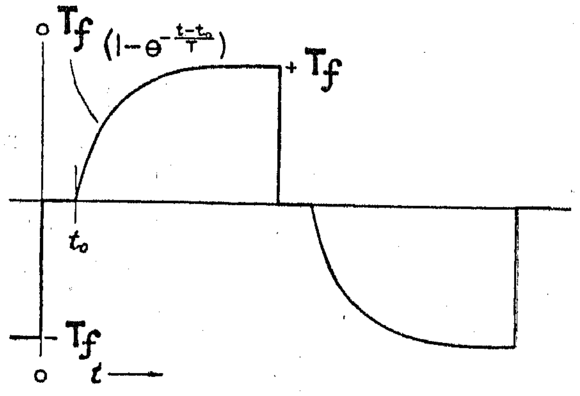

Fig. 11

Referring to Fig. 11, let the torque produced by the current through the line coil(s) be of the form $\mathbf{T}_f(1 - e^{-(t-t_0)/T})$ as before. (The time $t_1$ in Fig. 7 will not necessarily be identical with the instant $t_1$ mentioned in the present Section of the paper.) At time 0 the current through the line coil(s) is suddenly interrupted from its maximum negative value (the tongue of the primary relay leaves its negative contact screw) and does not start to rise until time $t_0$, when contact is established between the tongue of the primary relay and its positive contact screw. $t_0$ may vary between the limits 0 and $\infty$ but not assume the value 0. In the latter case the current would not fall from its maximum negative value to zero as it is not interrupted, but it would rise aperiodically from its maximum negative to its maximum positive value on account of the secondary current produced by the e.m.f. due to the self-inductance of the relay. In order to limit the problem to the wave-form of Fig. 11, which is the more interesting one because it represents actual conditions in a general form, we shall in the following discussion, when discussing very small values of $t_0$, assume that $t_0$ is so great that an interruption of current actually takes place, but at the same time so small that it may be put equal to 0 in the equations given. The duration of a half-cycle of the signal is assumed so long in comparison with $T$ that the value of the torque produced by the current at the end of each half cycle is equal to its maximum value $\pm \mathbf{T}_f$. Consequently, from time 0 the motion of the armature is given by

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* See p. 33 of Bibliography (10).

* See p. 32 of Bibliography (10).

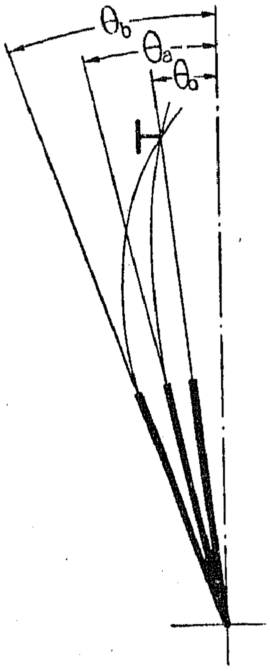

Fig. 12

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta + K\big(\theta - (-\theta_0)\big) = 0$$

or

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta + K(\theta + \theta_0) = 0 \quad . \quad . \quad . \quad . \quad (1)$$

where $K$ is a constant depending on the springiness of the tongue. $K$ must always be greater than $\mathbf{T}$ in order that the tongue shall not bend so much that its motion is limited by the pole-shoes. The tongue is assumed to be weightless.

The deflection of the armature under the influence of the torque $-\mathbf{T}_f$ is given by

$$-\mathbf{T}\theta + K(\theta + \theta_0) = -\mathbf{T}_f$$

$$\theta = \frac{-\mathbf{T}_f - K\theta_0}{K - \mathbf{T}} = -\theta_b$$

while the deflection for $\mathbf{T}_f = 0$ is

$$-\theta_a = -\frac{K\theta_0}{K - \mathbf{T}} \text{ (see Fig. 12).}$$

At time 0, $\theta = -\theta_b$ and $d\theta/dt = 0$, and the solution of (1) becomes

$$\theta = -\frac{K}{K - \mathbf{T}}\theta_0 - \frac{\mathbf{T}_f}{K - \mathbf{T}}\cos\gamma t \quad . \quad . \quad (1a)$$

where $\gamma = \sqrt{[(K - \mathbf{T})/\mathbf{I}]}$. This expression represents the motion of the *armature* from time 0 until $t_0$.

In equation (1) damping is neglected, but it is important to remember that damping is present; indeed the very assumption that the deflection is equal to the steady value $-\theta_b$ at time 0 is based on the fact that the oscillations of the armature during the preceding half-cycle are assumed to have died away by the time the current in the line coil(s) is interrupted. It will be shown later that, although neglected in the calculations, damping is very helpful—so much so that a relay with a springy tongue will not function satisfactorily in the absence of damping.

Equation (1$a$) represents an oscillatory motion of amplitude $\mathbf{T}_f/(K - \mathbf{T})$ superposed on the deflection $-K\theta_0/(K - \mathbf{T})$. $\mathbf{T}_f/(K - \mathbf{T})$ will always be so high that the tongue will leave the contact screw if the current-less interval ($t_0$) be too long. This will be dealt with later. In the following we shall assume that this is not the case.

At time $t_0$ the current starts to rise in the line coil(s), and the motion of the armature is now given by

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta + K(\theta + \theta_0) = \mathbf{T}_f\left(1 - e^{-\frac{t-t_0}{T}}\right) \quad . \quad (2)$$

the solution of which is

$$\theta = \frac{1}{K - \mathbf{T}}\left\{\mathbf{T}_f\left(1 - \frac{\gamma^2T^2}{1+\gamma^2T^2}e^{-\frac{t-t_0}{T}}\right) - K\theta_0\right\}$$
$$+ Q_1 \sin\gamma(t - t_0) + Q_2\cos\gamma(t - t_0)$$

or

$$\theta = -\frac{K}{K - \mathbf{T}}\theta_0 + \frac{\mathbf{T}_f}{K - \mathbf{T}}\left(1 - \frac{\gamma^2T^2}{1+\gamma^2T^2}e^{-\frac{t-t_0}{T}}\right)$$
$$+ \sqrt{(Q_1^2 + Q_2^2)}\sin\left[\gamma(t - t_0) + \beta\right] \quad . \quad (2a)$$

where $\beta = \arctan\frac{Q_2}{Q_1}$

The constants $Q_1$ and $Q_2$ are easily found when it is remembered that from equation (1$a$) we have, at time $t_0$,

$$\theta = -\frac{K}{K - \mathbf{T}}\theta_0 - \frac{\mathbf{T}_f}{K - \mathbf{T}}\cos\gamma t_0$$

$$\frac{d\theta}{dt} = \gamma\frac{\mathbf{T}_f}{K - \mathbf{T}}\sin\gamma t_0$$

We get

$$Q_1 = \frac{\mathbf{T}_f}{K - \mathbf{T}}\left(\sin\gamma t_0 - \frac{\gamma T}{1+\gamma^2T^2}\right)$$

$$Q_2 = -\frac{\mathbf{T}_f}{K - \mathbf{T}}\left(\cos\gamma t_0 + 1 - \frac{\gamma^2T^2}{1+\gamma^2T^2}\right)$$

The torque produced by the line current is now increasing and at some instant, say $t_1$, the deflection of the armature equals $-\theta_0$ and is increasing, and the tongue therefore leaves the negative contact screw. It is not possible to deduce an expression for $t_1$, the only means of determining this instant being to plot the curve of motion.

From time $t_1$ tongue and armature move together towards the positive contact screw, and the equation of motion is now the same as that of an armature with a stiff tongue, namely

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta = \mathbf{T}_f(1 - e^{-\frac{t-t_0}{T}}) \quad . \quad . \quad . \quad (3)$$

the solution of which is

$$\theta = -\frac{\mathbf{T}_f}{\mathbf{T}}\Big(1 - \frac{\lambda^2T^2}{\lambda^2T^2 - 1}e^{-\frac{t-t_0}{T}}\Big) + W_1e^{\lambda(t-t_1)} + W_2e^{-\lambda(t-t_1)} \quad . \quad . \quad . \quad . \quad (3a)$$

where $\lambda = \sqrt{\frac{\mathbf{T}}{\mathbf{I}}}$

$$2W_1 = -\theta_0 + \frac{\mathbf{T}_f}{\mathbf{T}}\Big(1 - \frac{\lambda^2T^2}{\lambda^2T^2 - 1}e^{-\frac{t_1-t_0}{T}}\Big) + \frac{\mathbf{T}_f}{\lambda T}e^{-\frac{t_1-t_0}{T}}\Big(\frac{1}{K - \mathbf{T}}\cdot\frac{\gamma^2T^2}{1 + \gamma^2T^2} + \frac{1}{\mathbf{T}}\cdot\frac{\lambda^2T^2}{\lambda^2T^2 - 1}\Big) + \frac{\gamma}{\lambda}\surd(Q_1^2 + Q_2^2)\cos\{\gamma(t_1 - t_0) + \beta\}$$

and

$$2W_2 = -\theta_0 + \frac{\mathbf{T}_f}{\mathbf{T}}\Big(1 - \frac{\lambda^2T^2}{\lambda^2T^2 - 1}e^{-\frac{t_1-t_0}{T}}\Big) - \frac{\mathbf{T}_f}{\lambda T}e^{-\frac{t_1-t_0}{T}}\Big(\frac{1}{K - \mathbf{T}}\cdot\frac{\gamma^2T^2}{1 + \gamma^2T^2} + \frac{1}{\mathbf{T}}\cdot\frac{\lambda^2T^2}{\lambda^2T^2 - 1}\Big) - \frac{\gamma}{\lambda}\surd(Q_1^2 + Q_2^2)\cos\{\gamma(t_1 - t_0) + \beta\}$$

At time $t_2$ the tongue strikes the positive contact screw, the springiness of the tongue comes again into play, and the motion of the armature is given by

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta + K(\theta - \theta_0) = \mathbf{T}_f(1 - e^{-\frac{t-t_0}{T}}) \quad . \quad (4)$$

the solution of which can be shown to be

$$\theta = \frac{K}{K - \mathbf{T}}\theta_0 + \frac{\mathbf{T}_f}{K - \mathbf{T}}\Big(1 - \frac{\gamma^2T^2}{1 + \gamma^2T^2}e^{-\frac{t-t_0}{T}}\Big) + \surd(G_1^2 + G_2^2)\sin\{\gamma(t - t_2) + \eta\} \quad (4a)$$

where $\eta = \text{arc tan}\,\frac{G_2}{G_1}$

$$G_1 \doteq -\Big(\frac{\mathbf{T}_f}{\mathbf{T}}\cdot\frac{\lambda^2T^2}{\lambda^2T^2 - 1} + \frac{\mathbf{T}_f}{K - \mathbf{T}}\cdot\frac{\gamma^2T^2}{1 + \gamma^2T^2}\Big)\frac{1}{\gamma T}e^{-\frac{t_2-t_0}{T}} + \frac{\lambda}{\gamma}[W_1e^{\lambda(t_2-t_1)} - W_2e^{-\lambda(t_2-t_1}]$$

$$G_2 = \theta_0 - \frac{K}{K - \mathbf{T}}\theta_0 - \frac{\mathbf{T}_f}{K - \mathbf{T}}\Big(1 - \frac{\gamma^2T^2}{1 + \gamma^2T^2}e^{-\frac{t_2-t_0}{T}}\Big)$$

In order not to make the equations too unwieldy, the values of the different constants have not been inserted, but even so the expressions for $\theta$ are very complicated and do not give a simple picture of the motion.

Beyerle* has discussed the motion of an armature provided with a springy tongue for the torque due to the line current of the form $\mathbf{T}_f(1 - 2e^{-\frac{t}{T}})$. This is the form which the torque of Fig. 11 assumes for $t_0$ = zero. Beyerle finds that if $\theta_0 = 0$ there exists—in the absence of damping—a lower limit for $\gamma T$ below which the armature will cross the neutral axis in a backward direction subsequent to its forward crossing of the axis under the influence of the torque produced by the current. In other words, if $\theta_0$ be infinitesimally small, the tongue will rebound if $\gamma T$ is too low.

A similar investigation of the motion given by equation (2*a*), with $t_0$ and $\theta_0$ both put equal to zero, reveals that whatever the value of $\gamma T$ the tongue will always rebound. It should be borne in mind that even with $t_0$ put equal to zero in the equation, it is still finite but so small that it may be neglected so far as the equations of motion are concerned.

The discrepancy between this result and that arrived at by Beyerle is due to the fact that different forms for the torque produced by the line current are assumed in the two cases.

It will be shown later that requisite damping of the motion and a *low* value of $\gamma$ are essential conditions for a satisfactory operation of the relay under practical conditions.

In practice $\theta_0$ is not so small that it can be neglected, and it will be interesting, therefore, to discuss equation (4*a*), which gives the complete motion of the armature in the absence of damping.

Equation (4*a*) shows that the motion has an oscillatory component of amplitude $\surd(G_1^2 + G_2^2)$, and that as $t$ increases the axis about which the oscillations take place tends to the value $\theta_b = \frac{K}{K - \mathbf{T}}\theta_0 + \frac{\mathbf{T}_f}{K - \mathbf{T}}$.

In order that the tongue shall not leave the positive contact screw on its own accord after time $t_2$ it is necessary that the amplitude of the two superposed motions shall be less than $(\theta_b - \theta_0)$.

As before, let the aperiodically decreasing deflection be neglected. In this case

$$\begin{aligned} -G_2 &= -\theta_0 + \frac{\mathbf{T}_f + K\theta_0}{K - \mathbf{T}} - \frac{\mathbf{T}_f}{K - \mathbf{T}}e^{-\frac{t_2}{T}} \\ &= \theta_b - \theta_0 - \frac{\mathbf{T}_f}{K - \mathbf{T}}e^{-\frac{t_2}{T}} \\ &\simeq \theta_b - \theta_0 \end{aligned}$$

i.e. $G_2$ alone equals the limiting value of the amplitude of the oscillation, so that in order that the tongue shall not leave the contact screw $G_1$ must be relatively small and the motion must be sufficiently damped.

The most important term in the expression for $G_1$ is

$$\frac{\lambda}{\gamma}[W_1e^{\lambda(t_2-t_1)} - W_2e^{-\lambda(t_2-t_1)}].$$

The condition for the above expression to be small is that the interval $(t_2 - t_1)$ is small, and this will be the case only if the velocity of the armature is already high at time $t_1$. (It is true that a high value of $\lambda$ will also reduce the time interval $(t_2 - t_1)$, but a high value of the velocity at time $t_1$ is more effective.) Now, the velocity of the armature at time $t_1$ is dependent mainly on the velocity of the oscillatory motion of equation (2*a*) and is proportional to

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* See Bibliography (12).

$$\gamma\sqrt{(Q_1^2 + Q_2^2)} = \gamma\frac{\mathbf{T}_f}{K - \mathbf{T}}\sqrt{\left(1 + \frac{3}{1 + \gamma^2T^2}\right)}$$

$$= \frac{\mathbf{T}_f}{\mathbf{I}\gamma}\sqrt{\left(1 + \frac{3}{1 + \gamma^2T^2}\right)} \quad (\text{for } t_0 = 0)$$

i.e. for $\mathbf{T}_f$ fixed it increases for a decrease in the value of $K - \mathbf{T}$. If $K - \mathbf{T}$ be low we obtain low values for $\gamma$ and for $\gamma T$.

With condition (*a*) fulfilled we obtain, in addition, short transit time.

It is noted that it is the value of $\gamma$ rather than $\gamma T$ with which we are now concerned.

In Fig. 13 is shown the motion of an armature under several different conditions. The tongue-travel from $-\theta_0$ to $+\theta_0$ is indicated by two horizontal chain-dotted lines.

Curves A–D have been computed from equations (2*a*)–(4*a*) for the following constants:

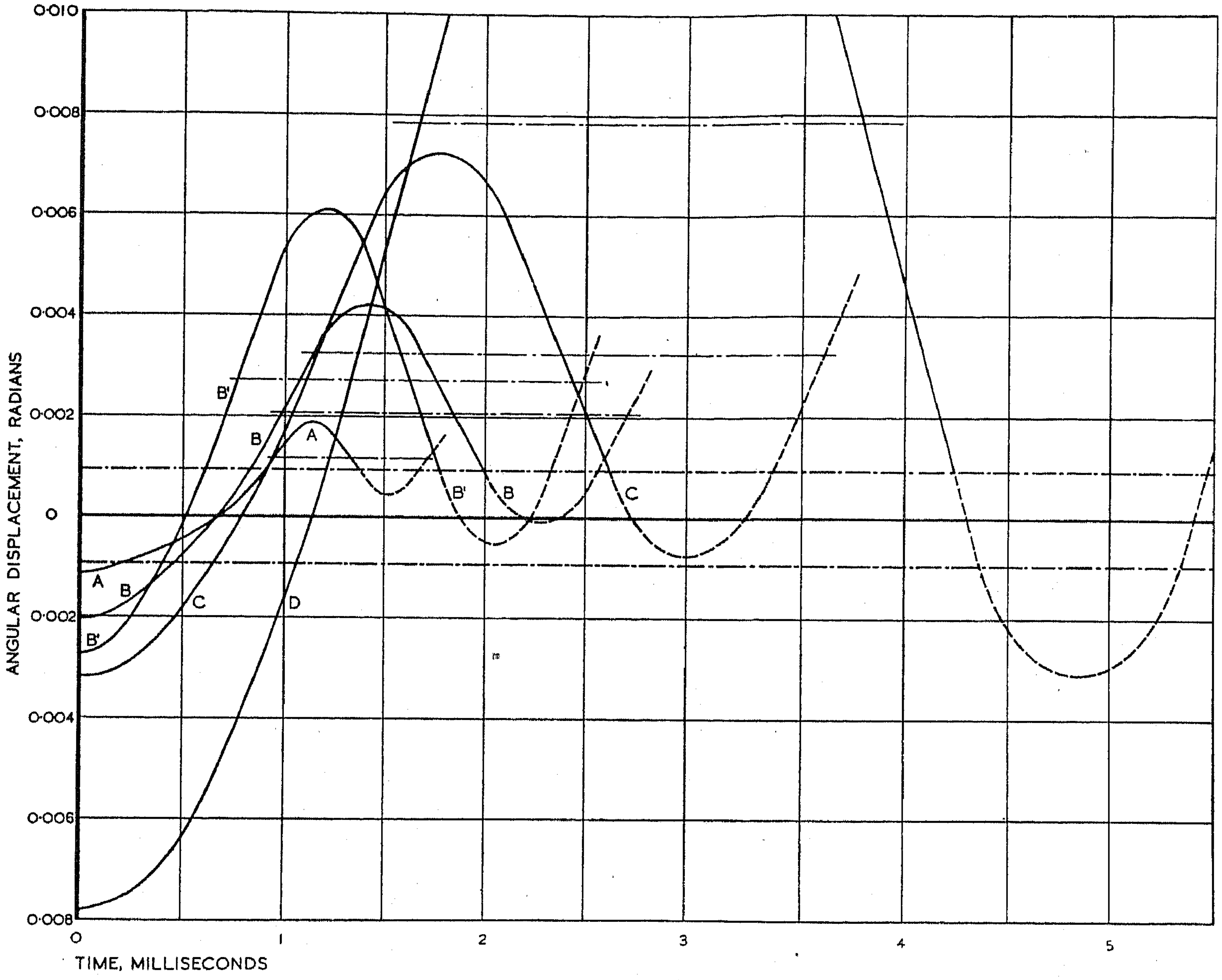

**Fig. 13**

It should be realized that $(t_2 - t_1)$ is the time taken by the tongue to complete its travel, i.e. the transit time, and since it is one of the two outstanding requirements for secondary relays—the other being reboundless performance—that the transit time should be short it is seen that by keeping $K - \mathbf{T}$ low both these requirements are met.

We may put the argument for a low value of $K - \mathbf{T}$ in another form: $G_1$ will be small in comparison with $G_2$ only if $G_2$ be relatively large, i.e. if $K - \mathbf{T}$ be small.

It is seen, therefore, that in order to avoid rebound two conditions must be fulfilled, namely

(*a*) $K - \mathbf{T}$ (and $\gamma$) must be small,
(*b*) damping must be sufficient.

$\mathbf{T} = 229 \times 10^6$ dyne-cm. per radian,
$\mathbf{T}_f = 0\cdot317 \times 10^6$ dyne-cm.,
$T = 0\cdot3 \times 10^{-3}$ sec.,
$\lambda = 2\,570$,
$\theta_0 = 0\cdot000925$ radian, corresponding to a tongue-travel of $0\cdot1$ mm.,
$t_0 = 0$.

Only the value of $K$ has been varied. The variations have been such that

$$\text{for curve A, } \frac{K}{K - \mathbf{T}} = 1\cdot1,\ \gamma = 8\,130,$$

$$\text{for curve B, } \frac{K}{K-\mathbf{T}} = 1\cdot 5, \ \gamma = 3\,640,$$

$$\text{for curve C, } \frac{K}{K-\mathbf{T}} = 2, \quad \gamma = 2\,570,$$

$$\text{for curve D, } \frac{K}{K-\mathbf{T}} = 4, \quad \gamma = 1\,482$$

The transit times $(t_2 - t_1)$ found from the curves are

For curve A, 0·69 millisec.
For curve B, 0·37 millisec.
For curve C, 0·265 millisec.
For curve D, 0·15 millisec.

It is seen that the armature experiences an increase in velocity during the interval $(t_2 - t_1)$ which is greater the smaller the initial velocity at time $t_1$, i.e. the smaller

$$\mathbf{T}_\mathfrak{f}/(K - \mathbf{T}).$$

As damping is neglected in the equations the armature must necessarily—as has been mentioned before—cross the line $+\theta_0$ a second time in the curves of Fig. 13. From the point of view of the amount of damping necessary in order that the tongue shall not leave the positive contact screw it will be interesting to compute the ratio

$$\frac{\dfrac{\mathbf{T}_\mathfrak{f} + K\theta_0}{K - \mathbf{T}} - \theta_0}{\sqrt{(G_1^2 + G_2^2)}}$$

for the four curves. The result is

| Curve | $\dfrac{\frac{\mathbf{T}_\mathfrak{f} + K\theta_0}{K - \mathbf{T}} - \theta_0}{\sqrt{(G_1^2 + G_2^2)}}$ |
|---|---|
| A | 0·319 |
| B | 0·533 |
| C | 0·579 |
| D | 0·629 |

It is seen that as the value of $\gamma$ is decreased the above ratio increases, and consequently the amount of damping required becomes less.

As the tongue is made stiffer, however, the above ratio becomes smaller until the point is reached—and this may be the case for curve A—where it is impossible to provide the necessary amount of damping; the tongue, therefore, will rebound.

The motion of the armature is damped mainly mechanically due to friction at the pivot and internal friction due to bending of the springy tongue; this latter component is likely to be the more prominent one and one which, it appears, cannot be adjusted in any way.

It follows from the above discussion that by keeping $\gamma$ low the smallest amount of damping is necessary in order to avoid rebound, and the shorter transit time is obtained.

In practice, however, $\gamma$ cannot be reduced indiscriminately. It must not be reduced so much that with the highest values of $\mathbf{T}_\mathfrak{f}$ which may be expected the armature will engage with the pole-shoes. Another consideration which must not be lost sight of is that for very low values of $\gamma$ the superposed oscillatory motion of the armature may not have died away at the end of a half-cycle because of its long periodic time and large amplitude, and if this be the case it is to be expected that the performance of the relay will be irregular as the oscillatory motion may possibly affect the instants at which the tongue starts to travel from one contact screw to the other for signals of short duration—dots—whereas this effect will not be noticed on the longer signals—dashes. In other words, distortion may be produced if $\gamma$ be very low.

The practical design, therefore, is likely to be a compromise.

The effect of increasing the operating current is shown in curve B′, which is similar to curve B except that $\mathbf{T}_\mathfrak{f}$ is doubled. The transit time is 0·205 millisec. and the ratio $\dfrac{\frac{\mathbf{T}_\mathfrak{f} + K\theta_0}{K - \mathbf{T}} - \theta_0}{\sqrt{(G_1^2 + G_2^2)}}$ is 0·553.

For all the examples worked out above $t_0$ was made equal to zero in the equations. The question of $t_0$ not being so small that it can be neglected will now be investigated.

### (7) Motion During the Interval $0$–$t_0$

It was mentioned that the tongue may leave the negative contact screw during the interval $0$–$t_0$ if $\mathbf{T}_\mathfrak{f}$ and $t_0$ be sufficiently large. The amplitude of the oscillations of the armature may be so high that the tongue not only leaves the contact screw but the armature may even pass the neutral axis of the relay and be attracted towards the positive contact screw during the current-less interval.

Let the time interval $0$–$t_0$ and the amplitude of the oscillations of the armature be so large that the armature passes the neutral axis before $t = t_0$.

Then, at time $t_a$, $\theta = -\theta_0$ and we have, from equation (1*a*):—

$$-\theta_0 = -\frac{K}{K-\mathbf{T}}\theta_0 - \frac{\mathbf{T}_\mathfrak{f}}{K-\mathbf{T}}\cos\gamma t_a$$

$$\frac{d\theta}{dt} = \gamma\frac{\mathbf{T}_\mathfrak{f}}{K-\mathbf{T}}\sin\gamma t_a$$

After time $t_a$ the only torque acting on the armature is that due to the permanent field alone, and the equation of motion is now given by

$$\mathbf{I}\frac{d^2\theta}{dt^2} - \mathbf{T}\theta = 0 \quad . \quad . \quad . \quad . \quad (5)$$

the solution of which is

$$\theta = \frac{1}{2}\left(\frac{\gamma}{\lambda}\cdot\frac{\mathbf{T}_\mathfrak{f}}{K-\mathbf{T}}\sin\gamma t_a - \theta_0\right)e^{\lambda(t-t_a)}$$
$$-\frac{1}{2}\left(\frac{\gamma}{\lambda}\cdot\frac{\mathbf{T}_\mathfrak{f}}{K-\mathbf{T}}\sin\gamma t_a + \theta_0\right)e^{-\lambda(t-t_a)} \quad . \quad (5a)$$

At time $t_b$ the tongue strikes the positive contact screw, and the motion is now given by

$$\mathbf{I}\frac{d^2\theta}{dt^2} + (K - \mathbf{T})\theta = K\theta_0 \qquad . \quad . \quad . \quad (6)$$

which gives

$$\theta = \frac{K}{K - \mathbf{T}}\theta_0 + \frac{\lambda}{\gamma}\left\{\frac{1}{2}\left(\frac{\gamma}{\lambda}\cdot\frac{\mathbf{T}_f}{K - \mathbf{T}}\sin\gamma t_a - \theta_0\right)e^{\lambda(t_b - t_a)} + \frac{1}{2}\left(\frac{\gamma}{\lambda}\cdot\frac{\mathbf{T}_f}{K - \mathbf{T}}\sin\gamma t_a + \theta_0\right)e^{-\lambda(t_b - t_a)}\right\}\sin\gamma(t - t_b) + \left(\theta_0 - \frac{K}{K - \mathbf{T}}\theta_0\right)\cos\gamma(t - t_b) \quad . \quad . \quad (6a)$$

i.e. an oscillatory motion superposed on the deflection $\frac{K}{K - \mathbf{T}}\theta_0$.

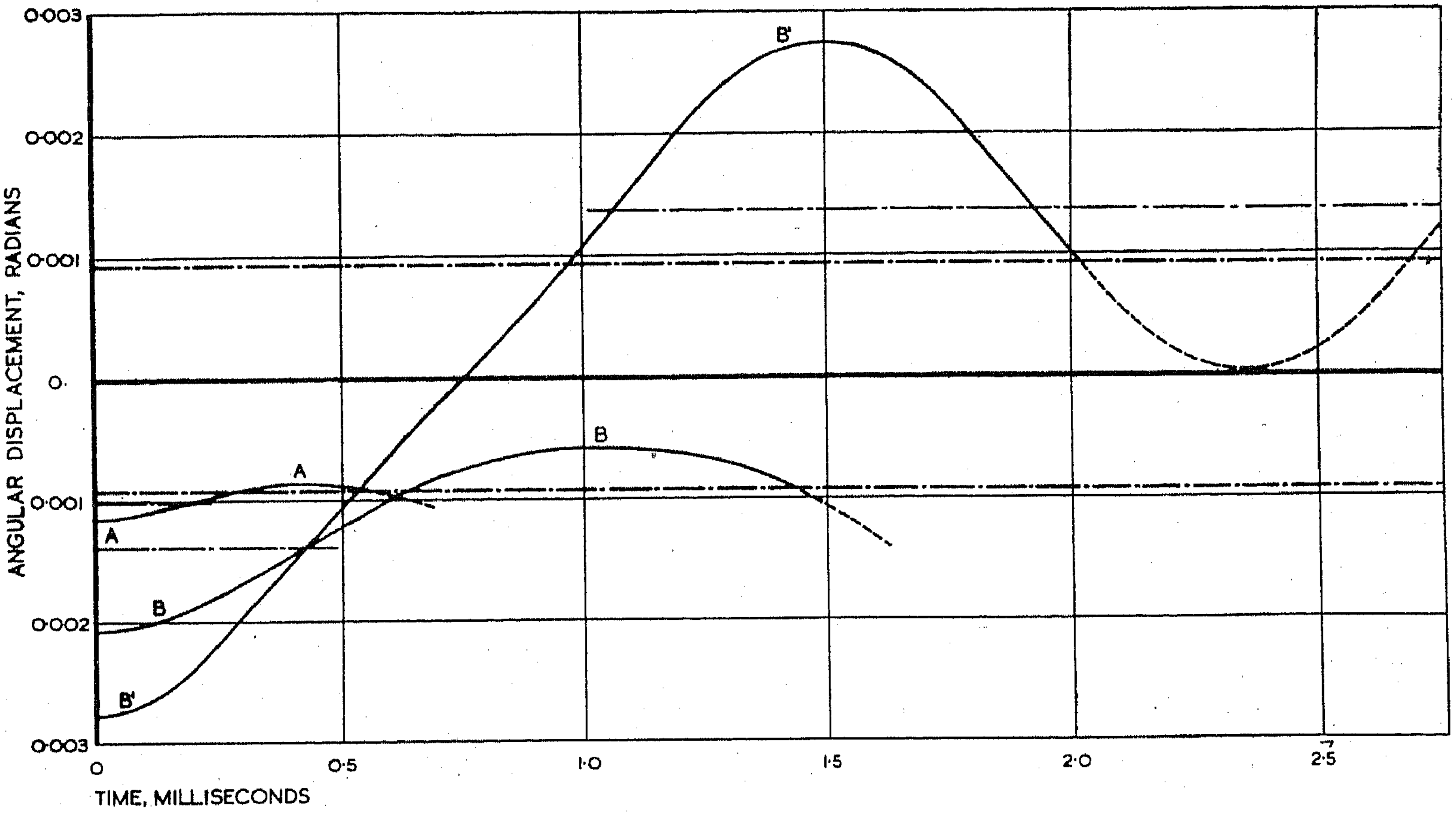

Fig. 14

Investigation of these formulae shows that it is possible for the tongue to complete its travel in the current-less interval, but the velocity of the armature when the tongue leaves the negative contact screw must then be so high that after striking the positive contact screw the tongue will rebound if the motion be not damped.

In practice the motion is damped and it will therefore be possible to let the tongue perform its functions in the current-less interval, provided the value of $\mathbf{T}_f$ be so chosen that it is sufficient to give the armature the motion described above on interrupting the current and not so high that, with the particular amount of damping at disposal, the tongue will be repelled from the contact screw after contact has been established for the first time.

In Fig. 14 are shown some curves (A, B, and B′) which have been worked out with the same constants as those used for the corresponding curves of Fig. 13, except that equations (1*a*), (5*a*), and (6*a*) have been used, i.e. within the time interval considered the line current has not yet started to rise in a positive direction.

For curves A and B the value of $\frac{\gamma}{\lambda}\cdot\frac{\mathbf{T}_f}{K - \mathbf{T}}\sin\gamma t_a$ is less than $\theta_0$, so that some time after leaving the negative contact screw the tongue returns, while for curve B′ the value of $\mathbf{T}_f$ is so great that the above product is greater than $\theta_0$ and the armature, therefore, continues its motion to $\theta = 0$ and beyond this point until it strikes the positive contact screw, from which it will rebound unless sufficiently damped.

The fact that the tongue may chatter on the contact screw during the time interval $t_0$ if $\mathbf{T}_f$ be not high enough is very objectionable not only directly but indirectly also, because at the instant when the positive current starts to rise in the line coil(s) the deflection of the armature is not big enough to assist in bringing the tongue across the contact gap quickly. The result will be that the time lost will be longer than the transit time $t_0$ of the primary relay.

Of course, it would be no serious matter if the value of $\mathbf{T}_f$ for which the tongue may chatter in the current-less interval were so small as to be less than the torque required for good operation of the relay, but this is not the case, for, as seen from Fig. 13, the motion given for instance in curve B (for which the value of $\mathbf{T}_f$ used is identical with that used for curve B in Fig. 14) is quite satisfactory.

It is clear that the above-mentioned chatter may be avoided if the discontinuities due to the current-less interval in the curve of Fig. 11 can be eliminated. Fortunately, this may easily be accomplished by shunting the series resistance and the line coil(s), or the line coil(s) alone, with a condenser. The limiting value of this condenser for the discharge not to be oscillatory is

$$\frac{R^2}{4L^2} > \frac{1}{LC} \quad \text{or} \quad C > \frac{4L}{R^2}$$

$R$ denotes the total resistance in circuit in the first case, while in the latter case the value of the resistance of the line coil(s) alone should be used for $R$.

### (8) Some Special Forms of the Torque Curve

#### (a) Condenser Impulses.

In addition to being required to operate on line current of the form shown in Fig. 11, secondary relays may be called upon to operate for condenser impulses. In Fig. 19 (IV) (B) is shown the discharge current from a condenser through the line coil(s) and a series resistance.

There should be no difficulty in computing formulae for the torques acting on the armature due to a current of this kind, but with the knowledge gained from the previous discussion this is hardly necessary in order to form a picture of the motion.

The current falls to a very low value at the end of the signal, and it is to be expected that at this instant $\theta$ will not be widely different from $\pm\,\theta_a$. In the examples which have been computed in the previous Sections the velocity of the armature was high, mainly because of the fact that the motion started at $-\,\theta_b$, but as $\theta_a$ is much smaller than $\theta_b$ the initial velocity of the armature will be correspondingly smaller when the relay is operated by condenser impulses.

The velocity with which the tongue strikes the contact screw will be governed, therefore, mainly by the increase in velocity experienced during the interval $(t_2 - t_1)$. For a condenser impulse this increase may be high, although, so far as the author's experience goes, the interval $(t_2 - t_1)$ (the transit time) for an impulse-operated armature is not so short as that for a direct-current (Fig. 11) operated armature of a springy-tongue relay of a particular design when convenient values of impulse and current are chosen in the two instances.

It would appear that the motion will be an oscillation the amplitude of which depends also on the velocity of the armature at time $t_2$ and a motion of a shape similar to that of the impulse, both these motions being superposed on the steady deflection $\theta_a$. A high value of the torque corresponding to the maximum value of the impulse, and a high value of $\theta_a$, are both effective in avoiding rebound, but it will be realized that if $\theta_a$ be large the impulse need not be so strong, i.e. in this way a more sensitive relay is produced and the transit time is improved. A high value of $\theta_a$ is obtained by keeping $\gamma$ low.

If the contacts across which the discharge operating the relay takes place chatter to any extent, the behaviour of the relay is likely to be affected adversely.

#### (b) Isolated Signals.

In some special telegraph circuits the relays are required to operate for isolated signals of the same form as that of a half-cycle as shown in Fig. 11, but without the signal in the reverse direction starting immediately after the cessation of the preceding signal, i.e. $t_0$ is of a variable duration and possibly not less than that of a half-cycle. The tongue is required to occupy one of its two extreme positions during and after each signal until it is reversed by the next signal. It is therefore necessary to prevent the armature being thrown from one side to the other at the interruption of the current as shown in curve B′ of Fig. 14, and it must be arranged that the tongue does not chatter as shown in curves A and B of Fig. 14.

The first condition, therefore, is that $\mathbf{T}_f$ must be so small that the factor of $e^{\lambda(t-t_a)}$ in equation (5*a*) becomes negative. In this case, however, the tongue may still chatter on the contact screw on which it is required to remain stationary. The only possible solution of this problem seems to be to modify the shape of the current-wave in such a way that the decrease of the current from its maximum value to zero at the end of each signal becomes more gradual.

One method of doing this is to shunt the relay plus its series resistance with a condenser. On disconnecting the battery in the line circuit the current through the relay will not fall abruptly to zero but will decrease gradually according to the equation

$$i = \frac{E}{r}e^{-\alpha t}\left\{\left(\frac{r}{\beta L} - \frac{\alpha}{\beta}\right)\sinh \beta t + \cosh \beta t\right\}$$

Here $E$ is the potential across the condenser at time 0, $r$ is the resistance of the relay plus its series resistance, $R$ is equal to $r$ plus a retardation resistance in series with the condenser; such a resistance may be necessary in order to avoid an oscillatory current instead of a slowly decreasing one.

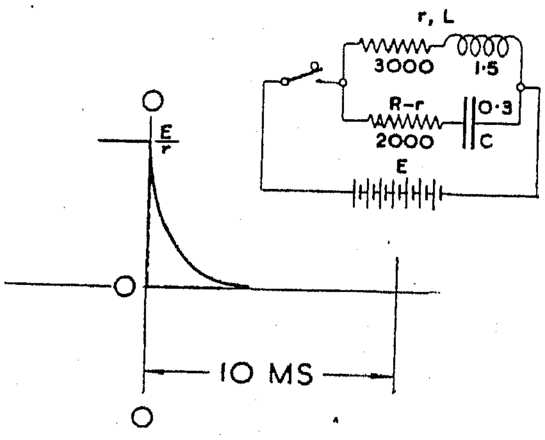

Fig. 15

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \beta = \sqrt{\left[\frac{R^2}{4L^2} - \frac{1}{LC}\right]}$$

In Fig. 15 the curve corresponding to the expression for $i$ is plotted. The values chosen for calibrating the curve are: $r = 3\,000\ \Omega$, $L = 1\cdot5$ H, $C = 0\cdot3\ \mu$F, $R = 5\,000\ \Omega$.

### (9) Factors Affecting Design

The sensitivity as a property of the relay was discussed in Part I only for want of an expression for the current required to operate the relay under fixed conditions with regard to the transit time. For a springy-tongue relay it is possible to deduce such an expression by making an approximation which is, admittedly, a rough one, namely that the velocity from $-\,\theta_0$ to $+\,\theta_0$ is equal to the maximum velocity $\gamma\dfrac{\mathbf{T}_f}{K - \mathbf{T}}$ of equation (1*a*). In this case the current required for a given performance of the relay is limited to a wave-form similar to that of Fig. 11, where the decrease from the maximum value to zero is sudden.

If $\gamma\frac{\mathbf{T}_f}{K - \mathbf{T}}$ be the velocity of the tongue during the interval from $t_1$ to $t_2$ we get

$$\text{Transit time } t_t = t_2 - t_1 = \frac{2\theta_0}{\gamma\frac{\mathbf{T}_f}{K - \mathbf{T}}} = \frac{2\theta_0(K - \mathbf{T})}{\gamma\mathbf{T}_f}$$

$$\mathbf{T}_f = \frac{1}{8\pi A} \cdot 2\Phi_L\Phi(l - \tfrac{1}{2}b)$$

from Part I, ($l - \frac{1}{2}b$ being the arm at which the force produced by the line field is acting),

and from the above

$$\mathbf{T}_f = \frac{2\theta_0(K - \mathbf{T})}{\gamma t_t} = \frac{2\theta_0\sqrt{[\mathbf{I}(K - \mathbf{T})]}}{t_t}$$

Equating these two expressions for $\mathbf{T}_f$ we get

$$\frac{2\Phi_L\Phi}{8\pi A}\left(l - \frac{b}{2}\right) = \frac{2\theta_0\sqrt{[\mathbf{I}(K - \mathbf{T})]}}{t_t}$$

Now, as in Part I, except that $i$ does not denote the minimum operating current but the value of the operating current which corresponds to $\mathbf{T}_f$ above,

$$\Phi_L = \frac{F}{\frac{2a}{A}}; \quad F = \frac{4\pi iN}{10}$$

$$\Phi_L = \frac{4\pi iNA}{20a}; \quad N = \sqrt{\frac{10^9 2aL}{4\pi A}}$$

$$\Phi_L = \frac{4\pi iA}{20a}\sqrt{\frac{10^9 2aL}{4\pi A}}$$

$$\text{i.e.} \quad \frac{4\pi iA}{10a}\sqrt{\left[\frac{10^9 2aL}{4\pi A}\right]} \cdot \frac{\Phi l}{8\pi A}\left(1 - \frac{b}{2l}\right) = \frac{2\frac{x_1}{l}\sqrt{[\mathbf{I}(K - \mathbf{T})]}}{t_t}$$

Putting $K - \mathbf{T} = p\mathbf{T}$ and inserting the expressions for $\mathbf{I}$ and $\mathbf{T}$, we get, after some reductions,

$$i10^3 = \frac{2\theta_0}{t_t}\sqrt{\left[\frac{l^3}{3} \cdot \frac{A_1}{10L}\sigma p\frac{\left(1 - \frac{b}{3l}\right)}{\left(1 - \frac{b}{2l}\right)}\right]}$$

$$= \frac{2x_1}{t_t}\sqrt{\left[\frac{lA_1}{30L}\sigma p\frac{\left(1 - \frac{b}{3l}\right)}{\left(1 - \frac{b}{2l}\right)}\right]}$$

The above expression for the current required to operate the springy-tongue relay in order to obtain a certain transit time is very similar in form to that arrived at in Part I for the least current to operate the stiff-tongue relay.

With regard to the possibility of defining a "specific minimum operating current," it will be noted that in the case of a springy-tongue relay also, account should be taken of the self-inductance, but the minimum operating current in itself is of small importance compared with the current required for a certain transit time. The contact pressure as such is of no importance; for a properly designed relay it will generally be higher than any normal requirements for stable operation. The operating current per unit self-inductance for a given transit time and a given tongue travel (contact distance) seem to yield a better characteristic of any relay under consideration. Unfortunately, the above formula is only approximate so that it is not possible to deduce the exact operating current for any value of transit time, having given the current required to operate the relay for a stated value of transit time.

The formula shows that $p$ should be low. From the discussion in Section 8($a$) it was found that $\theta_a$ should be high, but

$$\theta_a = \frac{K}{K - \mathbf{T}}\theta_0$$

$$= \frac{1 + p}{p}\theta_0$$

i.e. $p$ should be low also from a consideration of this point.

It is worth noticing that the expression for $i$ is independent of $\mathbf{T}$, $A$, and $a$. Since $A_1$ should be as small as possible, it is seen from the expression for $\mathbf{T}$, viz.

$$\mathbf{T} = \frac{1}{8\pi A} \cdot \frac{l^2}{a}\left(1 - \frac{b}{3l}\right)\left(1 - \frac{b}{2l}\right)B^2A_1^2$$

that for $A_1$ small, $\mathbf{T}$ will be small also unless $A$ and $a$ be small or $l$ be high. In practice it is found that it is preferable that $\mathbf{T}$ should be fairly high, this no doubt being due to the fact that the velocity of the armature in the interval $-\theta_0$ to $+\theta_0$ is increased for a high value of $\lambda$, i.e. for a high value of $\mathbf{T}$.

It seems worth while emphasizing the fact that for a springy-tongue relay the operating current for a given transit time is independent of the value of $\mathbf{T}$, i.e. independent also of the value of the sensitivity as formulated for a stiff-tongue relay, and once saturation in the armature is approached it is, therefore, only possible to reduce the operating current for a given transit time by varying a few mechanical dimensions.

The contact pressure for no current through the line coil(s) is proportional to $\mathbf{T}$ for the stiff tongue relay—it is simply given by $\mathbf{T}\theta_0/l_1$, where $l_1$ is the length of armature plus tongue—and it is, therefore, easy to ascertain the value of $\mathbf{T}$ by a measurement of the contact pressure and a knowledge of some simple dimensions. The contact pressure of the springy-tongue relay, on the other hand, is not so simply related to $\mathbf{T}$. The torque acting on the contact screw for no current through the line coil(s) is $K(\theta_a - \theta_0)$.

$$\text{But} \qquad \theta_a = \frac{K}{K - \mathbf{T}}\theta_0$$

$$\text{hence} \qquad K(\theta_a - \theta_0) = \frac{K}{K - \mathbf{T}}\mathbf{T}\theta_0$$

As the tongue is made stiffer, i.e. $K$ increased, the torque tends to the value $\mathbf{T}\theta_0$.

The value of $\mathbf{T}$, therefore, can be found only if an armature with a stiff tongue be at disposal for a measurement of the contact pressure. By repeating the measurement with the springy tongue armature it is possible to evaluate $K$.

The springy tongue will most conveniently be manufactured from a piece of flat spring steel or other suitable material. For a flat spring of length $l_d$, width $w_d$, and thickness $t_d$, the force required at one end to produce a deflection $d$ with the other end fixed is given by

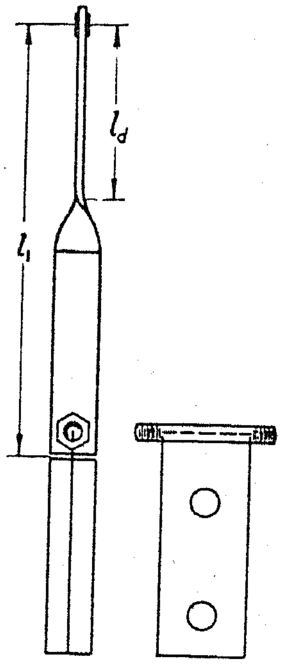

Fig. 16

$$F_d = \frac{d w_d t_d^3 E}{4 l_d^3}; \quad \frac{F_d}{d} = K_d$$

where $E$ is the modulus of elasticity of the material.

With no current through the line coil(s) the armature assumes the deflection $\theta_a$. The distance $d$ becomes equal to $(\theta_a - \theta_0)l_1$ and the force acting at the end of the armature is, therefore, $(\theta_a - \theta_0)l_1 K_d$, corresponding to a torque $(\theta_a - \theta_0)l l_1 K_d$. This is balanced by the torque produced by the permanent field, i.e.

$$(\theta_a - \theta_0)l l_1 K_d = \frac{K}{K - \mathbf{T}}\mathbf{T}\theta_0 = K(\theta_a - \theta_0)$$

hence

$$K = K_d l l_1$$

$$= \frac{w_d t_d^3 E l l_1}{4 l_d^3}$$

By adopting the form shown in Fig. 16 several tongues of different effective lengths $l_d$ and, therefore, different values of $K$ can easily be made for experimental purposes by variation in the distance from the armature at which the 90 degree twist is made. When the tongue is constructed in this way $l$ would be replaced by $(l_1 - l_d)$ in the equations above.

### (10) A Practical Design

In Figs. 17 (*a*) and (*b*) are shown some details of a springy-tongue relay designed according to the principles embodied in the present paper. The electromagnet, which is of the form of a rectangle with a gap for the armature as in Fig. 1, is placed in the horizontal plane.

When attempting to apply the foregoing theory a good deal of experimental work was necessary—indeed, it may be said that a very substantial part of the previous theoretical discussion is directly inspired by practical experience.

In order that the current required for a given performance shall be as small as possible, $l$ and $A_1$ should be small. In the present design $A_1 = 0 \cdot 6$ cm$^2$ and $l = 2 \cdot 85$ cm. In order to keep $x_1$ small the tongue should not be too short and the length chosen is about equal to $l$. The free length of the tongue, $l_d$, was not fixed until after thorough tests.

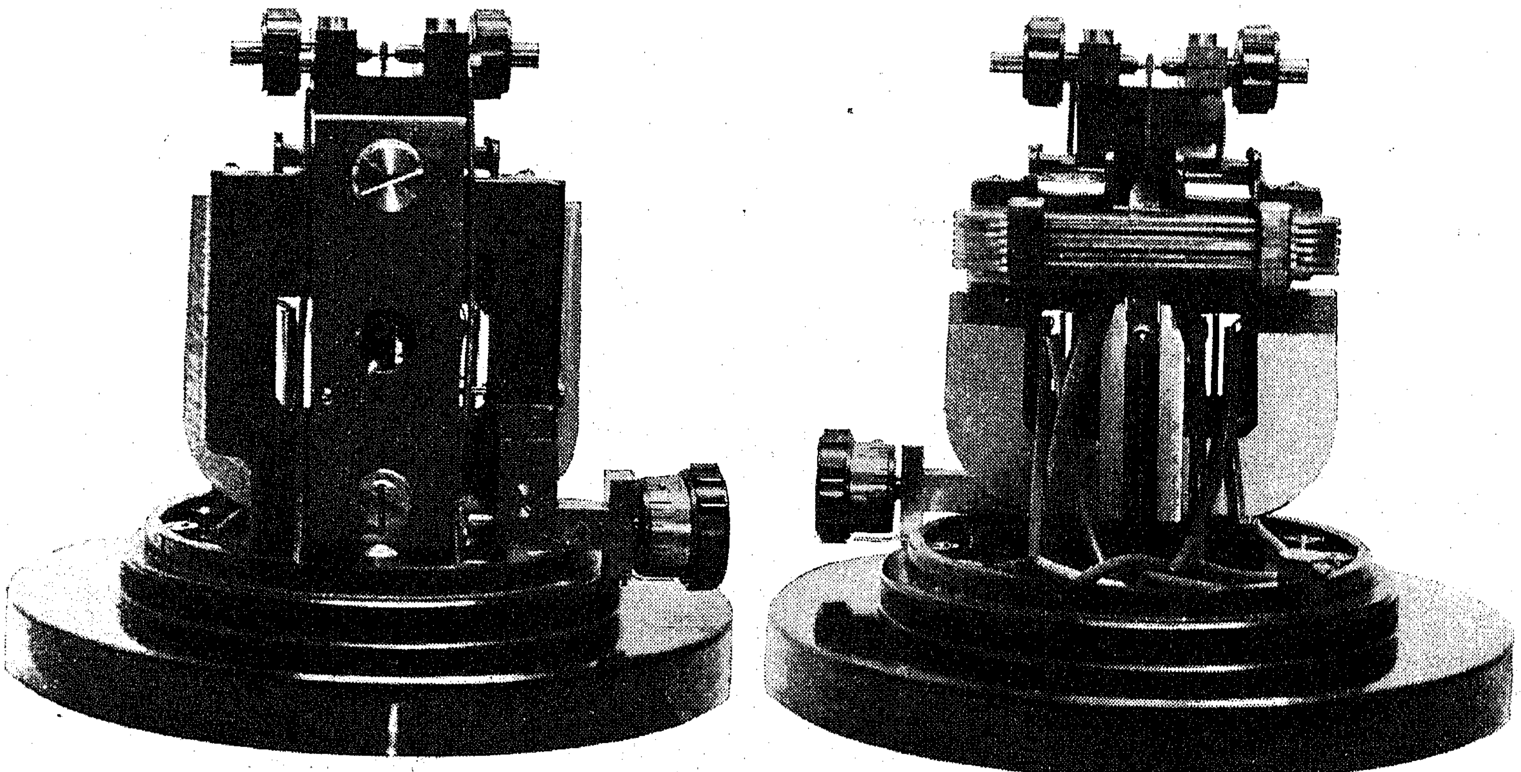
(*a*) (*b*)

Fig. 17

For the configuration of the magnetic circuit shown in Fig. 17 leakage plays a rather important part and for this reason it was necessary to base the calculation of the dimensions of the permanent magnet on a terminal conductance which was fairly much in excess of that of the air gaps. The dimensions of the permanent magnet follow from the design equation quoted in the Appendix. For economy in the use of steel the flux density $B_s$ is so chosen that the product $B_sH_s$ is a maximum.

Strictly speaking, leakage along the magnet should be taken into account, thus making the cross-sectional area of the magnet larger at the mid-section,* but in this particular case a fairly uniform cross-sectional area offered advantages in manufacture.

Owing to the large cross-sectional area required for the permanent magnet it was convenient to use two similar magnets in parallel rather than a single larger one. By arranging the electromagnet and the permanent magnets as shown a slight advantage compared with the usual Siemens design is obtained. Whereas half the total permanent flux traverses the whole length of the electromagnet in the usual design, in the present form only the difference between $\Phi_1$ and $\Phi_2$ traverses the greater part of the magnetic circuit of the electromagnet. Owing to the smaller superposed steady flux density the incremental permeability of this part of the magnetic circuit is increased and the magnetomotive force necessary to drive the flux through the iron circuit is reduced.

Only in the pole-shoes is the flux density due to the permanent field of some magnitude, each pole-shoe carrying a flux approximately equal to $\frac{1}{2}\Phi$. The magnetic alloy selected for the electromagnet should be one the incremental permeability of which is sufficiently high at the flux density obtaining in the pole-shoes.

Ultra-high-permeability alloys may be employed for the manufacture of the electromagnet, but for this particular relay it was found that the decrease in operating current which might be expected theoretically from the employment of this kind of alloy worked out at only about 10 %. Practical tests, moreover, did not show that there was any appreciable decrease in the operating current. However, owing to the considerations mentioned in Section 11(*d*) an ultra-high-permeability alloy was nevertheless selected for the core.

Naturally, the alloy selected for the manufacture of the armature should be one which allows of a high flux density, but the permeability at this value of the flux density is unimportant since the total flux through the armature is practically constant.

Both electromagnet and armature are made from laminations.

With a solid armature the transit time was slightly longer than that obtained with a laminated armature. Definite conclusions with regard to the different behaviour of the two kinds of armature could not be arrived at by a study of the intervals between the reversals of the line current and the times at which the tongue left either contact screw, but other evidence shows that the effect of using a solid armature is such as to flatten the line-flux curve to such an extent that this form of armature cannot be used in conjunction with low-resistance alloys for the electromagnet stampings unless these latter are made thinner than used in the experiments.

The line windings are split up into two halves, each wound on a separate bobbin, and the two bobbins are placed on either side of the air-gaps. Owing to the permanent magnets being two in number and placed as shown, the employment of a single spool placed on the yoke connecting the two parallel limbs of the electromagnet is possible, but however appealing such a design is from the point of view of simplicity and easy removal of a defective coil without interfering with the pole-shoes, it will be realized that only part of the field produced by the line current would in that case reach the air-gaps, on account of leakage. This alternative was tested but it was found that only about two-thirds of the line field reached the air-gaps and for this reason the double-bobbin design was adopted.

Pivots are always troublesome and it has been found that play in the pivots may affect the performance of telegraph relays adversely. It was, consequently, preferred to connect armature and armature pole-faces directly by a thin flat spring acting as a hinge, the free length of which it was found necessary to make very short.

In order to test the hinge spring for fatigue, the relay was operated continuously day and night for a week by alternating current of a frequency of 50, corresponding to a total of about 30 million complete cycles of operation, with the largest possible deflection of the armature, and the hinge spring withstood this test successfully.

The hinge spring is clamped by two pieces of bakelite in order to insulate armature with tongue from the rest of the relay, and with these bakelite pieces attached the hinge spring is inserted between two armature pole-faces and secured by screws. The armature pole-faces overlap the armature on either side in order to reduce the reluctance at the hinge.

The main parts of the relay are attached to a casting which is mounted on a bakelite base provided with spring jacks for electrical connections.

Bias adjustment is of the conventional type and the cover is a brass cylinder provided with a glass top.

The results obtained with the relay just described will be dealt with in the next Section.

## (11) Relay Tests

### (a) Oscillograms.

The behaviour of telegraph relays can best be studied by means of an oscillograph, preferably one with at least two loops, permitting the simultaneous recording of line current and of the current in the secondary circuit which is fed from a battery joined to the contact screws of the relay. If the secondary circuit be one containing resistance only, the oscillograph will trace a square-topped curve the top and bottom horizontal lines of which correspond to the times during which the tongue is in contact with either the positive or the negative contact screw. If the transit time be nil and rebound absent, these horizontal portions of the oscillogram will be joined by vertical traces [Fig. 18(*a*)], whereas a horizontal step on the vertical traces will indicate by its relative length the time taken by the tongue in travelling from one contact screw to the other, i.e. the transit time [Fig. 18(*b*)]. Rebound, if present, will be indicated by a tooth-form on the curve in Fig. 18(*c*).

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* See p. 804 of Bibliography (8).

In Figs. 19 (I–VI) are given some oscillograms which are representative of the performance of the relay just described. The curves of each oscillogram are: (A) the current in the secondary circuit, (B) the line current, and (C) the current in the secondary circuit when both contact screws are connected to the same pole of a battery while the tongue, through a resistance and the oscillograph, is connected to the other pole as shown in Fig. 20.

In all the oscillograms the duration of a half-cycle is $0 \cdot 010$ sec. The contact gap is $0 \cdot 10$ mm.

The self-inductance of the practical design relay was $1 \cdot 5$ H, and in the case of oscillograms I–III and V–VI the series resistance in the line circuit was 3 000 ohms.

The line current (B) in oscillogram I is reversed by a perfect telegraph transmitter, i.e. one having practically no transit time and being free from rebound. The line current rises to a maximum of about 15–16 mA ($i\sqrt{L} \simeq 19$) and the transit time of the tongue, according to curve (A), appears to be very slight. There appears to be a complete absence of rebound, but a close study of the curve will reveal a very minute " tooth " at each reversal, indicating that, actually, the tongue does rebound slightly, this no doubt being due to the tongue not being weightless, as assumed in the theoretical discussion. The significance of curve (C) will be referred to in Sub-section (*c*).

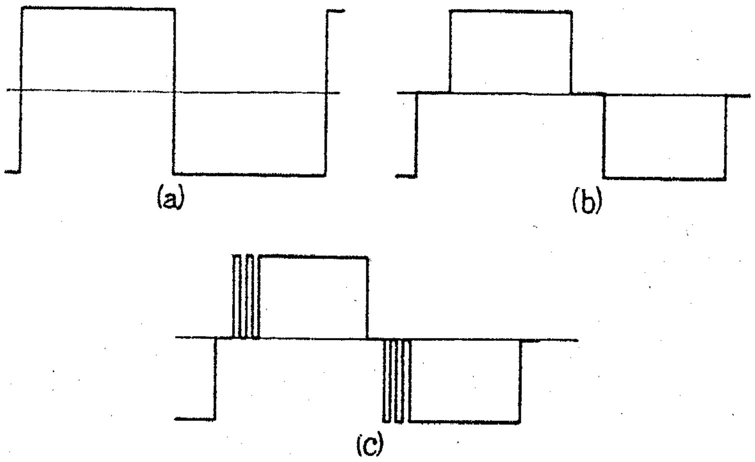


Fig. 18

Fig. 19 II shows the performance of the relay under the same conditions except that the transmitter in the primary circuit is a defective one, exhibiting both transit time and a considerable amount of chatter. The regularity of curve (A) is seen to have suffered from the bad shape of curve (B). An important point may be noted when comparing this oscillogram with oscillogram I: Whereas in oscillogram I the travel of the tongue must have started immediately after the reversal of the line current, in oscillogram II the initial deflection of the armature has, presumably, not been large enough to make the tongue leave the contact screw directly after the time of interruption of the current and, consequently, the irregularity of the current wave makes itself felt in the uneven lengths of positive and negative half-cycles as delivered by the relay.

In oscillogram III the performance of the defective transmitter has been improved by shunting relay plus series resistance (3 000 ohms) by a condenser of $0 \cdot 3\ \mu$F.

Oscillogram IV depicts the performance of the relay when operated by condenser impulses. The details of the line circuit are: 7 000 ohms, $0 \cdot 3\ \mu$F, ± 47 volts.

When recording oscillogram V a solid armature was substituted for the laminated armature used in the previous tests.

Oscillograms have been taken, but not recorded, of the performance of the relay with a transmitter which had a definite transit time and which was at the same time free from rebound. The transit time of the relay was somewhat increased, but otherwise the performance was not affected. When transit time on the transmitter becomes pronounced it would be advisable to shunt the relay circuit by a condenser as done for Fig. 19, III.

Oscillogram VI shows the performance of the relay on alternating current. The r.m.s. value of the current was $15\frac{1}{2}$ mA and the relay was fitted with the normal laminated armature.

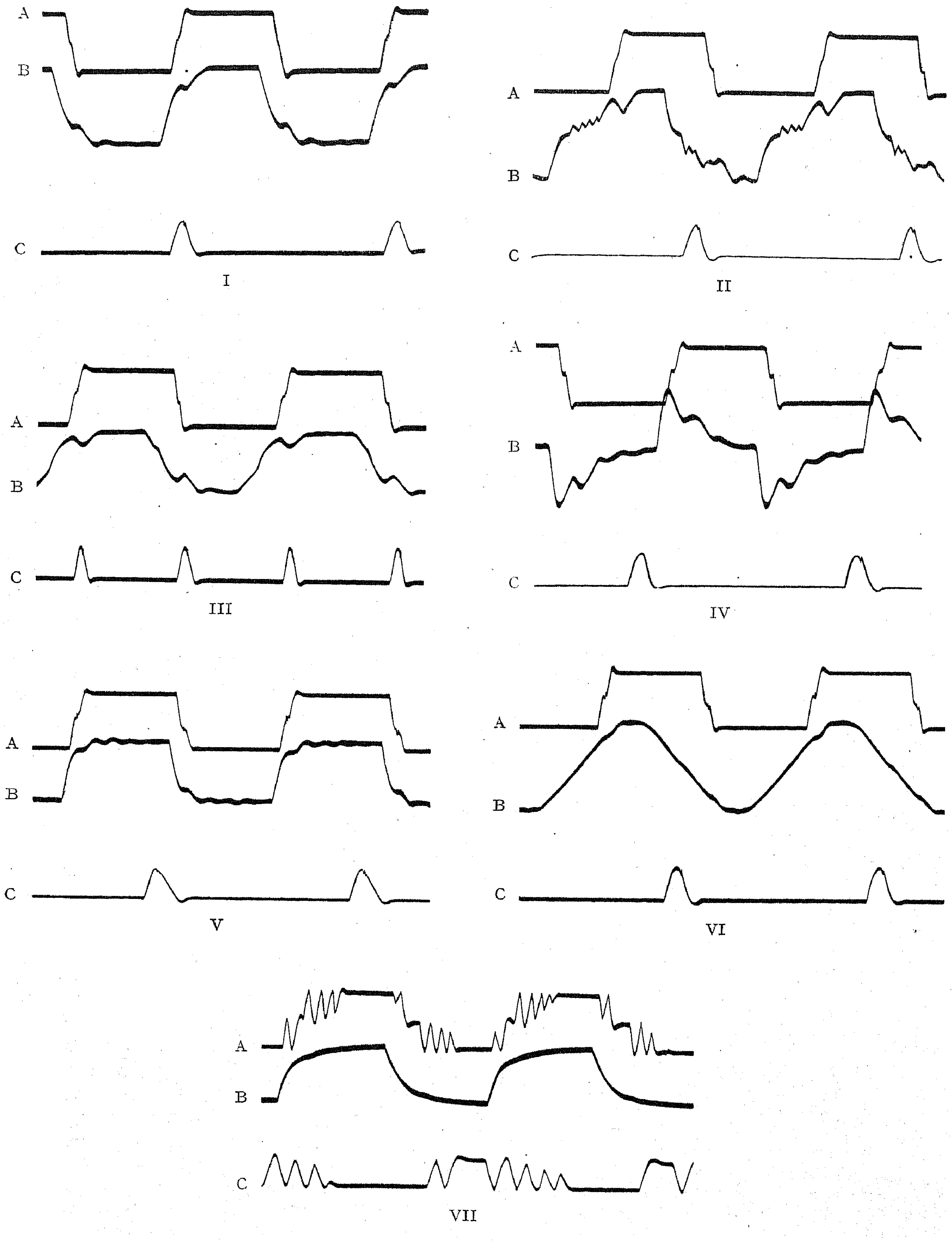
A striking contrast to the performance of the relay which has been described is given in oscillogram VII. This shows the performance of a certain manufacture of a stiff-tongue relay the magnetic circuit of which is the same as shown in Fig. 2; the current is about 15 mA (max.) as in most of the previous examples. A perfect transmitter was used. An interesting property of this relay is borne out by these oscillograms, viz. that a rebound occurs at the end of each signal. This seems to be of the same nature as the rebound which was theoretically predicted in the curves of Fig. 14, although in the present instance the line current was reversed without transit time. It appears that the tongue must have chattered owing to the sudden interruption of current and that the armature had not started its proper travel until the current reached a certain value. It should be noted that a modification of the current wave, for instance by shunting the series resistance with a condenser, would possibly have reduced the time lost in rebound. The performance here shown is perhaps rather worse than the average for this type of relay.

In most of the oscillograms there will be noted a kick in the front part of the line-current curve. This is due to the back e.m.f. induced in the line coils by the alteration of the permanent field which is caused by the movement of the armature. In oscillogram VII this kick is absent on account of the change of the permanent field caused by the movement of the armature being small; this is due to the fact that both the permanent field employed and the travel of the armature (the tongue being stiff) are small.

Another interesting property of the relay used for oscillograms I–VI is revealed by the ripple in the line current, most clearly borne out by oscillogram V. This ripple is caused by the oscillation of the armature about the contact screw, and its period of oscillation is equal to $1 \cdot 67$ millisec., as can readily be computed from the curve. In the case of oscillogram V, $\gamma$ equals 3 768, corresponding to a frequency of 600 cycles.

### (b) Measurement of Self-Inductance.

The self-inductance of a relay is very difficult to measure by ordinary methods for, whether it be measured



I II III IV V VI VII

Fig. 19

ballistically or by a.c. bridge methods, it is likely to vary with current and it might be doubtful which value should be chosen. If the relay is to work for a current of a sinusoidal wave-form an a.c. bridge measurement performed with the value of current in question should give the correct answer for that current, but if it is a question of assessing the value of the self-inductance under working conditions when a current of the form shown in Fig. 11 is employed, the value found from an a.c. measurement cannot be applied readily.

One important point has to be noted in connection with the measurement of self-inductance of a springy-tongue relay with a strong permanent field, i.e. that in this case there is a difference between the true and the effective self-inductance of the relay (when by effective self-inductance is understood that self-inductance which corresponds as nearly as possible to the actual shape of the current wave-form). The reason for this difference is that an e.m.f. is induced in the line coils during the motion of the armature, as mentioned above. It is very difficult to define the true self-inductance of a magnetic circuit consisting mainly of iron, but the effective self-inductance may be measured and this, fortunately, is the quantity with which the user of a relay is more directly concerned.

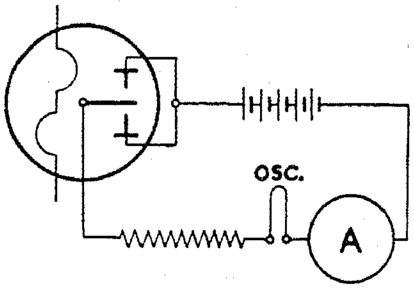


Fig. 20

A practical method used by Franzke,* which is very simple when an oscillogram is available, is to deduce the self-inductance from a measurement of the time at which the current has reached 0·63 of its maximum value. This time, as is well known, is equal to the time constant $T = L/R$ of the circuit. In carrying out this method it is usual to idealize the curve a little (see Fig. 21), but the result is, in the author's opinion, the best approximation possible for the effective self-inductance under working conditions.

Whatever method is employed, the results agree, generally, within about 10 % when appropriate values of testing current are employed.

### (c) Measurement of Transit Time.

The transit time, i.e. the time taken by the tongue in travelling from one contact butt to the other, is a property of fundamental importance and several methods are available for its measurement.

If an oscillogram of the current through a resistance is obtained as in Figs. 19 I–VII(A) the transit time is represented by a step of zero current and can be computed when the period of the signal wave is known. It will be realized, however, from inspection of the oscillograms of Fig. 19 that when the transit time is short—and the natural period of oscillation of the loop of the oscillograph not very low—an accurate determination by this direct method is very difficult.

Franzke* has used the method illustrated in Figs. 19 I–VII(C) and explained in Section 7(*a*) above. In the oscillograms the scale has been doubled in most cases in comparison with curves (A) and (B); the kicks (from negative to zero current) correspond to the time when the tongue does not rest on either contact screw, and the length of kick relative to the distance between the kicks gives the transit time of the relay.

The corners of the start and finish of the kicks of curve (C) are not too well defined, on account of the natural period of oscillation of the loop not being infinitely low, and a small inaccuracy must therefore be contended with.

A second method which is very widely used for the determination of the transit time of a relay makes use of the same circuit as that used in the above-mentioned oscillogram test; the oscillograph, however, is replaced by a moving-coil milliammeter. With the tongue at rest the full deflection of the meter is noted, and another observation of the deflection is taken when the armature vibrates under the action of dot signals of known frequency. This latter deflection is less than the full deflection, and the transit time is given by the duration of a half-cycle multiplied by the ratio of the difference of the two deflections to the full deflection.

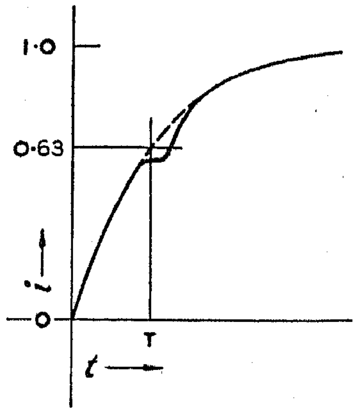


Fig. 21

This test is often referred to as an efficiency test, the "efficiency" being expressed as the percentage of the reduced deflection of the full deflection for a given speed of working.

This method is both accurate and simple to perform when rebound is absent. It is important, however, to use a moving-coil meter, because the deflection of this type of meter is proportional to the mean value of the current wave; another type of meter, for instance, a moving-iron instrument which gives deflections proportion to the root of the mean-square value of the current wave, cannot be used.

Simultaneously with the recording of the oscillograms of Fig. 19 this method for determining the transit time was used and the results obtained are compared with the values of transit time computed from oscillograms (C) in Table 1.

It will be seen that the transit times deduced from the oscillograms are somewhat higher than those given by the moving-coil meter test. The author attributes the discrepancy to difficulty in reading the kick of the oscillogram correctly rather than to any fault in the meter method; in the case of oscillogram VII the discrepancy is clearly due to the long time of rebound.

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* See Bibliography (10).

* See Bibliography (10).

The C.C.I.T.* recommends the use of the moving-coil meter method for the determination of transit time, but adopts the circuit of Fig. 22. In this case the connection to the tongue is temporarily interrupted while the deflection of the meter is adjusted to 100 divisions. With this connection restored and the relay fed from dot signals of the working frequency (the C.C.I.T. recommends alternating current at 25 cycles) a small deflection equal to the transit time as a percentage of a half-cycle is observed.

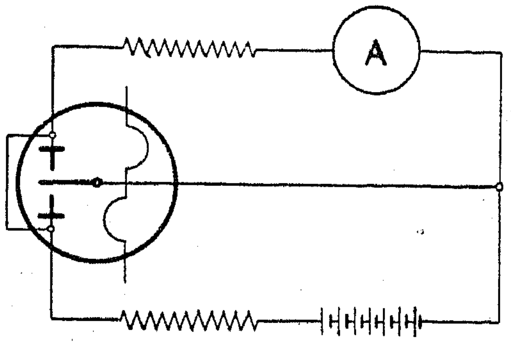

**Fig. 22**

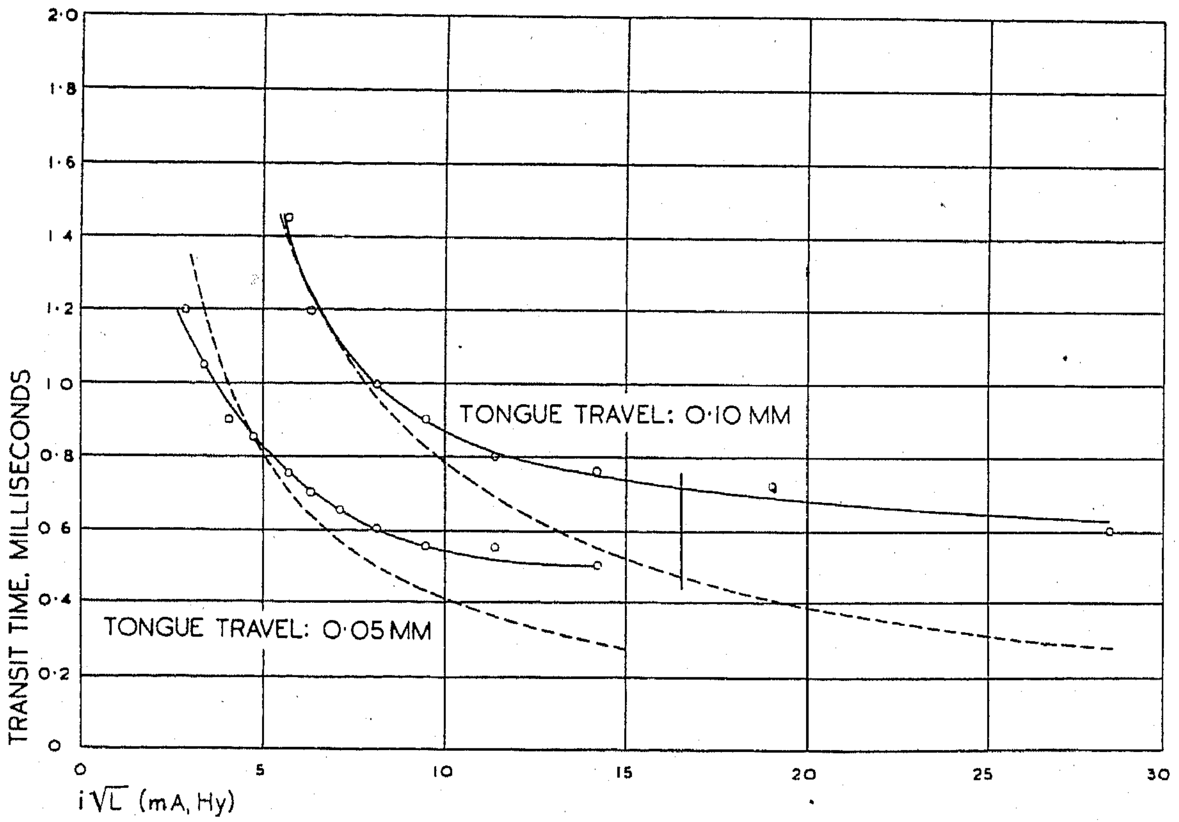

**Fig. 23**

It will be realized that only in the absence of rebound does the meter method give a correct measure of the transit time. When rebound is present the result of the test will be higher than the transit time and almost equal to the time lost in transit plus rebound, but as this time interval rather than the transit time alone is of importance in practice the apparent deficiency in the method as regards a determination of the transit time does not detract from the popularity of the method. In extreme cases such as that illustrated in Fig. 19, VII, the result given by the moving-coil meter test, however, is rather less than the total time lost in transit and rebound.

**Table 1**

| Reference to Fig. 19 | Transit time, millisec. | |
|---|---|---|
| | Moving-coil meter test | Oscillogram test |
| Oscillogram I .. .. | 0·8 | 1·17 |
| Oscillogram II .. .. | 1·0 | 1·24 |
| Oscillogram III .. .. | 1·1 | 1·39 |
| Oscillogram IV .. .. | 1·2 | 1·3 |
| Oscillogram V .. .. | 1·0 | 1·67 |
| Oscillogram VI .. .. | 0·95 | 1·32 |
| Oscillogram VII .. .. | 3·8 | 6·0* |

* Transit time alone: 2·0 millisec.

In Fig. 23 are given the results of the test using the meter method on the practical design of relay with currents of various strengths through the line coils. The current was increased by reducing the series resistance of the relay, keeping the battery e.m.f. constant. The theoretical curve to be expected from Section (9) is shown dotted, this curve having been fitted to a point on the front part of the experimental curve.

It is seen that the latter part of the experimental curve does not follow the theoretical curve. The reason for this is mainly that for very strong currents the tongue bends so much that the armature touches the pole-shoes; it was easy to ascertain that this was a reason for nonconformity with the theoretical curve by connecting a meter and a small battery between the insulated tongue and the pole-shoes, a deflection being observed when the armature touched the pole-shoes. In the test illustrated in the curve for 0·10 mm. tongue travel this occurred for $i\sqrt{L} = 16\cdot5$ (approx.). Another reason for non-conformity with the theoretical curve is that the latter is computed according to an expression which, as pointed out, is only approximate.

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* See p. 70 of Bibliography (13).

The above experiment was performed with a wave-form similar to that of Fig. 19 I (B) and at a dot frequency of 50 cycles. The self-inductance of the relay was 1·5 H and the travel of the tongue was 0·10 mm. and 0·05 mm. respectively.

A method in which a condenser is allowed to charge slowly through a high resistance for a certain time, the charge being measured in order to compute the time interval in question, has been suggested by Reybaud* in connection with tests on telephone relays, but when applied (with necessary modification) to the telegraph relay described the results were not satisfactory.

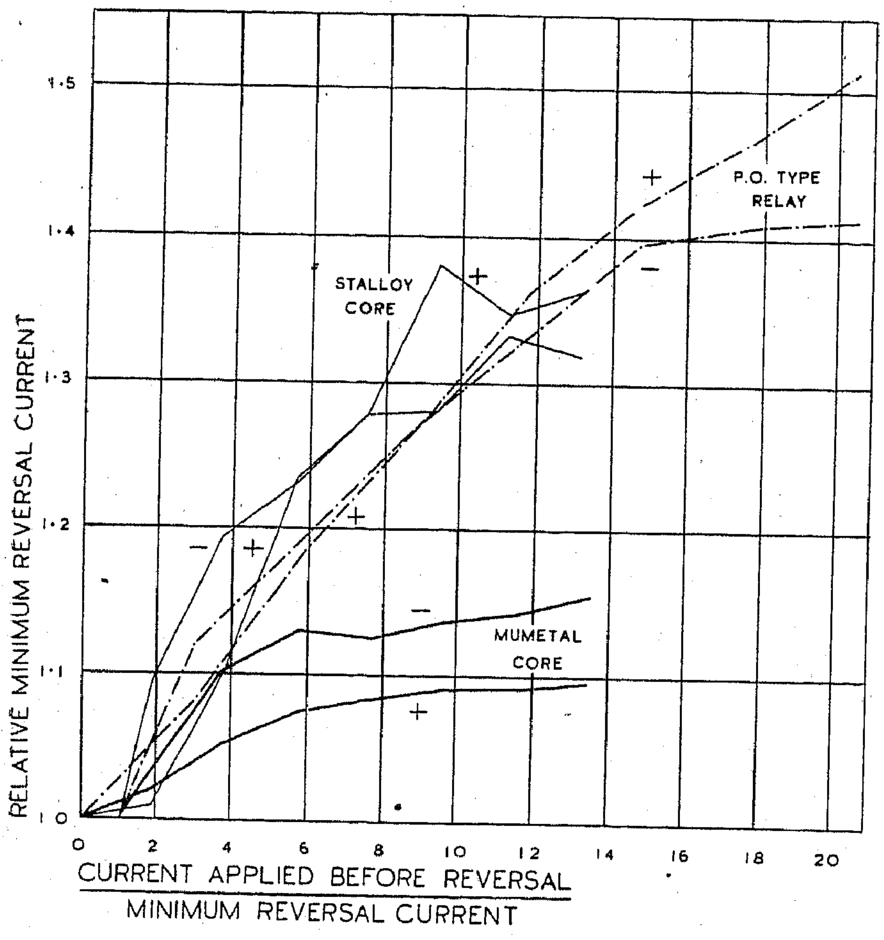

Fig. 24

## (d) Hysteresis

Collet† has suggested the following test in connection with polarized telegraph relays.

A circuit is made up consisting of battery, reversing key, a variable resistance, and the relay under test. The relay should be set neutral. From a very low value the current is slowly increased until the position of the armature is reversed. The value of the current necessary to reverse the armature is noted.

The current is now decreased and reversed when very low. The operations are repeated for the reverse current, and the current for which the relay operates should be almost the same as before.

Before decreasing the current prior to reversal it is *increased* a certain amount, then decreased to a low value, reversed, and slowly increased until the relay operates again. The amplitude of the current necessary to operate the relay now will be higher than before on account of hysteresis in the iron circuit of the electromagnet, due to the previous overdose of current in the reverse direction.

By repeating the experiment with increasing amounts of overdoses a " hysteresis " curve for the relay may be plotted.

In Fig. 24 are plotted some results from practical tests. Positive and negative currents necessary to reverse the armature were measured for various amplitudes of current before reversal, but in order to obtain curves allowing of comparison relative minimum currents are shown in Fig. 24 plotted against the ratio of the current applied before reversal to the minimum reversal current. One curve shows the results of a test on a Post Office type relay (double armature, electromagnet cores made of Swedish iron, coils in parallel, self-inductance about 4 H, minimum $i\sqrt{L} = 3\cdot4$); another curve is for the practical design evolved according to the present paper (coils in series, self-inductance about 6 H, minimum $i\sqrt{L} = 6\cdot4$) with cores of Stalloy; while the third curve is for the same relay with Mumetal cores for the electromagnet.

The hysteresis phenomenon for ratios of current applied before reversal to minimum reversal current equal to about 13 is represented by increases in the minimum operating current of respectively 35, 35, and 12 per cent approximately.

Due to the lower hysteresis obtained with Mumetal this material was adopted for the practical design of relay.

## (e) Distortion.

It is important that a telegraph relay should reproduce the received signals faithfully, i.e. distortion introduced by the relay should be low.

For the measurement of distortion several instruments exist, generally so arranged that at the instant a relay makes contact with either of its contact butts a flash of light is shown on a scale. If successive contacts are made at the correct instants the flashes are superposed on each other, but when distortion occurs either in the line in front of the relay or in the relay itself the flashes produced will spread over a band the width of which gives a measure of the distortion present.

Lack of neutrality (bias) in a relay will make alternate half-cycles of reversals too long and too short respectively, and the instrument will consequently show a spread of the flashes, half of which, expressed as a percentage of a half-cycle, is the percentage bias distortion.

Now it is generally possible while making a test to adjust the relay for neutrality when the transmitted signals are reversals, and it is therefore possible to eliminate this form of distortion, but, on the other hand, the flashes may not occur regularly but spread over a certain interval which may, or may not, be the same for reversals from positive to negative or negative to positive, depending on peculiarities in the performance of the relay. Half the maximum spread of the flashes, expressed as a percentage of a half-cycle, would then be the percentage distortion.

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* See Bibliography (14).

* See p. 109 of Bibliography (9).

It is found in practice that when a relay is tested on a distortion tester and adjusted for neutrality on a certain speed of reversals then, if the speed be varied, bias distortion ensues. This is contrary to the statements of Jipp.* For this reason, when measuring the distortion of the relay account must be taken of the bias distortion as this, of course, makes itself felt when mixed signals rather than reversals are transmitted.

In order to make a complete distortion test it is preferable, therefore, to make the observation of distortion while mixed signals are being transmitted. With Wheatstone signals the ratio of length of dot, letter-space, dash, and word-space signals, are as 1 : 3 : 3 : 7, and signals containing this combination would therefore be used in a distortion test.

The result of a distortion test carried out as above on the practical design relay with 0·1 mm. tongue travel is shown in Table 2.

**Table 2**

| Final value of current, mA | Total distortion | |
|---|---|---|
| | 120 w.p.m. (100 bauds) | 60 w.p.m. (50 bauds) |
| 11 ($i\sqrt{L} = 13 \cdot 5$) .. | approx. 4 % | approx. 4 % |
| 15 ($i\sqrt{L} = 18 \cdot 4$) .. | approx. 1½ % | approx. 2 % |

The result, it will be noted, is practically the same for both speeds. The results were practically identical with Stalloy and Mumetal cores.

When performing the above test the observation on the oscillogram that one minute rebound takes place at each reversal was confirmed. The time interval between the initial and the final contact was not more than 0·2 millisec.

It will be realized that for relays which exhibit the phenomenon of rebound to a marked degree, as in oscillogram VII of Fig. 19, the accurate determination of distortion will be almost impossible on account of the large number of flashes, spreading over a large interval, which occur at each reversal.

## CONCLUSION

It has been shown in the present paper that a satisfactory performance for a polarized telegraph relay, mainly as regards short transit time and, practically, freedom from chatter, can be obtained only by making the tongue springy. If the wave-form of the operating current is irregular this defect should be remedied by smoothing it, for example by means of a condenser shunt. The formulae obtained indicate the relative importance of the salient properties, but it has not been possible to take account of the effect of damping when evolving the theoretical expressions which have, moreover, been confined to an ideal relay, i.e. one without magnetic leakage. Damping, it was shown, must be sufficient but the experimental part of the paper has indicated that it is possible in practice to damp the motion sufficiently.

Only with a relay with this type of tongue is it possible to obtain an approximate theoretical expression for the current required for a certain transit time while at the same time the performance is practically free from chatter, and experiment has proved that the theoretical interdependence of transit time and current agrees fairly well with the experimental results.

The maximum transit time allowed by the C.C.I.T.* for polarized relays when determining the minimum operating current is 5 millisec. and it is recommended that the time lost in rebound should not exceed 1 millisec. for a transmitting relay and 2 millisec. for a relay used for reception. The total time lost in transit time plus rebound is about 1 millisec. for the practical design of relay evolved according to the present paper for values of $i\sqrt{L}$ which would be used in practice.

The results obtained for distortion are 2 %–4 % under practical working conditions, whereas the C.C.I.T.† allows 5 % when determining the minimum working current for the relay when operated by alternating current at a frequency of 25 cycles. The conditions stipulated by the C.C.I.T. are perhaps less severe than those under which the practical design of relay was tested. It has not been possible to predict theoretically the conditions for minimum distortion.

The magnetic circuit employed in conjunction with the use of a special ultra-permeability alloy for the electromagnet has made it possible to obtain a very low hysteresis effect.

The C.C.I.T. has recommended‡ that the electrical characteristics of a polarized telegraph relay should be (*a*) the d.c. resistance, (*b*) the effective resistance and reactance at 25 cycles for a specified current, and (*c*) the number of turns of the windings.

The user of a telegraph relay is not interested in the number of turns which the manufacturer has found to suit a particular relay, and the actual number of turns may vary enormously according to the dimensions of the air-gaps at the pole-shoes and the cross-sectional area of the latter without affecting the self-inductance (Part I, Section 5). The self-inductance of the relay, on the other hand, is of importance as it governs the shape and amplitude of the current wave and considerations of sparking at the contacts of any primary relay employed, and this property, therefore, rather than the number of turns, ought to be specified.

It is quite common to find the sensitivity of a relay specified by the minimum number of ampere-turns, but an example will illustrate how erroneous conclusions will be obtained by this method. In the case of the two relays used for the test of Fig. 24 the P.O. type relay operates for a value of $i\sqrt{L}$§ of 3·4, whereas for the practical design relay the value is 6·4, indicating that the former relay is the more sensitive of the two. (This is not a comparison of the relative merits of the two relays, as the question of transit time is not considered.) However, the corresponding numbers of ampere-turns are respectively 16·3 and 10·4, which would lead to exactly the opposite conclusion. Since it is possible for a springy-tongue relay to specify the current required for a given transit time and a given self-inductance (tongue travel fixed), this would give a better characteristic of any such relay, and by combining current and self-inductance in the expression $i\sqrt{L}$ direct comparison between different relays could readily be made.

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* See p. 226 of Bibliography (15).

* See pp. 72 and 73 of Bibliography (13).

† *Ibid.*

‡ See p. 73 of Bibliography (13).

§ In milliamperes and henrys.

Once the dimensions of a relay have been fixed, so that short transit time and practically chatterless performance are obtained, it will not be possible to alter the dimensions without detrimental effect upon the performance. It appears to be futile, therefore, to introduce adjustments for increasing the sensitivity by decreasing the permanent flux, as by such means the forces acting on the armature are, in general, reduced to such an extent that considerable chatter ensues.

To judge from many patent specifications and manufacturers' booklets, it seems to be a general belief that if the magnetic circuit of the electromagnet part of a relay be designed for low hysteresis a quick response and short transit time will be obtained. It is hoped that it is clear from the present paper that the mechanical design of the tongue, rather than the efficiency of the magnetic circuit, is instrumental in obtaining the desired performance from the relay.

## Acknowledgment

The author wishes to express his indebtedness to The Great Northern Telegraph Company, Ltd., for permission to publish the information contained in the present paper.

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## APPENDIX

### Reluctance Between Inclined Surfaces, and Constancy of Flux

The first assumption referred to at the end of Section (1) is clearly a question of the magnetic conductance between two inclined surfaces.

With reference to Fig. 25 it is seen that

$$dg_2 = \frac{1}{\dfrac{a-x}{hdy}} = \frac{hdy}{a-x}$$

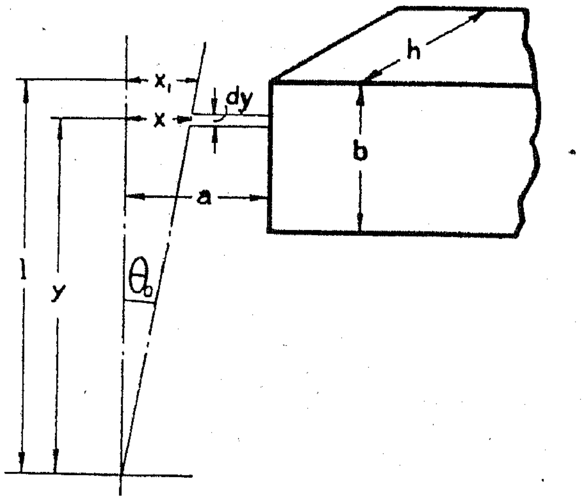

Fig. 25

Put $l = b$, which will simplify the calculations.

$$x = y \tan\theta_0; \quad x_1 = b \tan\theta_0$$

$$g_2 = \int_0^b \frac{hdy}{a - y\tan\theta_0}$$

$$= \left[ -h \frac{1}{\tan\theta_0} \log(a - y\tan\theta_0) \right]_0^b$$

$$= \frac{h}{\tan\theta_0} \log \frac{a}{a - b\tan\theta_0} = \frac{bh}{x_1} \log \frac{1}{1 - \dfrac{x_1}{a}}$$

This becomes

$$g_2 = \frac{bh}{x_1}\left(\frac{x_1}{a} + \frac{x_1^2}{2a^2} + \frac{x_1^3}{3a^3} + \ldots\right)$$

$$= \frac{bh}{a}\left(1 + \frac{x_1}{2a} + \frac{x_1^2}{3a^2} + \ldots\right)$$

The approximate formula used is (for $l = b$):

$$\frac{1}{S_2} = \frac{A}{a - \frac{x_1}{2}} = \frac{bh}{a}\left(1 - \frac{x_1}{2a}\right)^{-1}$$

$$= \frac{bh}{a}\left(1 + \frac{x_1}{2a} + \frac{x_1^2}{(2a)^2} + \frac{x_1^3}{(2a)^3} \cdots\right)$$

In a similar way it may be shown that

$$g_1 = \frac{bh}{a}\left(1 - \frac{x_1}{2a} + \frac{x_1^2}{3a^2} - \cdots\right)$$

and the approximate formula becomes

$$\frac{1}{S_1} = \frac{bh}{a}\left(1 - \frac{x_1}{2a} + \frac{x_1^2}{4a^2} \cdots\right)$$

For the case of a relay of the type under discussion $x_1/a$ may lie in the neighbourhood of 0·05, and the error introduced by the use of the approximate formulae, therefore, will be insignificant.

The constancy of the field depends on the constancy of the total reluctance of the magnetic circuit exterior to the magnet. If $S_0$ be the reluctance at the hinge of the armature, the total reluctance will equal $S_0 + S$, where $S$ is given by

$$\frac{1}{S} = g_1 + g_2 = \frac{A}{x_1} \log \frac{1 + \frac{x_1}{a}}{1 - \frac{x_1}{a}} \simeq 2\frac{A}{a}\left(1 + \frac{x_1^2}{3a^2}\right)$$

The value given by the approximate formulae is

$$\frac{1}{S_1} + \frac{1}{S_2} \simeq 2\frac{A}{a}\left(1 + \frac{x_1^2}{4a^2}\right)$$

As $S_0$ cannot in practice be made smaller than $S$ it will be seen that a variation in $S$ will mean less than half the variation in $S_0 + S$ when expressed as a percentage of the latter quantity. From the figures given it will be seen that the variation in $S_0 + S$ caused by the deflection of the armature will be less than 1 %.

Now, the " design equation "* for a permanent magnet, neglecting leakage, is

$$\frac{B_s}{H_s} = \frac{L_s}{A_s} G$$

where $B_s$ = flux density,
$H_s$ = magnetizing force per cm. length of steel,
$L_s$ = length of magnet,
$A_s$ = cross-sectional area of magnet,
$G$ = conductance of external magnetic circuit.

From the above $G = \dfrac{1}{S_0 + S}$

From the " design equation " it is seen that, for $L_s$ and $A_s$ fixed, $B_s/H_s$ must vary in the same manner as $G$. In Fig. 26 is shown a curve for the relationship between $B_s$ and $B_s/H_s$ and it is seen that since the curve is not a straight line the variation in $B_s$ caused by a variation of $B_s/H_s$ (which would equal the variation of $G$) depends upon the working point, i.e. the flux density $B_s$, of the magnet. In general the magnet will, for reasons of economy in the use of steel, be so designed that the working point is about $B_s$ = 6 000 for the particular steel (35 % cobalt) for which the curve is drawn, and to this, unfortunately, corresponds a high value of the slope of the $B_s - B_s/H_s$ curve. It has been shown above that the variation of $G$ is very small, but if, in a particular case, this does not hold and it is important to keep the total field constant, a flux density nearer the maximum value might be chosen.

In an experiment on the practical design of relay where the armature was deflected corresponding to a contact gap as large as 0·80 mm.—0·10 mm. being standard—a variation of 4 % in $\Phi$ was measured.

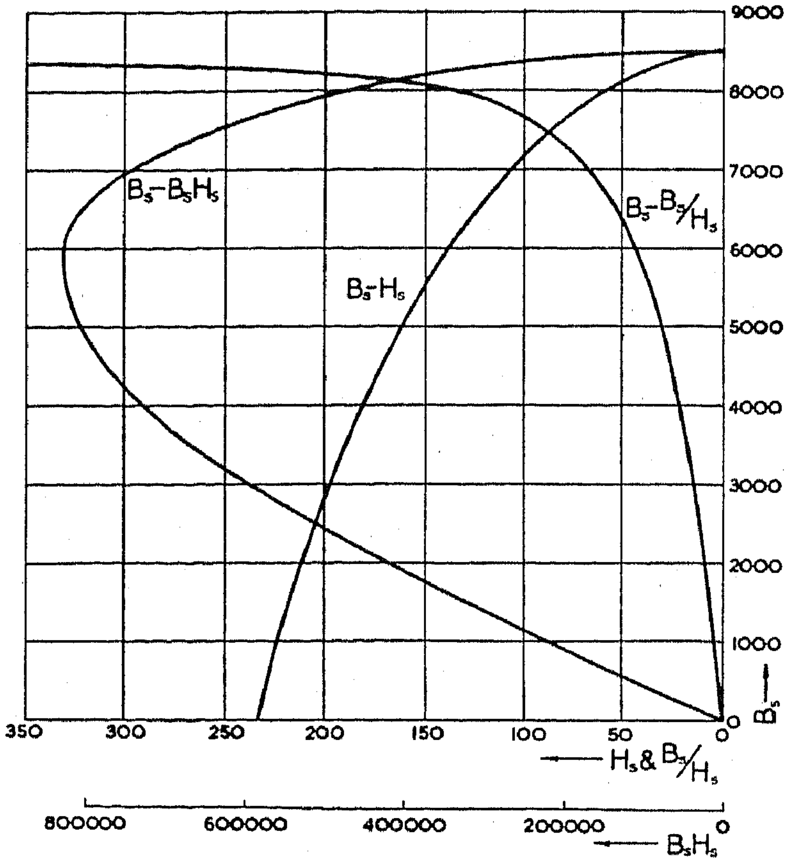

Fig. 26

From the $B_s - B_s/H_s$ curve of Fig. 26 it may also be inferred that the flux may not be considered proportional to a constant " equivalent magnetomotive force " for the permanent magnet, as some writers have done.*

For this reason the discussion has been conducted throughout in terms of fluxes only. Although the former line of attack may yield results consistent with the results above so long as $G$ may be considered constant, it fails when attempts are made to demonstrate how certain properties of a relay vary with $G$.

### Proportionality of Torque with Deflection

It was shown in Section (1) that the force exerted by the magnetic fields on the armature is proportional to the deflection of the latter, but it remains to be demonstrated that the torque varies in a like manner. This implies that the force may be considered to act at a point removed a fixed distance from the pivot of the armature.

As $G$ may be considered independent of the deflection of the armature, and as the total flux is constant, as shown above the magnetic potential, $V$, across either air-gap is also constant.

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* See p. 799 of Bibliography (8).

* See p. 163 of Bibliography (9) (paper by J. D. Tours and E. H. B. Bartelink on polarized relays).

Treated in this way we have (for $l = b$):—

$$F_m = \frac{1}{8\pi A}(\Phi_2^2 - \Phi_1^2)$$

$$= \frac{1}{8\pi bh} \cdot \frac{V^2b^2h^2}{a^2}\left[\left(1 - \frac{x_1}{2a}\right)^{-2} - \left(1 + \frac{x_1}{2a}\right)^{-2}\right]$$

$$\simeq \frac{V^2bh}{8\pi} \cdot \frac{2x_1}{a^3}$$

Referring to Fig. 25, we have ($l = b$):—

$$d\tau = y(d\Phi_2)^2\frac{1}{8\pi hdy} - y(d\Phi_1)^2\frac{1}{8\pi hdy}$$

$$= y\frac{1}{8\pi hdy}\left\{\left(\frac{Vhdy}{a - y\tan\theta_0}\right)^2 - \left(\frac{Vhdy}{a + y\tan\theta_0}\right)^2\right\}$$

$$\tau = \frac{V^2h}{8\pi}\left\{\int_0^b \frac{ydy}{(a - y\tan\theta_0)^2} - \int_0^b \frac{ydy}{(a + y\tan\theta_0)^2}\right\}$$

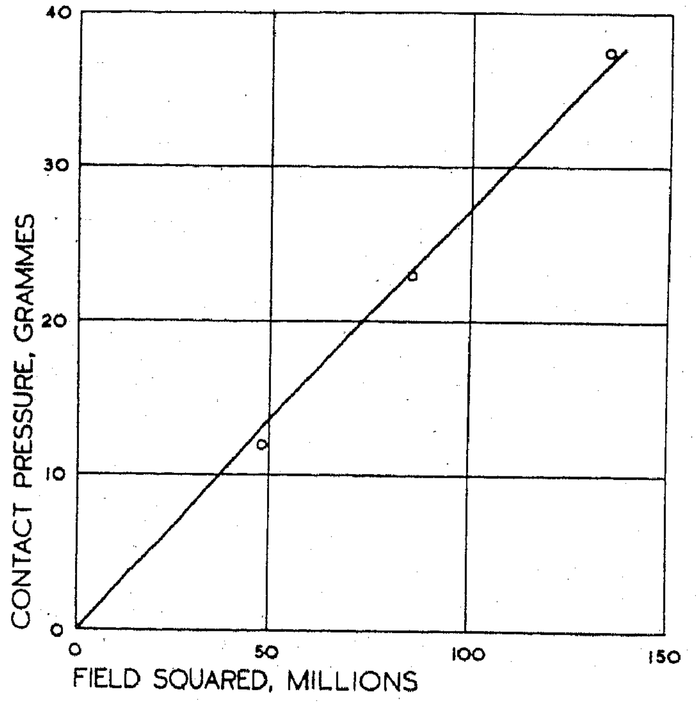

Fig. 27

which becomes

$$\tau = \frac{V^2h}{8\pi} \cdot \frac{1}{\tan^2\theta_0}\left\{\frac{a}{a - b\tan\theta_0} + \log(a - b\tan\theta_0) - \frac{a}{a + b\tan\theta_0} - \log(a + b\tan\theta_0)\right\}$$

$$\tau = \frac{V^2b^2h}{8\pi} \cdot \frac{4x_1}{3a^3} \text{ (approx.)}$$

Hence, $$r = \frac{\tau}{F_m} = \frac{2}{3}b,$$

i.e. the force may be considered to be concentrated at a point removed two-thirds of the pole-shoe breadth from the innermost edge of the pole-shoe.

For $l > b$ the formula for the torque becomes

$$\tau = \frac{1}{8\pi A} \cdot \frac{x_1}{a}\Phi^2\left(1 - \frac{b}{2l}\right)\left(l - \frac{b}{3}\right).$$

Practical tests, Figs. 27 and 28, confirm that the torque varies directly as $\Phi^2$ and directly as $x_1$, but the actual figures obtained differ considerably from the theoretical values, and the value of $r$, therefore, is more of academic than of practical interest. The determination of $\Phi$ was made by winding a search coil on the armature just inside the innermost edges of the pole-shoes, but since it is not possible to make such a search coil infinitely short the field measured will include some of the leakage flux, and will, therefore, be in excess of the actual value of $\Phi$. The discrepancy between theory and practice, therefore, may be attributed to unavoidable leakage-fields.

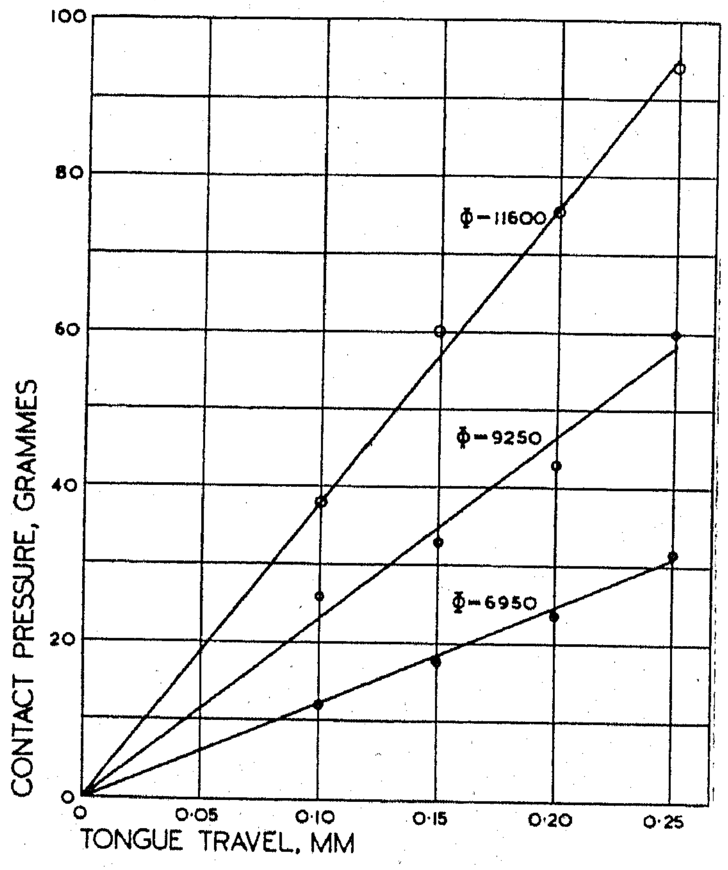

Fig. 28

# INSTITUTION NOTES

## NOMINATIONS FOR ELECTION TO THE COUNCIL

In addition to those members nominated by the Council (see vol. 82, page 699), the following have been nominated for ballot as Ordinary Members of Council:—

*Members.*

H. HOOPER (Nominated by W. Y. Anderson, W. Brown, N. M. Hill, F. W. Lawton, T. A. G. Margary, J. Nelson, G. R. J. Parkinson, C. H. Petford, R. H. Rawll, and G. Rogers.)

A. E. McKENZIE (Nominated by A. S. Blackman, A. P. MacAlister, E. A. Mills, A. J. Newman, F. Newey, F. N. Rendell-Baker, L. L. Robinson, W. A. Royle, C. W. Salt, and J. D. Spark.)

O. C. WAYGOOD [Nominated by J. Cormack, B.Sc., R. G. Devey, E. L. Morland, J. E. Nelson, W. Parry, M.Eng., G. K. Paton, P. J. Robinson, M.Eng., F. E. Spencer, E. G. Taylor, B.Sc.(Eng.), and Prof. F. J. Teago, D.Sc.]

*Associate Member.*

F. C. ORCHARD (Nominated by H. Brierley, F. F. Elliott, G. F. Gregory, A. F. Harmer, R. Illingworth, E. E. Jolly, W. E. Kidner, R. Lee, W. G. Trend, and B. C. P. Ward.)

## ELECTIONS AND TRANSFERS

At the Annual General Meeting of The Institution held on the 12th May, 1938, the following elections and transfers were effected:—

### Elections

*Associate Members*

Adams, Ralph Maxwell, Lieut.
Archavsky, Itzkhok.
Bannister, Reginald Harry,
Corkett, Harold.
Easton, Vivian, B.Sc.
Elsdale, Robinson, Col., O.B.E., M.C., M.A.
Fry, Donald William, M.Sc.
Greig, John Charles, B.Sc.
Harris, James Harold, B.Sc.(Eng.).
Idnani, Thakur Narainsing, B.Sc.Tech.
Keith, Douglas Neill.
Kemp, Herbert George.
Lepper, Gerald Harper.
McIntosh, James.
Murray, Lawrence John, B.Eng.
Pringle, Thomas Percival, B.E.E.
Richards, Idris Trevor.
Service, William John.
Taylor, Paul Harvard.
Weston, George Edward D., B.Sc.(Eng.).
Wheatley, Edwin Kenneth.
Wrigley, Frederick Richardson G.

*Associates*

Clark, Walter Charles.
Firth, Arthur Lawson.
Green, George Norman.
Hampton, Edward James.
Holt-Keene, Holford.
Hurworth, Harry.
Lythall, Reginald Tarlton.
Prendergast, Harold.
Starbuck, Leonard Wilson G.
Vaughan, Cyril Hugh.

*Graduates*

Arron, Redvers.
Barson, Albert Clifford.
Birrell, Andrew Bruce H., B.Sc.
Clare, Boris Arthur.
Cooper, Albert, B.Sc.
Cowell, Colin Hall, B.Sc. (Eng.).
Deshpande, Dattatraya, B.E.
de Silva, Gostinnawadu Francis.
Epstein, Edward Nathaniel, B.A.
Freedberg, William, B.Sc. (Eng.).
Geddes, George.
Gore, William Ernest, B.Sc.(Eng.).
Hayward, Doyle Walton.
Iengar, Naduvakkarai Srinivasa A., B.A.
Jones, Harold Alan.
McGill, John Stirling.
Mohanraj, Vellore Govindraj, B.Sc.(Eng.).
Pearson, John Archibald.
Rao, Basrur Venugopal, B.Sc.
Rao, Kaiwar Lakshmana, B.A.
Reece, Thomas Elved, B.Sc.(Eng.).
Sohrab, Bahman.
Storey, William Alan.
Thombre, Balkrishna Vithal.
Vanikar, Balkrishna Sakharam.
Varma, Hans Raj.
Vinsen, James Goy, B.Sc. (Eng.).
Westcombe, Norman George.
Whitie, Gordon.

*Students*

Abraham, Thomas
Agius, Joseph.
Aldridge, Albert Edward.
Ambler, Mark.
Anderson, William Stewart
Armour, Robert Martin.
Barnsley, Hudson Gordon.
Barraclough, John Francis.
Bell, John James.
Beresford, Donald Alfred.
Berridge, Lester Herbert.
Bhattacharyya, Susil Kumar.
Bickerdike, Stanley Ronald.
Brodie, Donald Arthur M.
Brown, Winston Herbert.
Burbidge, Charles William.
Burrage, Morriss Frederick
Burt, Frank Harry.
Carroll, John Walter.
Chatterjea, Mukul.
Collier, Ernest William.
Crompton, George Humphrey.
Crosby, William Arthur, B.Sc.
Dagg, Girja Dayal.
Dawe, Leonard Stanley.
Desai, Jayantilal Ambavidas.
Devaney, Henry John.
Dove, Ivan Raymond.
Dyson, Leonard.
Emms, Arthur George.
Farnsworth, Alfred.
Fartash, Nosratollah.
Francis, Stanley James.
Fraser, Malcolm Joseph.
Frazer, Thomas Athol, M.A.
Galaway, George Ronald.
Gardner, Thomas.
Ghose, Rabindra Nath.
Gray, John.
Green, Geoffrey.
Griffin, John Gaudern.
Hammond, William Herbert.
Harley, Leslie James.
Harris, John.
Hartgroves, Frederick Leslie G.
Hawkley, Leonard John.
Hewitt, Wesley.
Hoda, Serajul.
Hogg, James Douglas.
Howson, Frank William.
Huddlestone, John Nevil.
Huntriss, Harold Claude.

### *Students*—continued

Jambhekar, Eknath Dattatraya.
Kanjilal, Sanat Kumar.
Keep, James Frederick.
Kerr, George Edward.
Khan, Habib Ur Rahman.
Kirmani, Ehtisham Ali.
Kochak, Sharda Shanker.
Lakshmipathi, Parandapalli Venkatarama Iyer.
Lester, Jack Douglas.
Littler, Kenneth Woodier.
Luing, John Frederick.
Lumkin, Frederick Arthur.
Mahindroo, M. R.
Marks, Francis Edward.
Marriott, Martin George.
Melbourn, Peter.
Murthy, Manchiganti K.
Nanke, Felix Geoffrey.
Newstead, John Charles.
Oxley, George Clarence.
Parker, Denis William.
Parthasarathy, Prathivadhi Bavankaram.
Patterson, Leslie Simpson.
Pocock, Sydney Norman.
Price, Charles Kenneth J.
Ransley, Harold Thomas.
Rigby-Jones, Arthur.
Robertson, Ronald Hislop.
Robinson, James Arthur.
Rodbourn, Ivan Samuel.
Saksena, Virendra Kumar.
Saxena, Krishan Gopal.
Sen, Samarendra Nath.
Shimmin, Geoffrey Noel.
Sinha, Sudhanshu Shekhar.
Smith, Donald Albert.
Somers, Arthur Edward.
Stanley, Claude.
Tammadge, Ronald William.
Vatal, Sham Sunder N.
Venkataraman, Ardhanari.
Venkatvaradan, B. Ramanna.
Vowles, Alan Findley.
Walker, Donovan Knyvett, M.Sc.
Wassilissin, Anatole.
Watson, Kenneth Clyde.
Weeks, Maurice Edwin.
Willis, Horace Robert.
Willis, Stanley Bernard.
Wilson, William Leonard.
Window, Frederick Arthur.
Wisdom, Arthur.
Wooldridge, Albert Geoffrey.
Wyeth, Frederick Albert.

## Transfers

### *Associate Member to Member*

Aust, John Horsey.
Bennett, Abraham Francis.
Fielding, Alfred Leslie.
Gripper, Laurence Albert.
Harpham, Arthur Thomas.
Hines, Ronald John, B.Sc.
Latimer, Kenneth Eric, B.Sc.(Eng.).
Ray, Frederick Ivor, B.Sc.(Eng.).
Shilstone, Frank Walter.
Whitaker, Alfred, M.A.

### *Associate to Associate Member*

Caswell, Arthur Edward.
Gregory, Claude Barnard, B.A.
Heap, Hubert Leslie S., B.Sc.(Eng.).
Newing, Robert.

### *Graduate to Associate Member*

Alleway, Charles Edward.
Anderton, Leslie Claude, B.Sc.(Eng.).
Ashton, Norman.
Aspinall, Wilfred, B.Eng.
Brayshaw, George Stanley, B.Sc.(Eng.).
Carpenter, John Ernest, B.Sc.(Eng.).
Clapham, Harold Edward C.
Coffey, Albert William, B.Eng.
Coleman, John Peden.
Cooper, James Reginald, B.E.
Dall, William Christie.
Gelder, William Alfred E., B.Sc.
Green, Robert Alan.
Harrison, Kenneth Whiteley.
Havart, Reginald Francis.
Hillier, Walter Muspratt.
Hopkins, Bernard George.
Horley, Frank Albert.

### *Graduate to Associate Member*—continued

Hutchinson, Cecil Arundel.
Littlefair, Allan Fort.
Mason, Gerald Linford.
Mills, Harry Douglas.
Milne, Archibald George.
Moores, Bernard John.
Oram, Gavin Charles.
Pass, Charles William, B.Sc.Tech.
Pettitt, Leslie William.
Phillips, Cecil William M.
Platt, Geoffrey Hine, M.Sc.Tech.
Radford John Samuel.
Read, George Stuart J., B.Sc.
Robertson, Alan Stuart.
Sime, Thomas Livingstone, B.Sc.(Eng.).
Smith, Clive Gordon, M.A.
Spooner, Jack Walter.
Talbot-Jones, Ronald Victor.
Tanner, John Patrick.
Terry, John.
Ward, Laurence George, B.Sc.(Eng.).
Whatman, Amherst Barrow, B.A.
White, Herbert Ernest.

### *Student to Associate Member*

Kendall, Peter John O.

The following transfers were also effected by the Council at their meeting held on the 28th April, 1938:—

### *Student to Graduate*

Alexander, Nellumatil Thomas, B.A.
Balkrishnan, Narayan.
Biggs, John Norris.
Blackburn, Eric Albert.
Blackett, Cedric James R.
Chatterjee, Amiya Kumar.
Curtis, Donald Cuthbert.
Dastur, Naval Jamshedji.
Deshpande, Murlidhar Vinayak.
Dotivala, Hormuz Dadabhai.
Downey, James Arthur.
Fell, Philip James.
Gejji, Ramchandra Krishna.
Gibson, John George.
Homersham, Brian Ryder McC.
Hood, Charles, B.Sc.Tech.
Houldin, Joseph Eric, B.Eng.
Hussain, Lewis Emmanuel.
Jackson, George Stanley, B.Sc.Tech.
Jangalwala, Dara Dinshawji.
Karve, Keshav Raghunath.
Kerr, Harold Leslie, B.Sc.Tech.
Khanna, Chaman Lal.
Krishnaswamy, Vippodu, B.A., B.Sc.(Eng.).
Lane, Geoffrey Meredith, B.Sc.
Lodge, Ernest Charles.
Mahajan, Narayan Balwant.
Melville, Denis Hill.
Morgan, Hugh Gabriel B.
Morgan, Wilfred Francis R.
Munton, Richard Douglas.
Pattison, John Charles.
Pereira, Charles Anthony F.
Ramanathan, P., B.A.
Rane, Kumar Shridhar.
Reed, Anthony, B.Sc.
Russell, Ernest Sidney.
Sanharan, Venkitaram.
Seervai, Homi Pirojshaw.
Sivasamban, Palayavalam Sivaraxamier.
Soman, Narayan Shridhar.
Spens, Andrew Nathaniel H.
Splane, Edgar Charles.
Sreenivasamurthy, B. N.
Srinivasan, K., B.A.
Srinivasan, Kazhiur, B.A.
Sukhadia, Pratapchandra Uttamram.
Thomas, Ambrose Bramwell.
Toma, Robert.
Treasurer, Vinaykant Dhirajlal.
Veith, Franz Sigmund.
Vohra, Satya Pal.
Wagle, Moreshwar Mangesh.
West, Kenneth.
Whitwam, Maurice Grenville, B.Sc.(Eng.).
Williams, John Alan, B.Eng.
Wooster, Clement Bruce.

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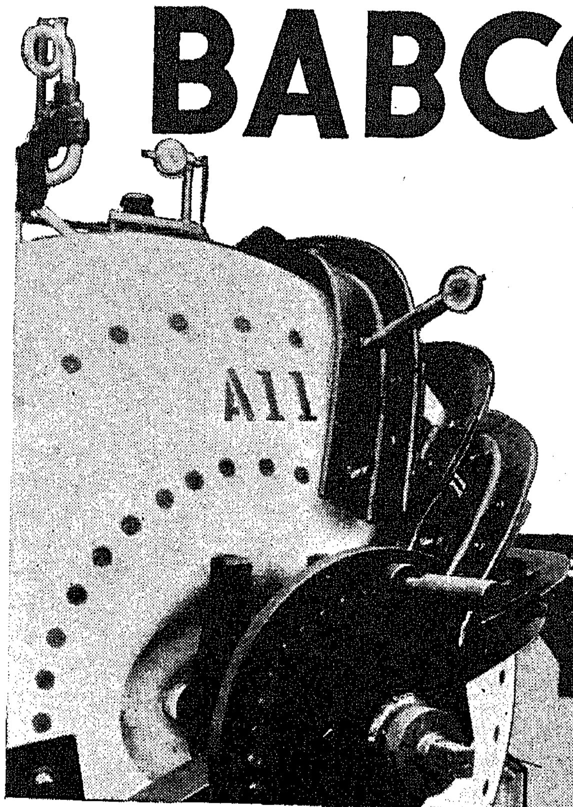

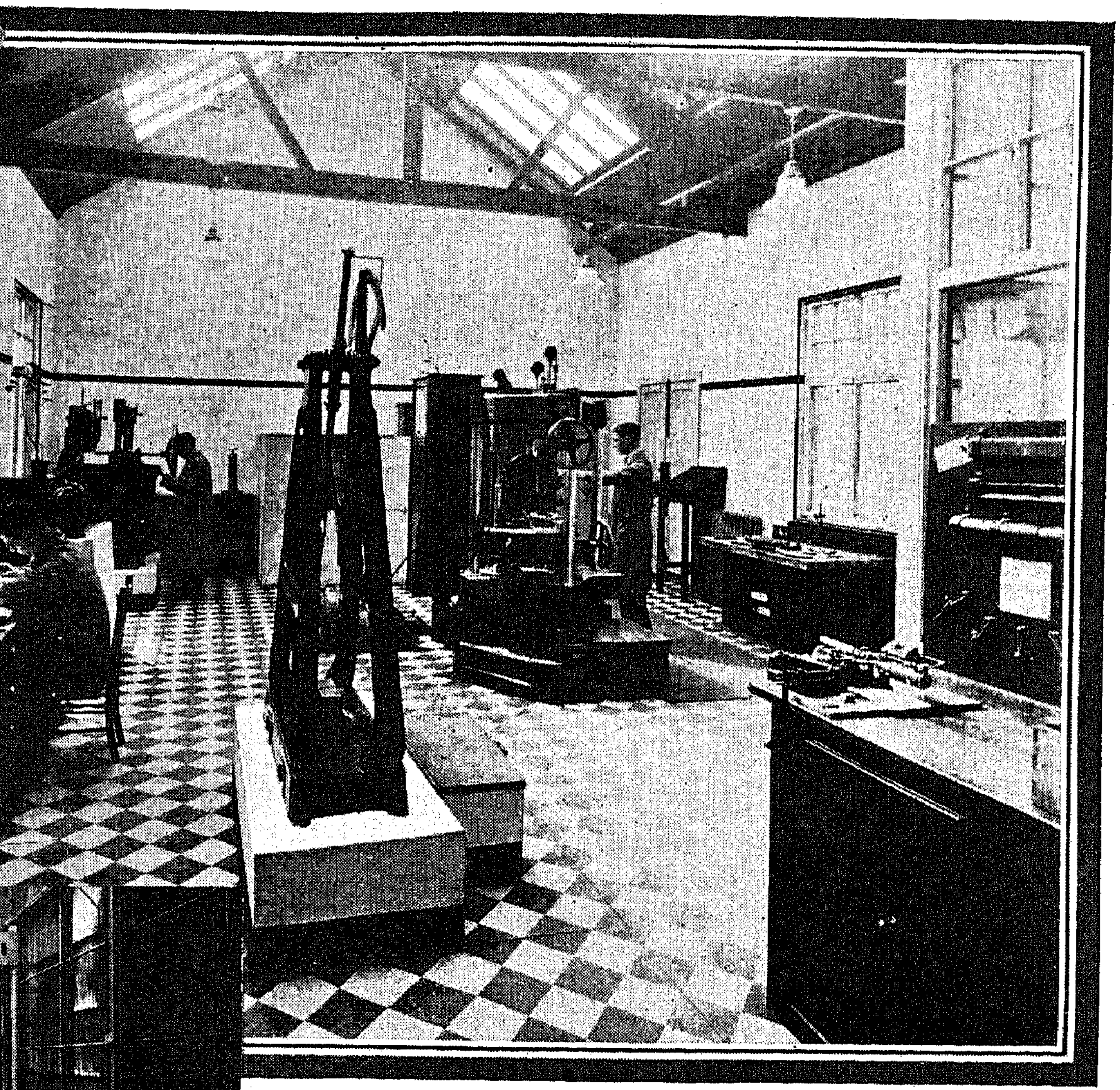

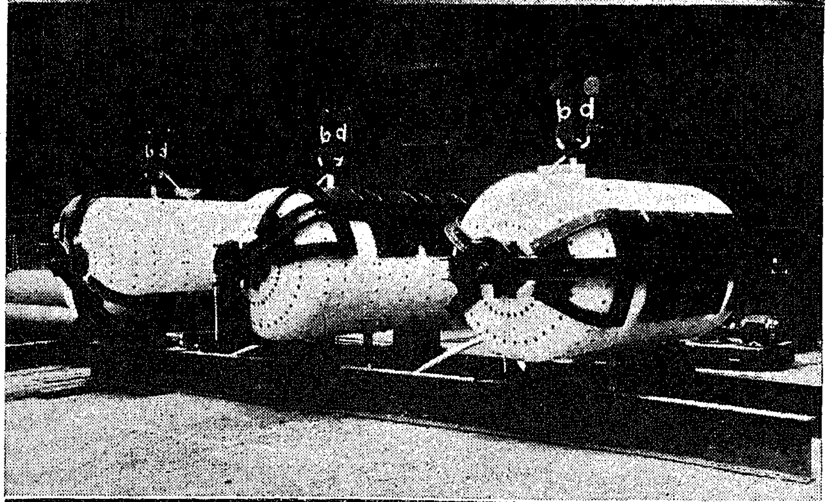

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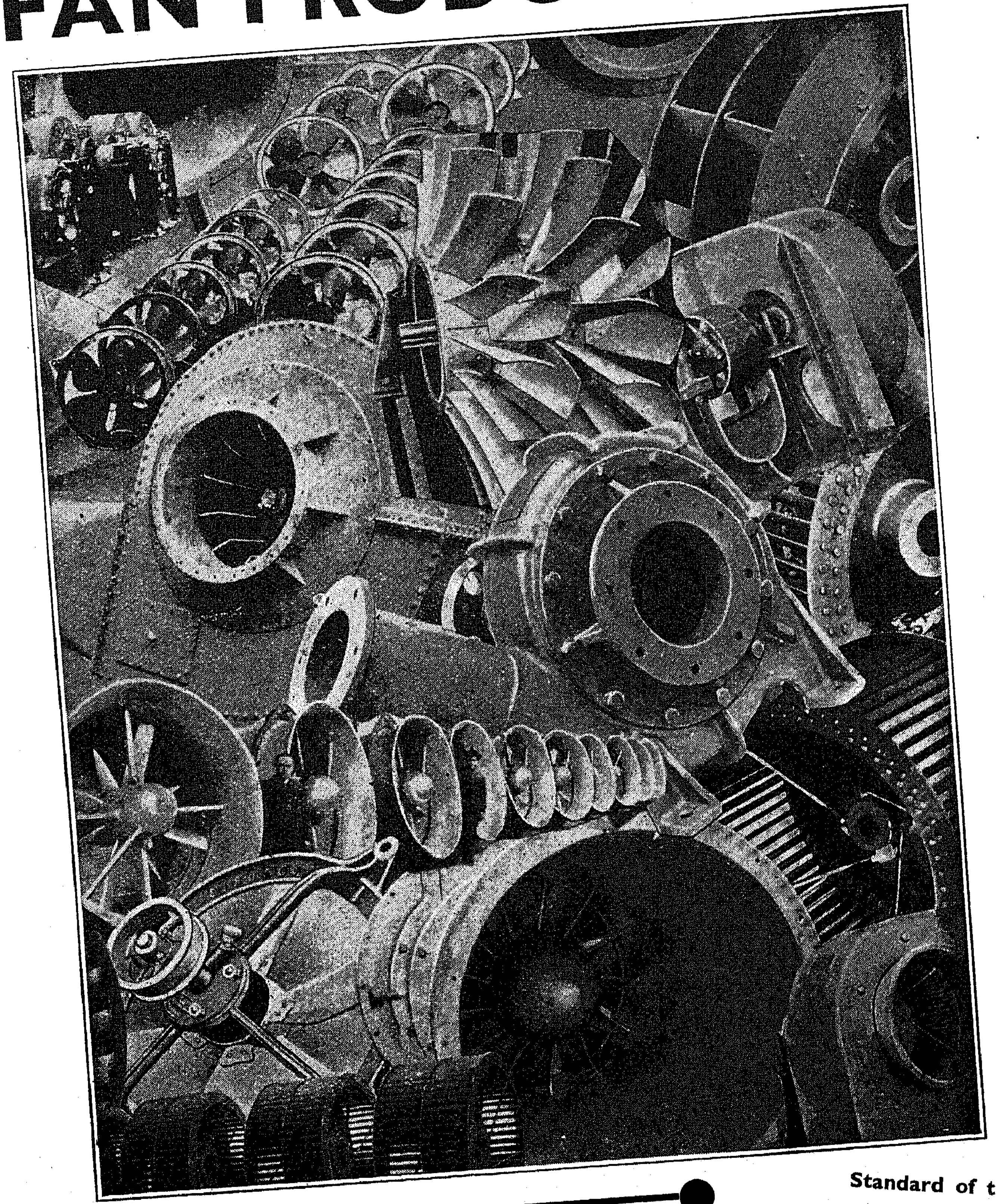

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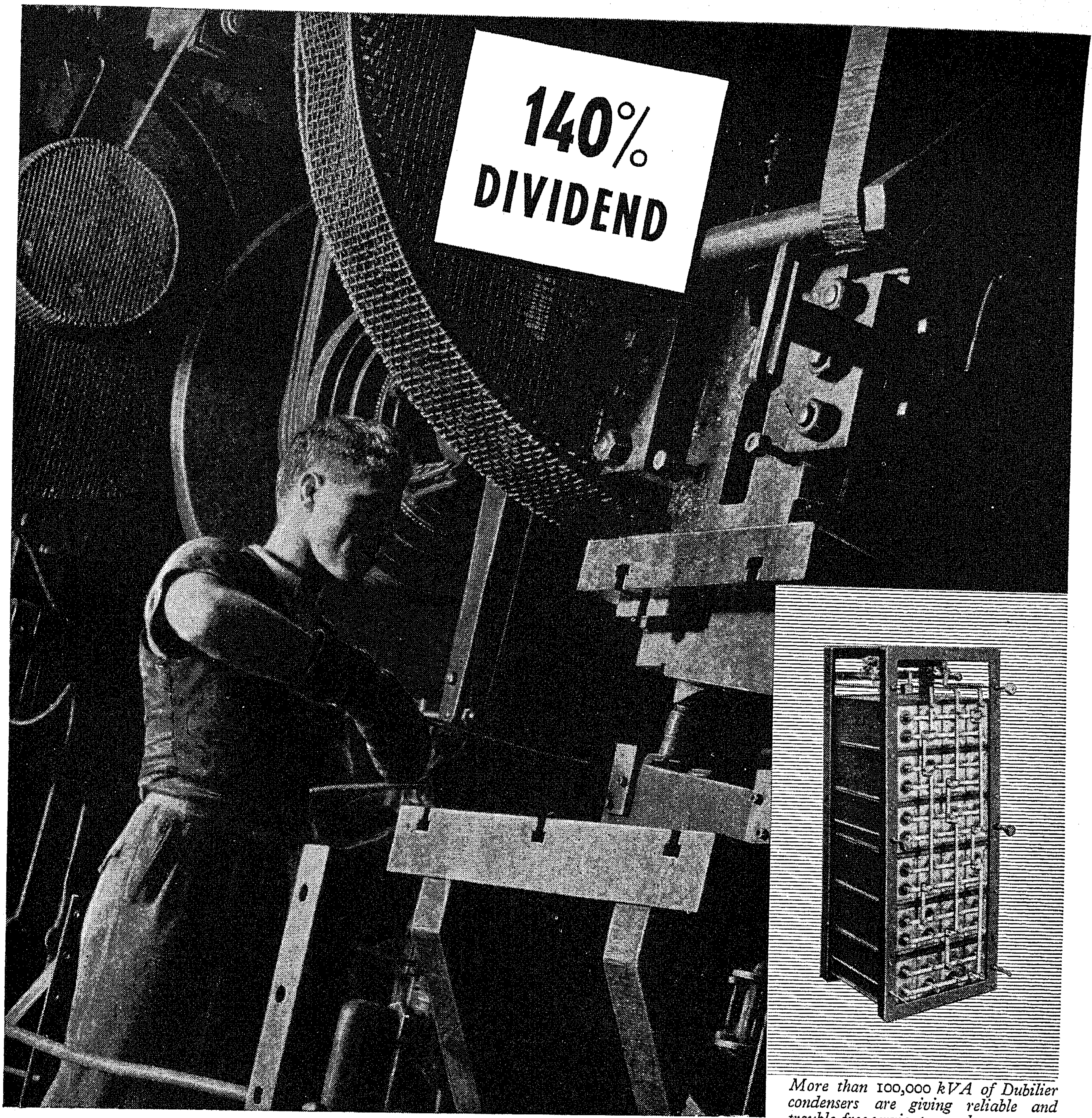

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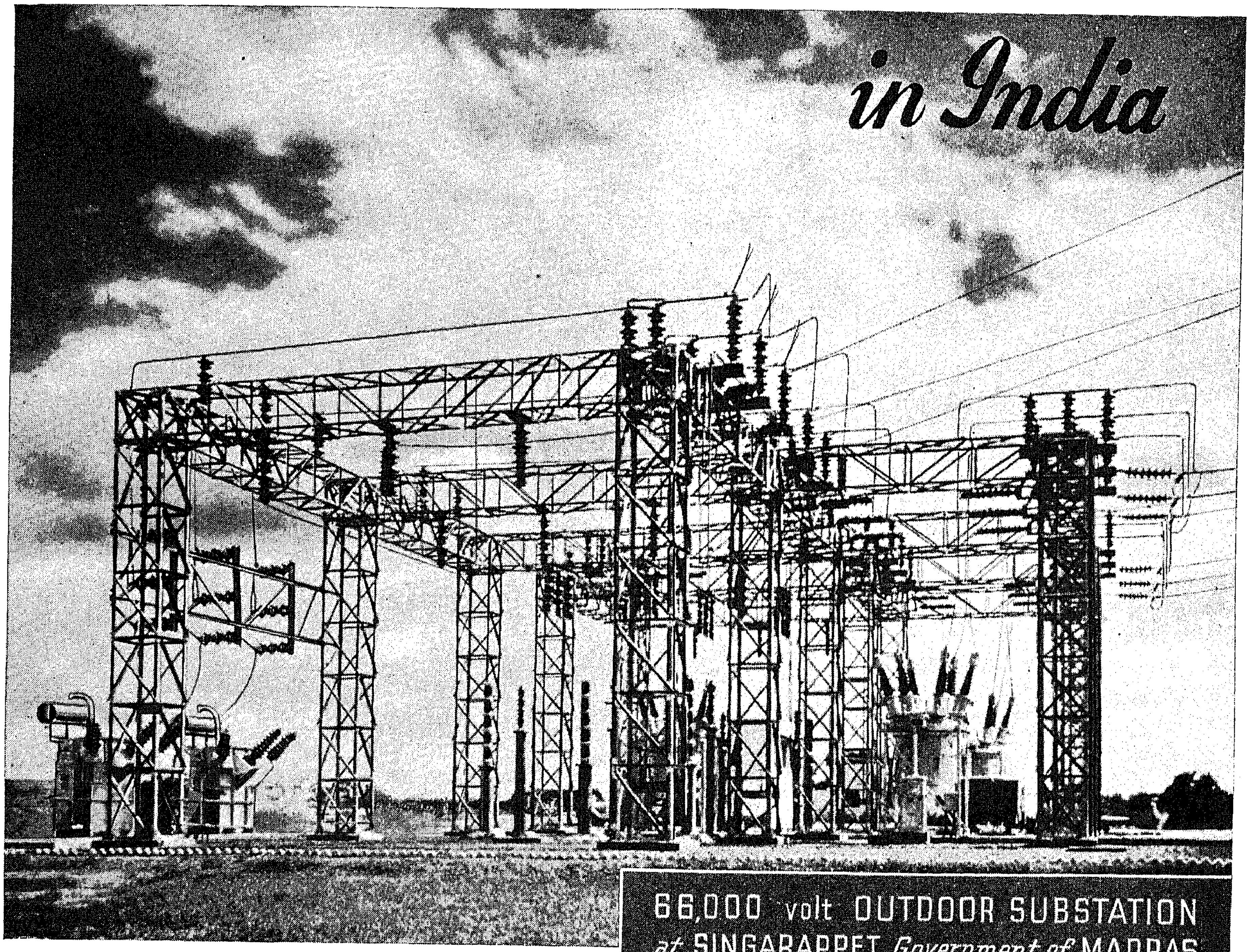

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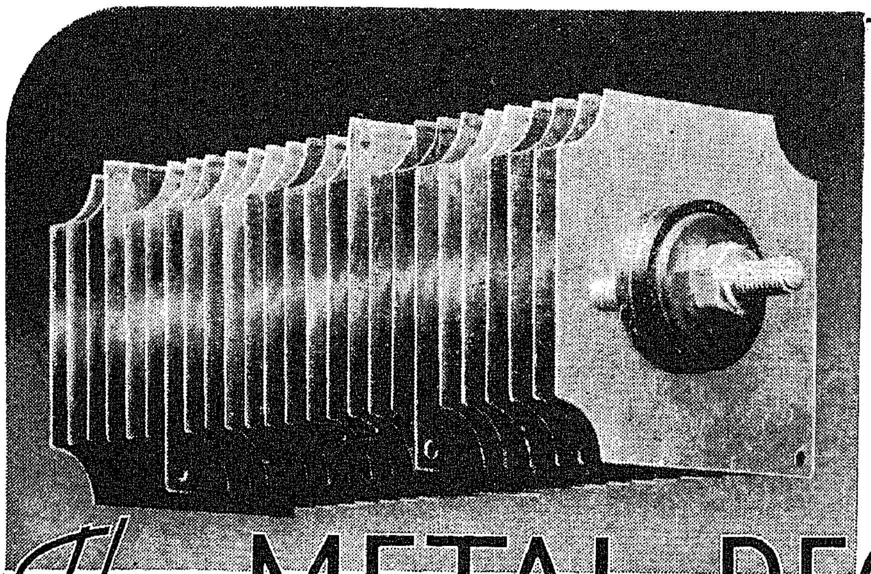

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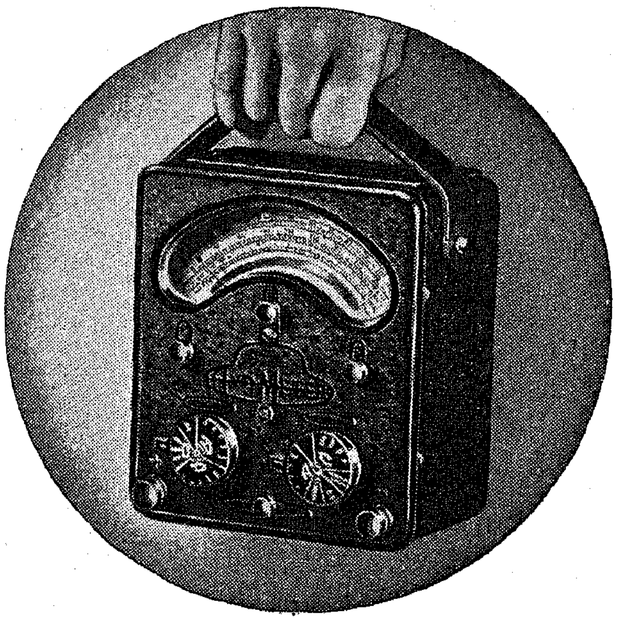

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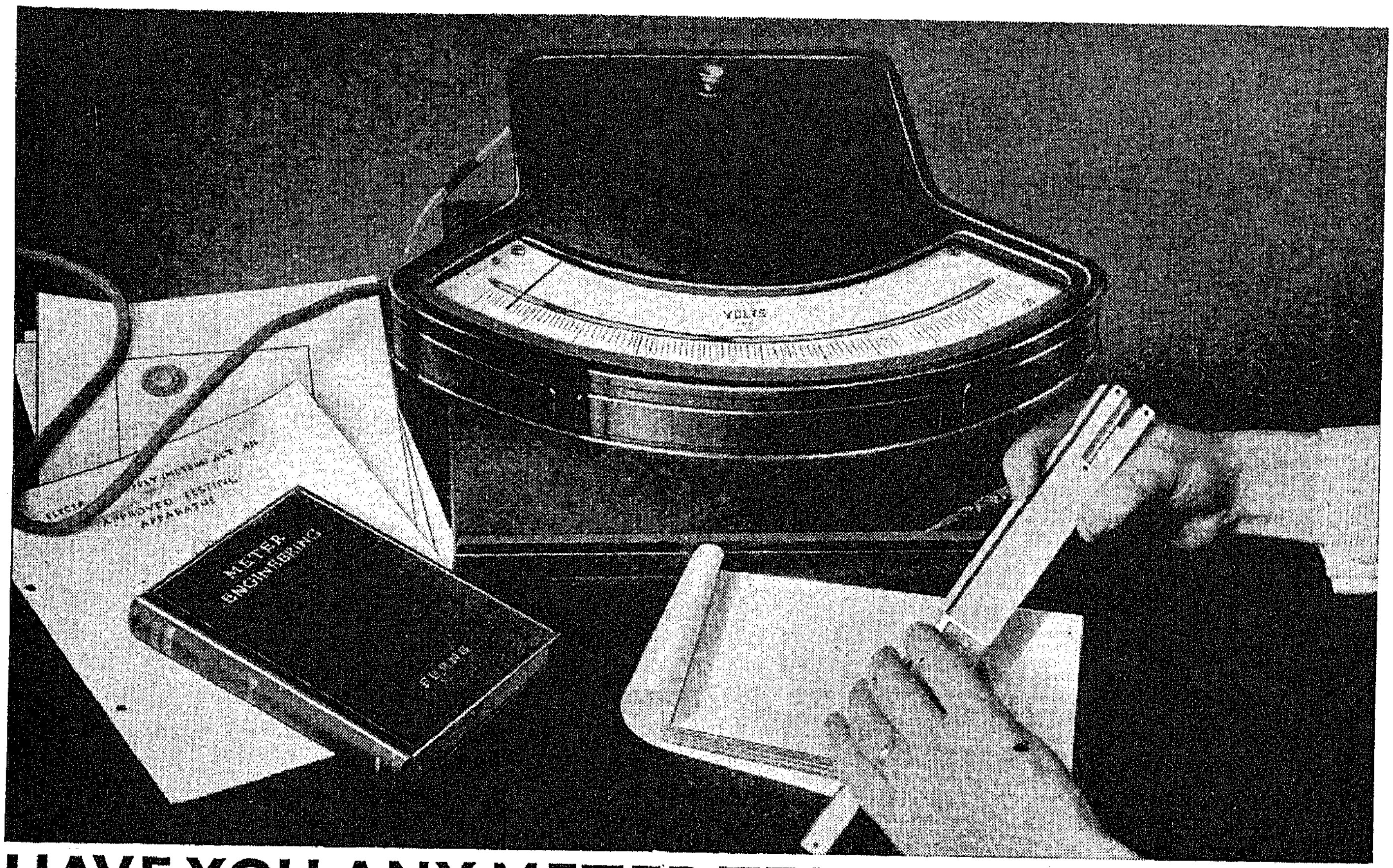

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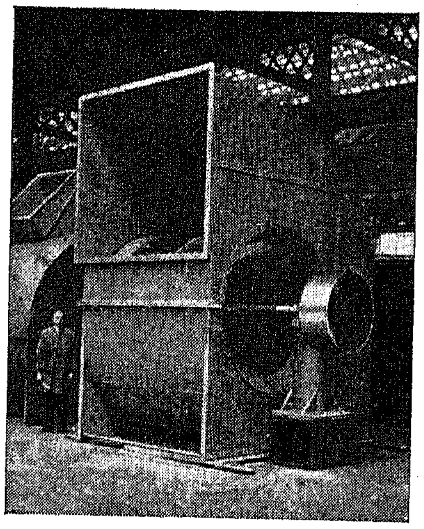

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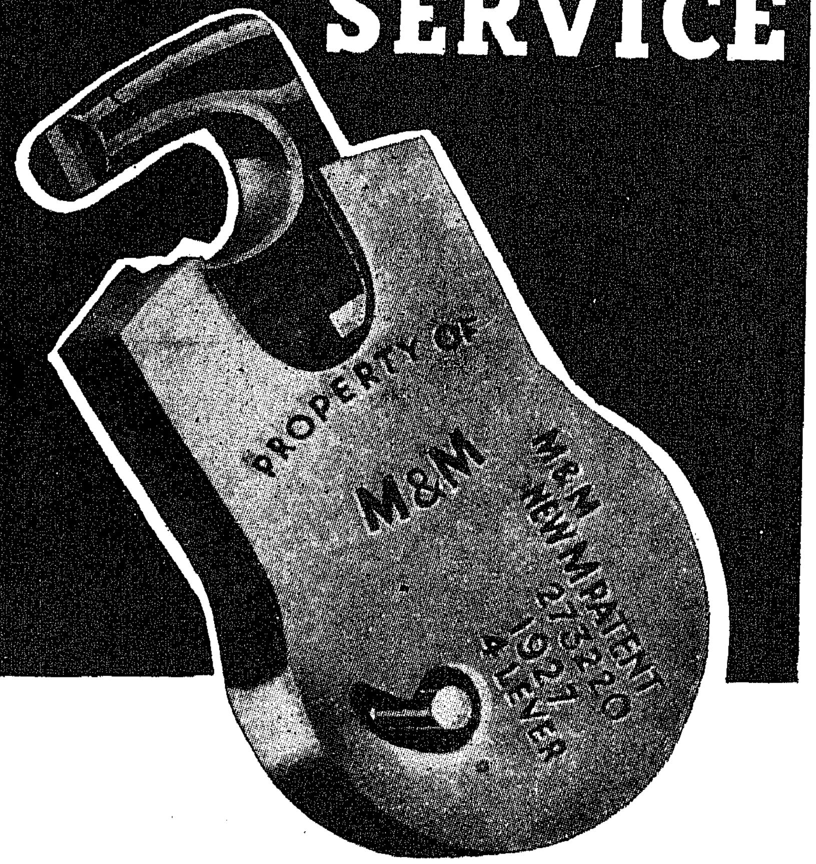

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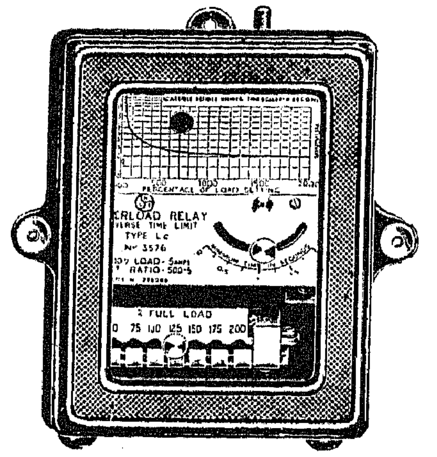

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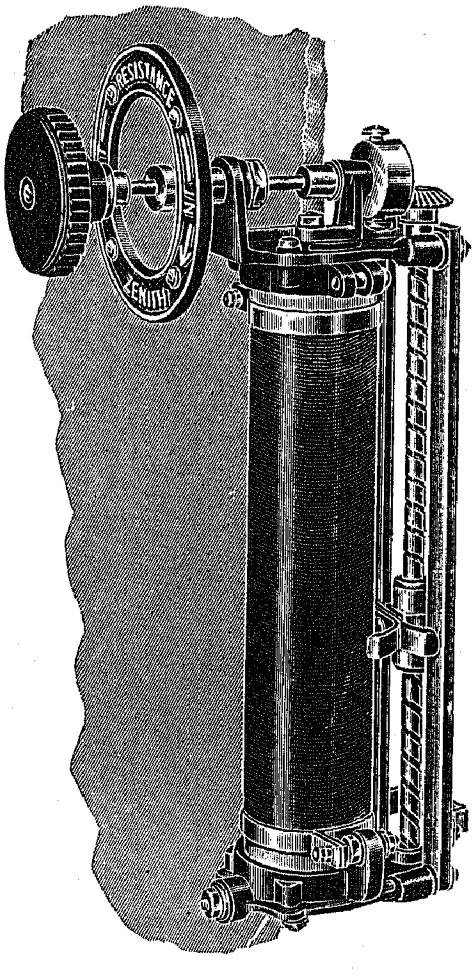

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